Adaptive Control in Problems with Dynamic Disturbance of Unknown Level

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The main words in the title are “adaptive” and “unknown level”. The latter means that there is a dynamic disturbance in the control system, but we do not know its level in advance. We tune the useful control according to the actual disturbance.

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Linear Control System with Unknown Level of Dynamic Disturbance

$$\dot{\mathbf{x}} = \mathbf{A}(t)\mathbf{x} + \mathbf{B}(t)u + \mathbf{C}(t)v,$$

$$t \in [\vartheta_0, \vartheta] = T, \qquad \mathbf{x} \in \mathbb{R}^m,$$

$$M, \qquad u \in P \subset \mathbb{R}^p, \qquad v \in \mathbb{R}^q$$

M is the target terminal set defined by n components of $\mathbf{x}$

The principal idea of our method is quite general. But its elegant realization was made for linear control systems only. Here, u is a useful control. A prescribed convex compact constraint P is given for it. The objective of the useful control is to lead the system to some given convex terminal set M at the terminal instant ϑ. The symbol v denotes the disturbance. There is no any constraint for v *a priori*. It is very typical that the terminal set M is determined only by some n ($n \leq m$) components of the phase vector $\mathbf{x}$.

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Adaptive Control

$$\dot{x} = B(t)u + C(t)v,$$

$$x \in \mathbb{R}^n, \quad t \in T = [\vartheta_0, \vartheta], \quad M \subset \mathbb{R}^n$$

$$u \in P \subset \mathbb{R}^p, \quad v \in \mathbb{R}^q$$

Requirements for the *adaptive* control $U(t, x)$:

1) guaranteed reaching the terminal set, if the disturbance is lower than some critical level;
2) a “weak” disturbance should be parried by a “weak” control.

weak disturbance => weak control *and* guaranteed reaching;
strong disturbance => strong control *and* guaranteed reaching;
very strong disturbance => maximal allowed control *and* some terminal miss (to be minimized).

Due to linearity of dynamics and fixed terminal instant, we can pass to the system without phase vector in the right-hand side. It is the standard way.

In this slide, non-formal requirements for the adaptive feedback control $U(t,x)$ are written.

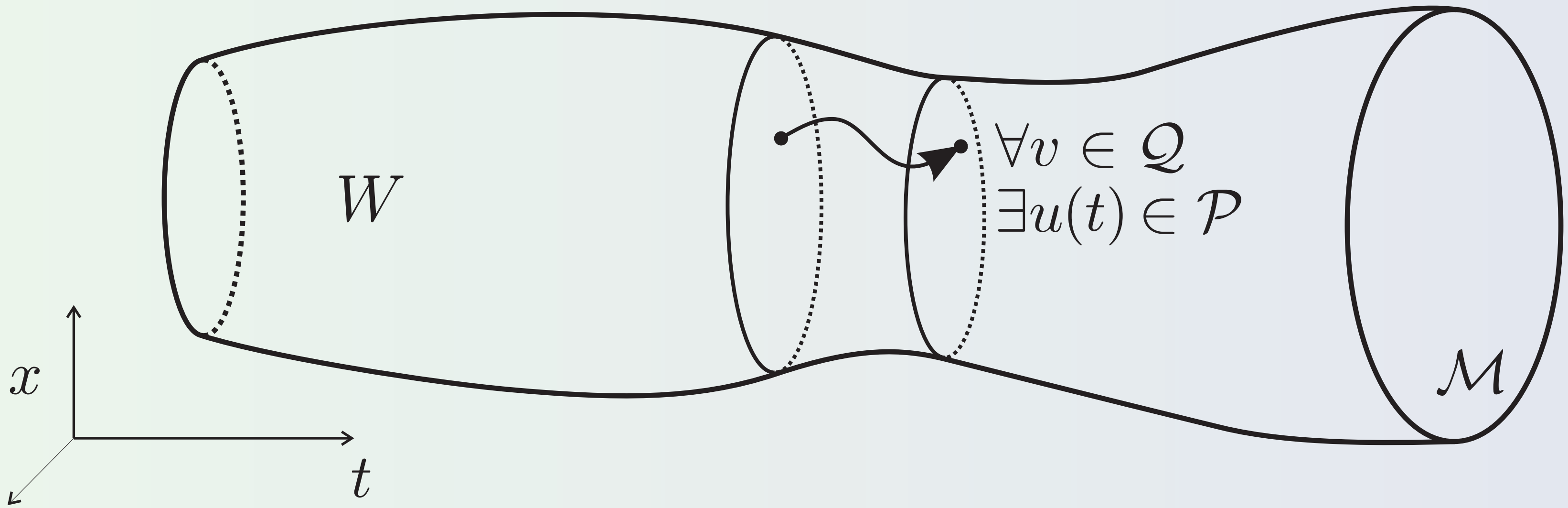
Stable Bridges

$$\dot{x} = B(t)u + C(t)v,$$

$$x \in \mathbb{R}^n, \quad t \in T, \quad \mathcal{M},$$

$$u \in \mathcal{P}, \quad v \in \mathcal{Q}$$

$(\mathcal{P}, \mathcal{Q}, \mathcal{M})$ – parameters of the game



maximal stable bridge, alternating integral, discriminating kernel

The main concept for our construction of the adaptive control is the stable bridge. This concept was introduced in the theory of differential games by N.N.Krasovskii and A.I.Subbotin.

Let us fix a convex constraint $\mathcal{P}$ for the control u, a convex constraint $\mathcal{Q}$ for the disturbance v, and let $\mathcal{M}$ be a convex terminal set. The stable property of the bridge means the following: any constant disturbance $v \in \mathcal{Q}$ can be parried by a control $u(t) \in \mathcal{P}$ so that the motion stays in the bridge and reaches the set $\mathcal{M}$ at the terminal instant. A set maximal by inclusion and possessing the property of stability is named as maximal stable bridge. Equivalent terms for it are alternating integral and discriminating kernel. Time sections of a maximal stable bridge in linear problems with convex terminal set are convex.

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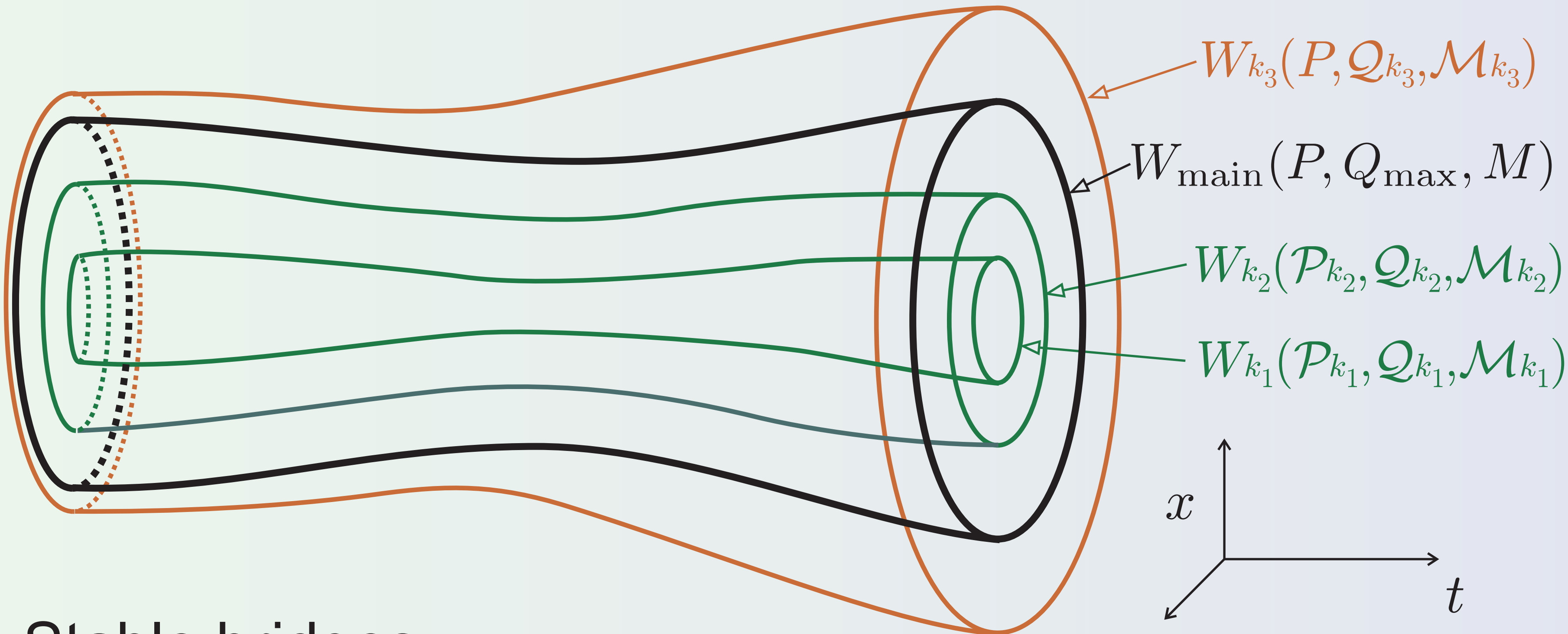
Bridges for the Adaptive Control



Stable bridges:

$$W_{k_1} \subset W_{k_2} \subset \ldots \subset W_{\text{main}} \subset \ldots \subset W_{k_3} \subset \ldots$$

Parameters of the bridges:

$$\mathcal{P}_{k_1} \subset \mathcal{P}_{k_2} \subset \ldots \subset P$$

$$\mathcal{Q}_{k_1} \subset \mathcal{Q}_{k_2} \subset \ldots \subset Q_{\max} \subset \ldots \subset \mathcal{Q}_{k_3} \subset \ldots$$

$$\mathcal{M}_{k_1} \subset \mathcal{M}_{k_2} \subset \ldots \subset M \subset \ldots \subset \mathcal{M}_{k_3} \subset \ldots$$

We consider an infinite family of stable bridges. These bridges are ordered by a numerical parameter k. With increasing the parameter, we get a family of stable bridges increasing by inclusion, where each greater bridge corresponds to a larger constraint for the disturbance.

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Some Properties of Stable Bridges

Multiplication by a scalar and summation of sets:

$$kW = \{(t,x) \in T \times \mathbb{R}^n : \; x \in kW(t)\}$$

$$W_1 + W_2 = \{(t,x) \in T \times \mathbb{R}^n : \; x \in W_1(t) + W_2(t)\}$$

Properties:

1. If F is a stable bridge with parameters $(\mathcal{P}, \mathcal{Q}, \mathcal{M})$ and $k \geq 0$, then kF is a stable bridge with parameters $(k\mathcal{P}, k\mathcal{Q}, k\mathcal{M})$.
2. If F is the maximal stable bridge with parameters $(\mathcal{P}, \mathcal{Q}, \mathcal{M})$, then kF is the maximal stable bridge with parameters $(k\mathcal{P}, k\mathcal{Q}, k\mathcal{M})$.
3. If F_1 and F_2 are stable bridges with parameters $(\mathcal{P}_1, \mathcal{Q}_1, \mathcal{M}_1)$ and $(\mathcal{P}_2, \mathcal{Q}_2, \mathcal{M}_2)$, then $F_1 + F_2$ is a stable bridge with parameters $(\mathcal{P}_1+\mathcal{P}_2, \mathcal{Q}_1+\mathcal{Q}_2, \mathcal{M}_1+\mathcal{M}_2)$.

Here, some specific operations of multiplication by a non-negative scalar and summation of sets located in the game space *time* $\times$ *phase vector* are introduced. Due to linearity of the dynamics, the stability property is preserved under these operations.

Constructing Adaptive Control

1) Choose a critical level $Q_{\max}$ for the disturbance.

2) Construct the main bridge W_{main} with parameters $(P,\ Q_{\max},\ M)$.

3) Construct an additional stable bridge W_{add} with parameters $(\{0\},\ Q_{\max},\ M_G)$.

4) Construct a family of sets

$$W_k = \begin{cases} kW_{\text{main}}, & 0 \le k \le 1, \\ W_{\text{main}} + (k-1)W_{\text{add}}, & k > 1. \end{cases}$$

Properties:

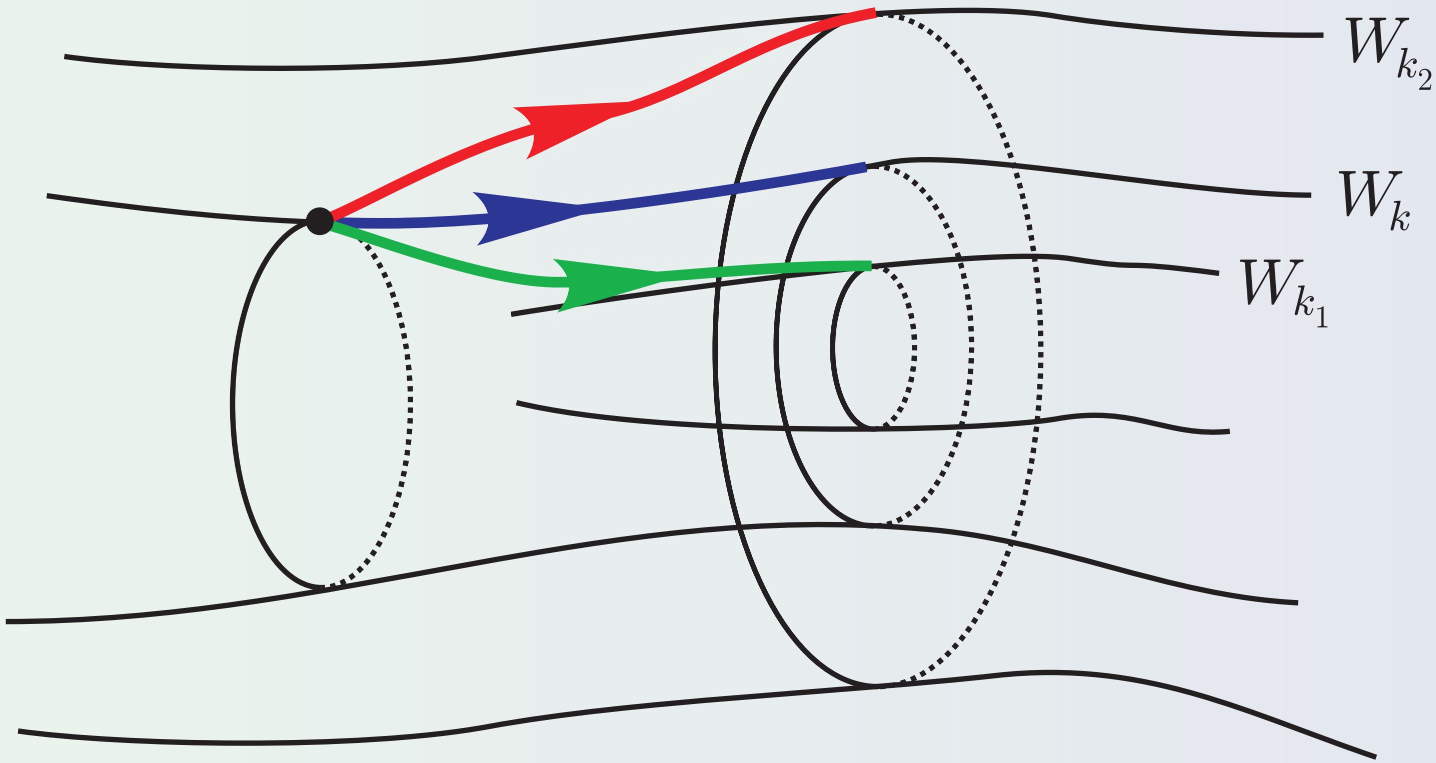
$W_k,\ k \quad 1$, are maximal stable bridges with parameters $(kP,\ kQ_{\max},\ kM)$;

$W_k,\ k > 1$, are stable bridges with parameters $(P,\ kQ_{\max},\ M + (k{-}1)M_G)$.

Here, the algorithm for constructing the family is shortly described. At first, two maximal stable bridges are built. One of them is the maximal stable bridge corresponding to the given terminal set M, the given constraint P for the useful control and some reasonable constraint $Q_{\max}$ for the disturbance. The set $Q_{\max}$ is chosen by us. Another bridge is the maximal stable bridge built for some small terminal set M_G, absent useful control and the same constraint $Q_{\max}$ for the disturbance. The main requirements for choice of $Q_{\max}$ and the terminal set M_G for the second bridge is non-emptiness of all time sections of both built bridges.

Bridges from the family are constructed on the basis of these two bridges by means of the set operations introduced above.

Behavior of Trajectories



– if $v(t)$ is stronger than $kQ_{\max}$

– if $v(t)$ is just of the level $kQ_{\max}$

– if $v(t)$ is weaker than $kQ_{\max}$

In this slide, the character of trajectories is shown with respect to the actual level of disturbance.

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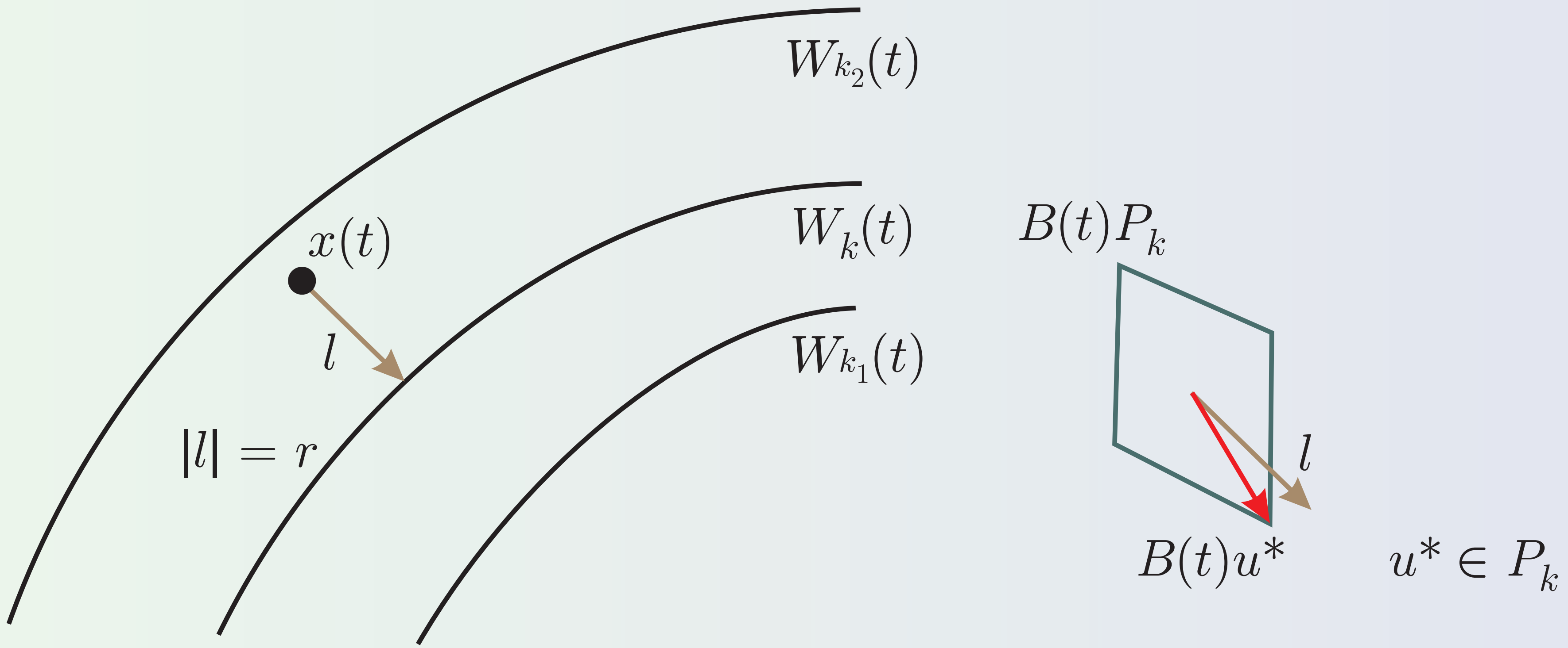
Extremal Aiming in the Scheme of Adaptive Control



N.N. Krasovskii, A.I. Subbotin *Positional Differential Games.* Nauka, Moscow, 1974 (in Russian)
N.N. Krasovskii, A.I. Subbotin *Game-Theoretical Control Problems.* Springer-Verlag, New York, 1988.

To construct adaptive control, we use extremal aiming rule described in the books by N.N.Krasovskii and A.I.Subbotin. Using a parameter r in our algorithm, we compute the bridge, distance to whose time section from the current phase position of the system equals r. The aiming control vector $u^* \in P_k$ is extremal on the vector l directed from the position to the bridge. The constraint P_k corresponds to the bridge W_k, on whose boundary the current position is suited. We use this constant control in the nearest step of discrete scheme of control. In the same way, we construct the control in the next steps.

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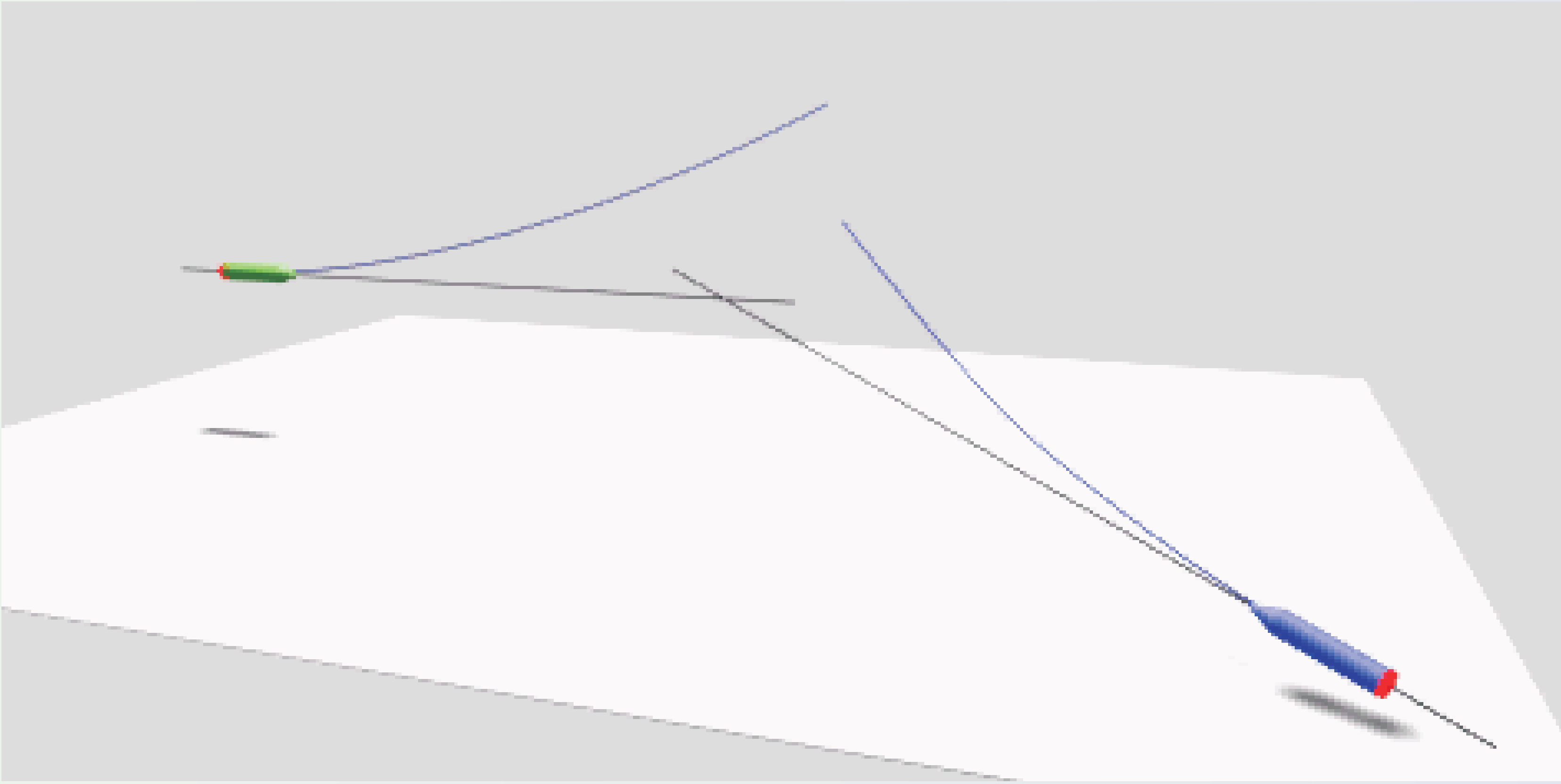
Interception of a Weak-Maneuvering Object by Another One



J. Shinar, M. Medinah, M. Biton *Singular Surfaces in a Linear Pursuit-Evasion Game with Elliptical Vectograms.* In: Journal of Optimization Theory and Applications, Vol.43, No.3, 1984, pp.431-458.

J.Shinar, M.Zarkh *Pursuit of a faster evader – a linear game with elliptical vectograms*. In: Proceedings of the Seventh International Symposium on Dynamic Games, Yokosuka, Japan, 1996, pp.855-868.

We tested our method of adaptive control on several problems. Now, we consider an interception problem. Model linear dynamics was given in papers by Josef Shinar and his colleagues.

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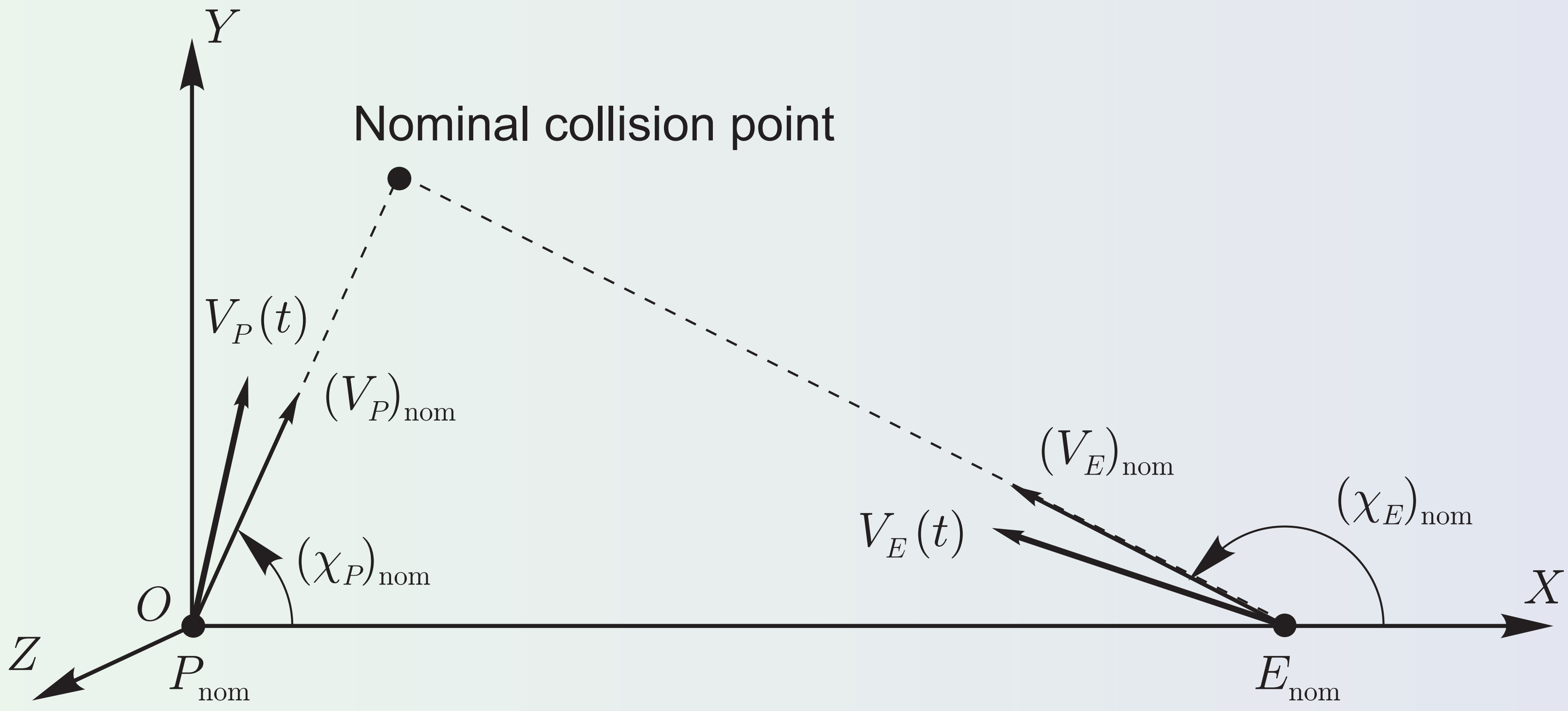
Interception Problem. Linear Dynamics



$$\ddot{z} = F,$$
$$\dot{F} = -(F - u)/\tau_P,$$
$$\ddot{y} = v$$

$$t \in [0, \vartheta], \quad z, y \in \mathbb{R}^2, \quad u \in P,$$
$$\varphi\big(z(\vartheta), y(\vartheta)\big) = \big|z(\vartheta) - y(\vartheta)\big|.$$

Here, a scheme of nominal motions of two objects in the original three-dimensional space is shown.

In the linear dynamics, z, y are the vectors of projections to the plane YZ of the pursuer's and evader's positions, respectively; the parameter u is the control of the pursuer; v is the control of the evader. The control u is bounded by an ellipse P. From our point of view, the control v is a dynamic disturbance. There is no any constraint for the disturbance v in advance. We take some level set of the payoff function as the terminal set for the method of adaptive control.

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Data for Simulation

$$\tau_P = 1.0 \text{ sec}, \quad \vartheta = 10.0 \text{ sec},$$

$$a_P = 1.3 \text{ m/sec}^2, \quad (\chi_P)_{\text{nom}} = 47.94°,$$

$$a_{E\max} = 1.0 \text{ m/sec}^2, \quad (\chi_E)_{\text{nom}} = 45°,$$

$$P = \left\{ u \in \mathbb{R}^2 : \frac{u_1^2}{0.87^2} + \frac{u_2^2}{1.3^2} \leqslant 1 \right\},$$

$$Q_{\max} = \left\{ v \in \mathbb{R}^2 : \frac{v_1^2}{0.71^2} + \frac{v_2^2}{1.0^2} \leqslant 1 \right\},$$

$$r = 0.01, \quad \Delta = 0.01 \text{ sec},$$

$$\Delta z_0 = z_0 - y_0 = (-3 \text{ m}, 0 \text{ m}),$$

$$\Delta \dot{z}_0 = \dot{z}_0 - \dot{y}_0 = (0 \text{ m/sec}, 2 \text{ m/sec})$$

Here, the parameter a_P is the constraint for the lateral acceleration of the pursuer. This value together with the cosine of the angle $(\chi_P)_{\text{nom}}$ between the nominal trajectory of the pursuer and the nominal line-of-sight defines the constraint P.

To apply the method of adaptive control, we should choose an auxiliary constraint $Q_{\max}$ for the disturbance. To do this, we take a reasonable value $a_{E\max}$ bounding the lateral acceleration of the evader. This value together with the cosine of the angle $(\chi_E)_{\text{nom}}$ between the nominal trajectory of the evader and the nominal line-of-sight defines the constraint $Q_{\max}$.

The initial relative position and velocity of the system are also given.

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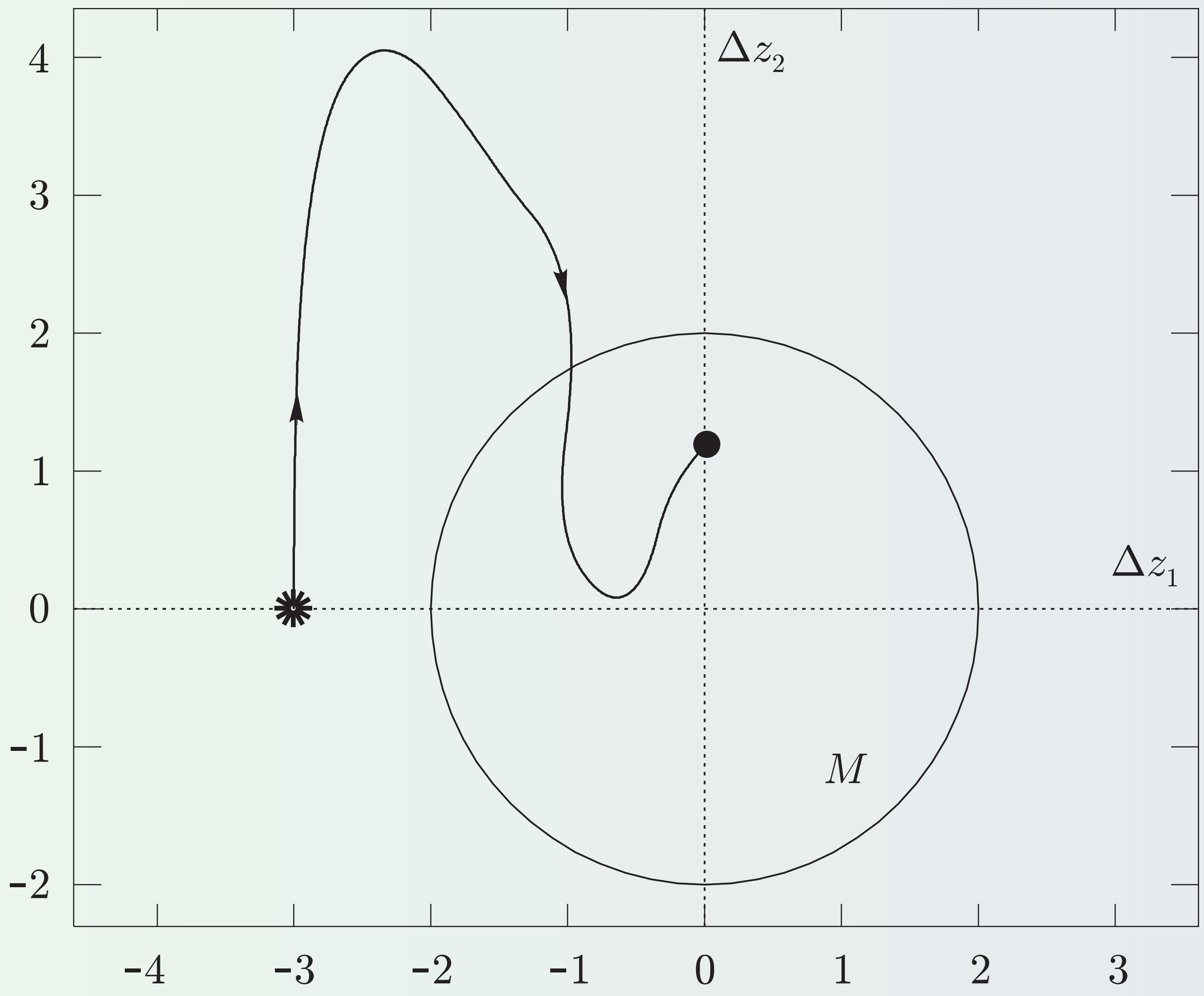
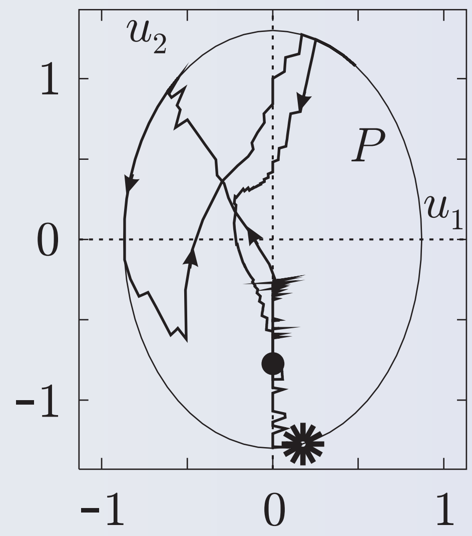
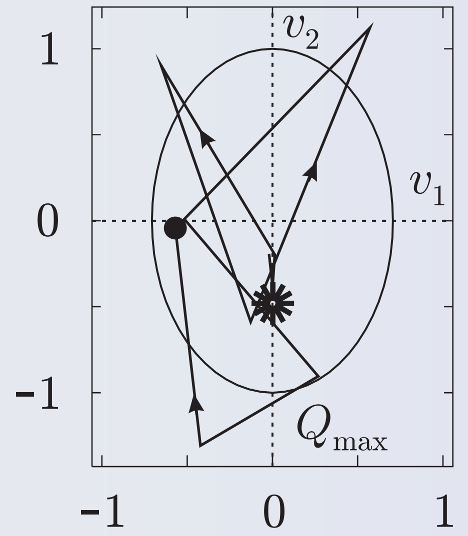
Results of Simulation







Here, the pictures of simulation results are shown. At the left, we see the phase trajectory of the system in difference geometric coordinates. At the right, the ellipses P and $Q_{\max}$ and hodographs of the realizations $u(t)$ and $v(t)$ are given. The disturbance v was generated as a piecewise-constant stochastic function, which values were taken from an ellipse that was greater than the ellipse $Q_{\max}$. The hodograph of the control u is inside the ellipse P.

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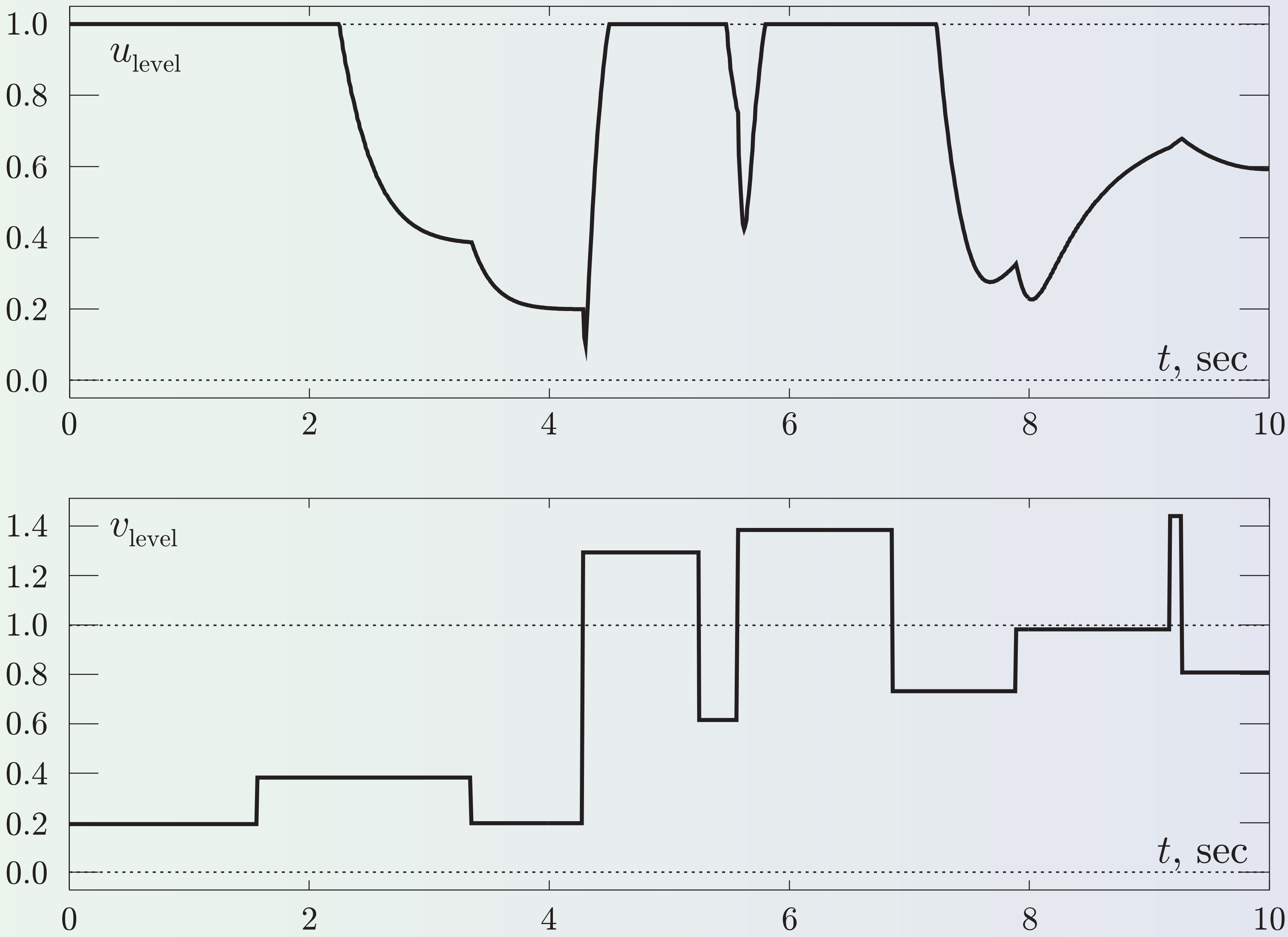
Results of Simulation (cont.)



Here are the realizations of levels of the control u and the disturbance v. We see that the level of the control u tunes to the actual deviation of the phase vector from the origin and the actual level of the disturbance.

Works of the Authors

1. **Ganebny S.A., Kumkov S.S., Patsko V.S., Pyatko S.G.** *Robust Control in Game Problems with Linear Dynamics. Preprint.* Institute of Mathematics and Mechanics, Ekaterinburg, 2005.
2. **Ganebnyi S.A., Kumkov S.S., Patsko V.S.** *Control design in problems with unknown level of dynamic disturbance* // Journal of Applied Math. and Mech., Vol. 70, 2006, pp. 680-695.
3. **Ganebny S.A., Kumkov S.S., and Patsko V.S.** *Constructing robust control in game problems with linear dynamics.* In: Game Theory and Applications, Vol.11, 2007, pp. 49-66.
4. **Ganebny S.A., Kumkov S.S., Patsko V.S., and Pyatko S.G.** *Constructing robust control in differential games. Application to aircraft control during landing.* In: Annals of the International Society of Dynamic Games, Advances in Dynamic Game Theory and Applications, Vol.9, S.Jorgensen, T.Vincent, M.Quincampoix (Eds.), Birkhauser, 2007, pp. 69-92.

Here, some references to the authors' papers on the adaptive control method are given.