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Model Problems of Aircraft Guidance in Education Program on Optimal Control and Differential Games

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AbstractModel aero-space problems are described, which are used by the authors in a lecture course on optimal control and differential games.

Keywords: education, optimal control, differential games

1. INTRODUCTION

For the last two years, the first author had read a one-semester course (14 lectures) on optimal control and differential games for master students of the first study-year at the Mathematical-Mechanical Department of the Ural State University, Ekaterinburg, Russia. Since the listeners were of different specialties, the delivered material did not suppose any preliminary acquaintance with the subject.

Six lectures on the optimal control were devoted to the concept of a reachable set of a control system and to the statement that open-loop control leading the system onto a point on the boundary of its reachable set satisfies the Pontryagin's maximum principle. After proving this statement, linear and nonlinear optimal control problems with some concrete optimality criteria (e.g., problem with a fixed termination instant and integral-terminal payoff function, optimal-time problem) were considered.

Other eight lectures concerning the differential game theory were based on the concept of a maximal stable bridge (MSB), which is a keystone of the Krasovskii's theory for solving problems of conflict control. The concept of MSB generalizes naturally the concept of tube of the reachable set in the control theory. In the game-control problems with a payoff function, MSB is built by the backward procedure from a level set (Lebesgue set) of the payoff function. In the game space, the bridge defines a solvability set for the problem with a result not greater than some constant c, where the value c corresponds to the chosen level set. Further, the following materials were considered: the concept of an extremal positional strategy, the scheme of control with a guide, methods of the positional control on the basis of the switch surfaces. For problems with small dimension of the state vector (or for ones that can be reduced to such problems), the main ideas of numerical methods are set forward.

Peculiarity of the course consists in the following. At the end of each lecture, results of computer simulation were demonstrated approximately during 20 minutes, and this material was connected to topic of the lecture. The authors had composed model aero-space guidance problems, which were earlier investigated in their scientific work. Four such problems are described in the paper.

2. THREE-DIMENSIONAL REACHABLE SET FOR A MODEL OF AIRCRAFT MOTION IN PLANE

For navigational computations, the following model of aircraft motion in the horizontal plane (Miele (1962); Pecsvaradi (1972)) is used:

$$\dot{x} = V\sin\theta,\ \dot{y} = V\cos\theta,\ \dot{\theta} = \frac{g}{V}\tan\gamma;\ |\gamma| \le 30^\circ. \quad (1)$$

Here, x and y are the Cartesian coordinates of the aircraft, θ is the angle of the velocity vector counted clockwise from the positive direction of the axis y, V is the magnitude of velocity, γ is the bank angle, and g is the gravity acceleration.

Assume $V = \text{const}$. Then, after a normalization, we pass from system (1) to the following system:

$$\dot{x} = \sin\theta, \quad \dot{y} = \cos\theta, \quad \dot{\theta} = u; \quad |u| \le 1. \quad (2)$$

Model (2) is used also in theoretical robotics; it is called "Dubins' car" (Laumond (1988)).

The reachable set $G(t; x_0, y_0, \theta_0)$ at a given instant t is the set of all positions (x, y, θ), which can be reached at the instant t from the initial point (x_0, y_0, θ_0) (taken at the starting instant $t_0 = 0$) by the trajectories of system (2) using admissible piecewise-continuous control $u(\cdot)$. Without loss of generality, suppose that $x_0 = y_0 = \theta_0 = 0$. Denote by $G(t)$ the reachable set from this point.

In paper (Patsko et al. (2003)), it was established (based on application of the Pontryagin's maximum principle to system (2)) that for any point $(x, y, \theta) \in \partial G(t)$ the control steering to this point is piecewise continuous and has at most two switches. In addition, there are only 6 variants of changing the control: 1) 1, 0, 1; 2) −1, 0, 1; 3) 1, 0, −1; 4) −1, 0, −1; 5) 1, −1, 1; 6) −1, 1, −1.

The second variant means that the control $u \equiv -1$ acts on some interval $[0, t_1)$, the control $u \equiv 0$ works on an interval $[t_1, t_2)$, and the control $u \equiv 1$ operates on the interval $[t_2, t]$. If $t_1 = t_2$, then the second interval (where $u \equiv 0$) vanishes, and we obtain a single switch from $u = -1$ to $u = 1$. In the case $t_1 = 0$, the first interval where $u \equiv -1$ vanishes; in the case $t_2 = t$ the third interval with $u \equiv 1$ is absent. The control has constant value if one of the following three conditions holds: $t_1 = t$, $t_2 = 0$, or both $t_1 = 0$ and $t_2 = t$. A similar property is true for the other variants.

The proposition on six variants of the control $u(t)$ steering to the boundary of the reachable set $G(t)$ is similar in its form to the Dubins' theorem (Dubins (1957)) on the variants of the controls steering to the boundary of the reachable set $\mathcal{G}(t)$ by given time t. The same variants are valid. However, due to the relation between the sets $G(t)$ and $\mathcal{G}(t)$ (the set $\mathcal{G}(t)$ is the union of the sets $G(s)$ over $s \in [0, t]$), the above mentioned properties of the controls leading to the boundary of the set $G(t)$ result in the analogous properties of the controls leading to the boundary of the set $\mathcal{G}(t)$, but the converse is not true.

We apply the above formulated result on the structure of the control $u(t)$ steering to $\partial G(t)$ to the numerical construction of the boundary $\partial G(t)$.

To construct the boundary $\partial G(t)$ of the set $G(t)$, we search through all controls of the form 1–6 with two switches at the instants t_1, t_2. For every variant of switches, the parameter t_1 is choosing from the interval $[0, t]$, and the parameter t_2 from the interval $[t_1, t]$. In addition, controls with one switch and without switches are also considered. Taking a specific variant of switching and searching through the parameters t_1, t_2 on some sufficiently fine grid, we obtain a collection of points generating a surface in the three-dimensional space x, y, θ.

Therefore, each of the six variants yields its own surface in the three-dimensional space. The boundary of the reachable set $G(t)$ is composed of the pieces of these surfaces. The six surfaces are loaded into the visualization program without any additional processing of data. Using this program, the boundary of the reachable sets is extracted. Some surfaces (in part or as a whole) are located inside of the reachable set. The visualization program does not plot such pieces. The three-dimensional sets has been drawn by the program "Cortona VRML Client" utilizing standard format VRML for the demonstration of interactive vector graphics.

Fig. 1 shows the boundary of the set $G(t)$ at time $t = 1.5\pi$ from two perspectives. The different parts of the boundary are marked with different colors. For example, part 2 is reachable for the trajectories with the control $u(t)$ of the form -1, 0, 1 with two switches. The sections of the reachable set by the plane $\theta =$ const are depicted with some step along the axis θ.

Fig. 2 shows reachable sets $G(t)$ at the same perspective but with different scales for four time instants t.

Note that during solving the problem, we assume that the angle θ can change in the range $(-\infty, \infty)$. This allows to see the laws of evolution of the sets $G(t)$ with growing t. It is possible to pass easily to the sets, for which the angle θ is calculated modulo 2π. Fig. 3 shows such a set $G(t)$ for $t = 2\pi$.

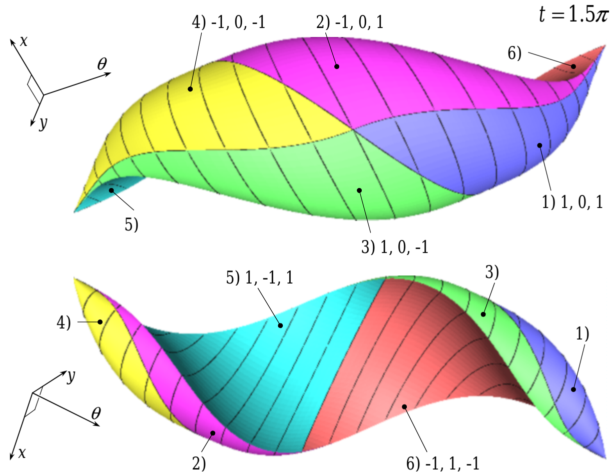

Figure 1. The set $G(1.5\pi)$ from the two perspectives

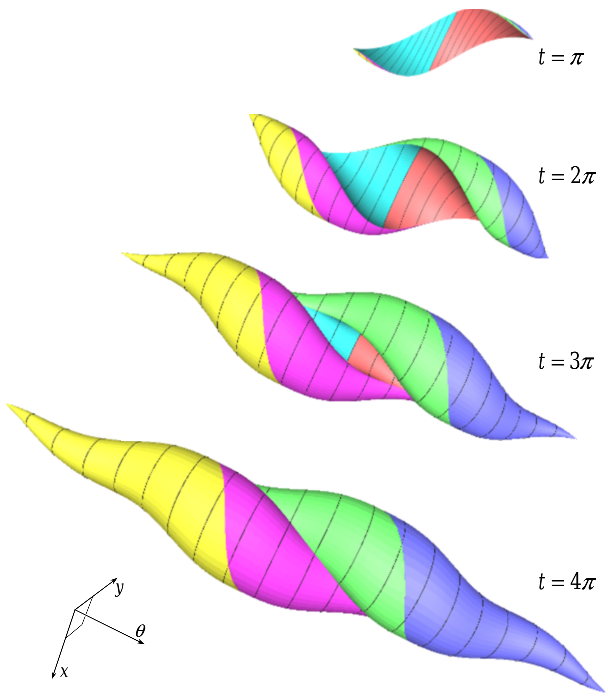

Figure 2. Development of the reachable set $G(t)$

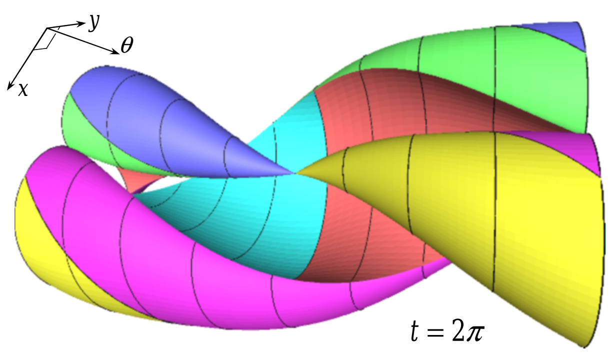

Figure 3. The set $G(2\pi)$ with θ computed modulo 2π

When demonstrating these three-dimensional sets at the instant for non-linear system (2), we emphasize that the Pontryagin's maximum principle is only a necessary condition for the controls leading the system to the boundary of the reachable set. Some parts of the obtained surfaces are located inside the reachable set, but the maximum principle is held for them too.

3. LINEAR DIFFERENTIAL GAMES WITH FIXED TERMINATION INSTANT

Linear differential games

$$\begin{aligned} \dot{x} &= A(t)x + B(t)u + C(t)v,\ t \in [t_0, T],\ x \in \mathbb{R}^n, \\ u &\in P \subset \mathbb{R}^p,\ v \in Q \subset \mathbb{R}^q,\ \varphi\big(x_i(T), x_j(T)\big) \end{aligned} \tag{3}$$

with fixed termination instant T and continuous payoff function φ depending on two components x_i, x_j of the phase vector are of a very important type of differential games. For such games, there are effective numerical procedures for constructing level sets of the value function. We suppose that the first (second) player governs the control u (v) choosing it from a convex compact set P (Q) to minimize (maximize) the value of the payoff φ at the instant T.

The variable change

$$\xi(t) = X_{i,j}(T, t)x(t)$$

provides a standard pass to an equivalent differential game. Here, $X_{i,j}(T, t)$ is a matrix combined of the ith and jth rows of the fundamental Cauchy matrix $X(T, t)$ for the differential equation $\dot{x} = A(t)x$. This equivalent game is

$$\begin{aligned} \dot{\xi} &= D(t)u + E(t)v, \quad t \in [t_0, T], \quad \xi \in \mathbb{R}^2, \\ u &\in P, \quad v \in Q, \quad \varphi\big(\xi_1(T), \xi_2(T)\big). \end{aligned} \tag{4}$$

In the lecture course, we describe a backward procedure for an approximate stepwise construction of time sections $W(t)$ of MSB W for game (4). The bridge is built from a terminal set M, which is taken as a polygon in the plane ξ_1, ξ_2. As the set M in game (4) with the payoff function φ, we take the level set $M_c = \big\{(\xi_1, \xi_2) : \varphi(\xi_1, \xi_2) \le c\big\}$.

4. LINEAR INTERCEPTION PROBLEM

Consider a differential game concerning an interception problem (Shinar et al. (1984); Shinar and Zarkh (1996)). The pursuer in this problem is an antimissile, the evader is a maneuvering aerial target. The natural payoff is the minimal approach distance, that is, the miss, which is minimized by the pursuer P and maximized by the evader E. The vectors of the initial nominal velocities $(V_P)_{\rm nom}$ and $(V_E)_{\rm nom}$ are directed such that there is an exact collision along the nominal rectilinear trajectories. The control of each object is orthogonal to the current velocity vector (the current direction of the building longitudinal axis). The maximal values of the lateral control accelerations are bounded by constants μ and ν. Assume that $\mu > \nu$. The evader controls its acceleration directly, but the pursuer has an additional inertial link with the time constant τ_P. Capabilities of the objects to change direction of their velocities during the motion are small (weak-maneuvering objects).

The choice of the coordinate axes is made in the following way. The origin O coincides with the nominal pursuer

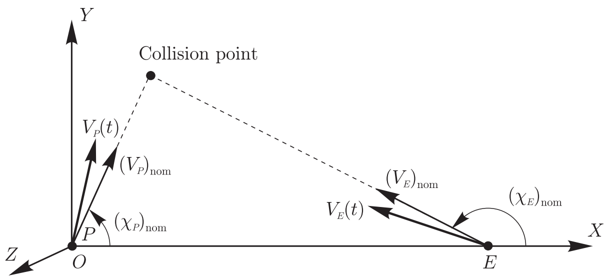

Figure 4. Coordinate system in the interception problem

position $P_{\rm nom}$ at the initial instant. The axis OX is directed along the initial nominal line-of-sight. The axis OY is orthogonal to OX and is located in the plane defined by the vectors of the nominal velocities (Fig. 4). The axis OZ is orthogonal to the mentioned two ones.

Since deviations of the actual velocities $V_P(t)$ and $V_E(t)$ from their nominal values $(V_P)_{\rm nom}$ and $(V_E)_{\rm nom}$ are small, the relative motion along the axis OX can be considered as uniform. Therefore, the miss can be computed at the instant of the nominal collision as the distance between the objects in the plane YZ. Thus, the problem of minimization of the closest spacial miss can be reduced to the minimization of the distance in the YZ plane at the fixed instant T of the nominal collision.

Linearizing the objects' dynamics with respect to the nominal motions, we get the following linear differential game (Shinar et al. (1984); Shinar and Zarkh (1996)):

$$\begin{aligned} &\ddot{x}_P = a_P, && t \in [0, T], \\ &\dot{a}_P = (u - a_P)/\tau_P, && x_P, x_E \in \mathbb{R}^2, \\ &\ddot{x}_E = v, && u \in P, \quad v \in Q, \\ &\varphi\big(x_P(T), x_E(T)\big) = \big|x_E(T) - x_P(T)\big|. \end{aligned} \tag{5}$$

Here, x_P is the position vector of the first (pursuing) player, x_E is the position vector of the second (evading) player, τ_P is the time constant characterizing the inertiality of the first player's control action. The sets P and Q bounding the first and second players' controls are ellipses:

$$P = \left\{ u \in \mathbb{R}^2 : \frac{u_1^2}{A_P^2} + \frac{u_2^2}{B_P^2} \leqslant 1 \right\},$$

$$Q = \left\{ v \in \mathbb{R}^2 : \frac{v_1^2}{A_E^2} + \frac{v_2^2}{B_E^2} \leqslant 1 \right\}.$$

The semiaxes A_P, B_P, A_E, B_E are parallel to the coordinate axes and can be computed on the basis of the constants μ, ν bounding the players' accelerations and cosines of the angles $(\chi_P)_{\rm nom}$ and $(\chi_E)_{\rm nom}$. The termination instant T is fixed. The payoff is the geometric distance between the objects at the termination instant. The first player minimizes the payoff, the second one maximizes it.

By introducing two-dimensional vector $y = x_E - x_P$, the system (5) can be rewritten as (3). Numerical constructions are made in the coordinates of system (4).

In Fig. 5, two level sets of the value function (two MSBs) are shown in the space t, ξ_1, ξ_2 for the case $(V_P)_{\rm nom} < (V_E)_{\rm nom}$. One can see that the larger set (corresponding to $c = 1.67$) has smooth boundary. The smaller set (corresponding to $c = 1.546$) has a narrow "throat" with complicated geometry of t-sections, which changes in time. A zoomed view of the throat is given in Fig. 6. The

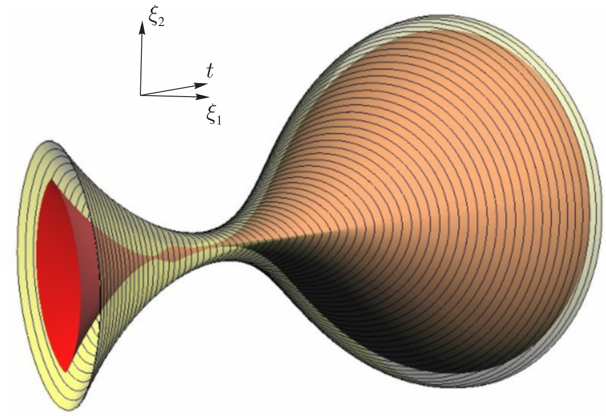

Figure 5. A level set with a narrow throat and a larger one

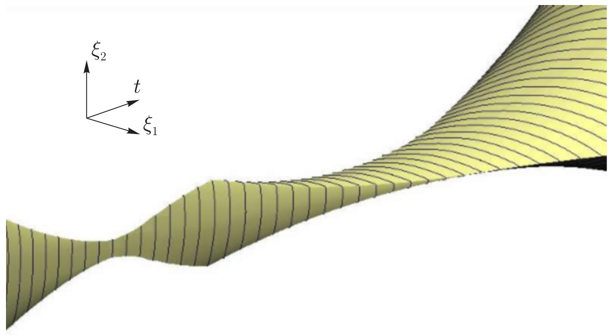

Figure 6. A zoomed view of the narrow throat

construction are made for the parameters $T = 7.0$, $\tau_P = 1$, $A_P = 0.871$, $B_P = 1.30$, $A_E = 0.71$, $B_E = 1.00$.

The main aim of involving this model problem in the course is to show how elliptic constraints P and Q can appear naturally. Also, this problem demonstrates how dramatically the solution can depend on the parameters of the game.

5. INTERCEPTION PROBLEM WITH TWO PURSUERS AND ONE EVADER

An interception problem similar to the previous one but with two pursuers is very difficult because if to keep the second dimension of the geometric miss between each of the pursuers and the evader, then equivalent game (4) has a 4-dimensional phase vector.

To simplify reasonably the problem, we assume that the vectors of the nominal velocities $(V_{P_1})_{\text{nom}}$, $(V_{P_2})_{\text{nom}}$, $(V_E)_{\text{nom}}$ are in the plane of the initial nominal geometric positions of all three objects and the pursuit takes place in this plane too. Such a situation can be considered if in the control scheme, the pursuers' controls are disjointed into two channels: "vertical" and "lateral". The miss in each of these two channels is one-dimensional.

From the mathematical point of view, in each channel, we have a problem, where two pursuers P_1, P_2 and the evader E move along a line. The dynamics description for pursuers P_1 and P_2 is (Le Ménec (2011))

$$\begin{aligned}
&\ddot{x}_{P_1} = a_{P_1}, && \ddot{x}_{P_2} = a_{P_2},\\
&\dot{a}_{P_1} = (u_1 - a_{P_1})/\tau_{P_1}, && \dot{a}_{P_2} = (u_2 - a_{P_2})/\tau_{P_2},\\
&|u_1| \le \mu_1, && |u_2| \le \mu_2,\\
&a_{P_1}(t_0) = 0, && a_{P_2}(t_0) = 0.
\end{aligned} \tag{6}$$

Here, x_{P_1} and x_{P_2} are the geometric coordinates of the pursuers, a_{P_1} and a_{P_2} are their accelerations generated by the controls u_1 and u_2. The time constants τ_{P_1} and τ_{P_2} define how fast the controls affect the systems.

The dynamics of the evader E is similar:

$$\begin{aligned}
&\ddot{x}_E = a_E, && \dot{a}_E = (v - a_E)/l_E,\\
&|v| \le \nu, && a_E(t_0) = 0.
\end{aligned} \tag{7}$$

Let us fix some instants T_1 and T_2. At the instant T_1, the miss of the first pursuer with respect to the evader is computed, and at the instant T_2, the miss of the second one is computed:

$$\begin{aligned}
r_{P_1,E}(T_1) &= |x_E(T_1) - x_{P_1,E}(T_1)|,\\
r_{P_2,E}(T_2) &= |x_E(T_2) - x_{P_2,E}(T_2)|.
\end{aligned} \tag{8}$$

Assume that the pursuers act in coordination. This means that we can join them into one player (which will be called the *first player*). This player governs the vector control $u = (u_1, u_2)$. The evader is counted as the *second player*. The resultant miss is the following:

$$\varphi = \min\{r_{P_1,E}(T_1),\ r_{P_2,E}(T_2)\}. \tag{9}$$

At any instant t, both players know exact values of all state coordinates $x_{P_1}, \dot{x}_{P_1}, a_{P_1}, x_{P_2}, \dot{x}_{P_2}, a_{P_2}, x_E, \dot{x}_E, a_E$. The first player choosing its feedback control minimizes the miss φ, the second one maximizes it.

Studying this problem, we pass to two one-dimensional relative geometric coordinates

$$y_1 = x_E - x_{P_1}, \quad y_2 = x_E - x_{P_2}$$

and, further, to the coordinates ξ_1 ξ_2, which are the forecasts of y_1 and y_2 to the instants T_1 and T_2, respectively. Level sets of the value function (MSBs) are constructed in the three-dimensional space t, ξ_1, ξ_2.

In Fig. 7, one can see a numerically obtained level set of the value function for the following parameters of the problem:

$$\mu_1 = \mu_2 = 1.1, \quad \nu = 1, \quad \tau_{P_1} = \tau_{P_2} = 1/0.6,$$
$$\tau_E = 1, \quad T_1 = T_2 = 20.$$

With growing of the backward time, the t-sections lose connectedness and disjoin to two parts, which join back with further growth of the backward time.

In the lectures, we show to the students the level sets for other variants of the problem parameters. We emphasize that the non-convexity of the t-sections of level sets of the value function and losing connectedness by them are stipulated by the concrete type of payoff function (9). If formula (9) would contain max instead of min, then for any $t \le \max\{T_1, T_2\}$ the value function would be convex.

6. ADAPTIVE CONTROL OF AIRCRAFT LANDING UNDER WIND DISTURBANCE

Take-off and landing of an aircraft under wind disturbance are the natural examples (Miele et al. (1986, 1988)) of application of modern methods of mathematical control

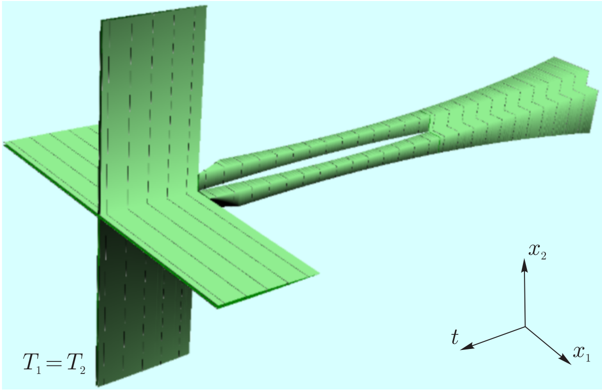

Figure 7. A level set of the value function with t-sections losing connectedness

theory and differential game theory to practical problems. But there are some difficulties during formalization of these problems.

The aircraft has four controls: thrust, elevator, rudder, and ailerons. Bounds for their control ranges are known. Therefore, during formalization, we can strictly describe the constraints for the useful control. But there are problems with the constraints for the disturbance. Even if to suppose some middle maximal level of it and to take into account the theoretically worst realization, the result of the process can be unacceptable. Also, if the actual wind disturbance is weak, then the result is good, but the realization of the control is technically bad: it switches from one extreme value to another instead of keeping some intermediate level.

So, during the formalization, we get a problem with fixed termination instant. With that, the level of the dynamic disturbance is bounded, but unknown *a priori*. To apply standard methods of control, we do the following. At first, choose some monotonically growing family $\{Q_k\}$, $k \geq 0$, of constraints for the disturbance. The set Q_k for $k = 1$ is called *critical*. It is chosen from some "reasonable" estimation of the disturbance. For each value k, we also define some constraint P_k for the useful control. It grows with $k \in [0, 1]$. When $k = 1$, it equals P, the maximal capability of the control. If $k \geq 1$, then $P_k = P$.

Each pair P_k, Q_k together with some terminal set M_k produces a *stable* tube (bridge) W_k in the space *time* $t \times$ *phase vector* x. It possesses the following stability property: if the initial position is in the tube and the realization of the disturbance is inside the set Q_k, then the useful control taking its value from the set P_k can keep the motion inside this tube.

If to take the family of terminal sets $\{M_k\}$ monotonically growing on k, then the system of stable bridges $\{W_k\}$ is also monotone. The system generates a control according to the following procedure. At the instant t, we measure the current phase state $x(t)$ and find the value $\bar{k}$ such that $x(t)$ is on the boundary of $W_{\bar{k}}$. Then we take an appropriate control with its value from the set $P_{\bar{k}}$ and keep it during some small period of time. If after the end of this period the motion of the system leaves the bridge $W_{\bar{k}}$, then the disturbance is actually of a level higher than $Q_{\bar{k}}$. Therefore, during the next period of time we shall apply a control of a higher level too (if it is possible). The value of the control will be taken from some set $P_{\tilde{k}}$, $\tilde{k} > \bar{k}$, which corresponds to a bridge $W_{\tilde{k}}$ whose boundary contains the new system position. If, vice versa, the motion comes into the interior of the bridge $W_{\bar{k}}$ and reaches some bridge of a lower level, then during the next period of time we shall apply a control of a lower level corresponding to some bridge $W_{\hat{k}}$, $\hat{k} < \bar{k}$, which is reached by the system.

Thus, it is possible to say that the control *adapts* itself to the actual level and "quality" of the disturbance. The final result depends on the maximal level of the disturbance during the process and on how "clever" it was.

The approach for constructing the growing family $\{W_k\}$, $k \geq 0$, of stable bridges for problems with linear dynamics is suggested in (Ganebny et al. (2009)). In the case of linear system, any bridge from the family can be easily built on the basis of two special stable bridges and the index k. One of these two bridges corresponds to the index $k = 1$. The another one is subsidiary and does not belong to the family. The generation of a certain bridge involves operation of algebraic sum (Minkowskii sum) and multiplication of a set by a non-negative scalar. Therefore, we can keep in the memory of a computer these two bridges only.

When a bridge is built, the control is produced by application of the *extremal shift* method (Krasovskii and Subbotin (1974, 1988)). Its realization in the case of convex sets is very simple.

In the framework of the studied problem, the landing of an aircraft is considered until passing the threshold of the runway (Patsko et al. (1994)). After linearization with respect to the nominal motion along the glide path, the dynamics disjoins in two almost separate systems: one is for the vertical motion and another is for the lateral motion. Both systems are considered as linear problems with unknown level of the dynamic disturbance. Terminal sets for them are taken from tolerances in the coordinates *vertical deviation* from the nominal position, *velocity of the vertical deviation* (for the vertical channel), and *lateral deviation*, *lateral velocity* (for the lateral channel). If at the instant of passing the runway threshold, the aircraft reaches these tolerances, then the following stage of landing is guaranteed to be safe.

The nominal glide path is rectilinear, its height over the runway threshold is 15 m. The adapting controls are produced in the framework of two linear systems mentioned above. The forecasted instant of passing the runway threshold is corrected during the landing process. The produced controls are put into the non-linear dynamics of the aircraft.

Results of simulations of the non-linear system for two variants of the wind disturbance taken as a wind microburst (Ivan (1985)) are given in Fig. 8. The figure contains results concerning the vertical channel. The initial position of the aircraft is taken in 8000 m far from the runway, in 40 m up and 80 m aside from the glide path. Trajectories and graphs for the weak microburst are shown by dashed lines. The results for the case of strong microburst are drawn in solid lines. Upper left subfigure gives a view of the trajectories in the coordinates *vertical deviation*,

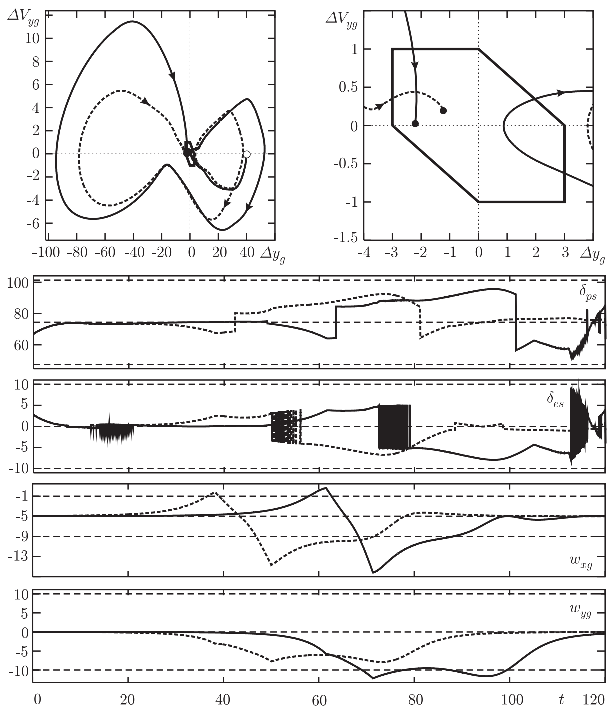

Figure 8. Simulation of the landing problem. At the top: trajectories in the phase plane *vertical deviation* Δ_{yg} (m) $\times$ *velocity of the vertical deviation* ΔV_{yg} (m/sec). Below: graphs of the command controls on the thrust δ_{ps} (deg) and elevator δ_{es} (deg); at the bottom: graphs of the longitudinal w_{xg} (m/sec) and vertical w_{yg} (m/sec) components of the wind velocity. The dashed lines correspond to the weak microburst, solid ones are for the strong microburst.

velocity of the vertical deviation. The upper right subfigure show a large view near the terminal set. In both situations, the process finishes successfully: the trajectories reach the terminal set. Two graphs below give the realizations of the controls on thrust and elevator. Two lower graphs show the realization of the disturbance along the trajectory. The thin dashed lines denote the nominal and extreme levels of the controls (in the graphs of the controls) and the zero level and critical constraints for the disturbance.

One can see that at the beginning, the disturbance grows and decreases later. The useful controls also increase their levels adapting to the disturbance. The maximal level of the useful control is not achieved despite the disturbance is out of the critical constraint (for some period of time). Also, there are some periods when the useful controls "chatter"; but the chattering is smoothed by the inertiality of the servomechanisms.

Studying this problem, students learn some complete description of the non-linear dynamics of the aircraft, linearized systems of the vertical and lateral motions, adapting control method, and different models of the wind disturbance.

CONCLUSION

Concepts of reachable sets and maximal stable bridges founded the basis of the course on optimal control theory and differential games. The considered material is illustrated by model problems from aero-space field. Numerical construction of reachable sets and maximal stable bridges allows to obtain new results, which can be useful in engineering practice.

ACKNOWLEDGEMENTS

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