Model Selection for Radar Registration Errors by Comparison of Radar and ADS-B Data

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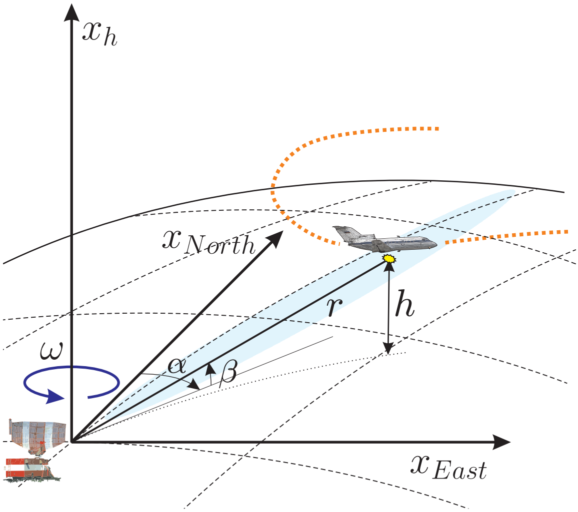
Words of speech

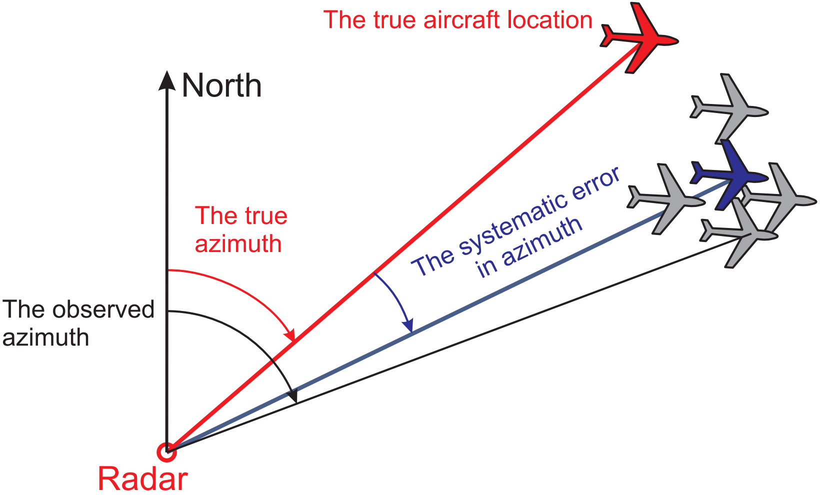
My report is devoted to the problem of model selection for radar systematic errors.

At first, I would like to recall some facts about radar observations.

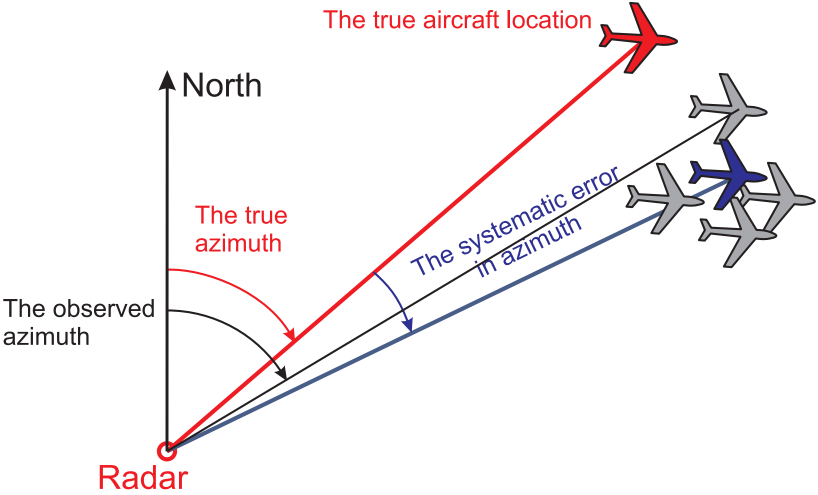
Radar measurement of an aircraft location



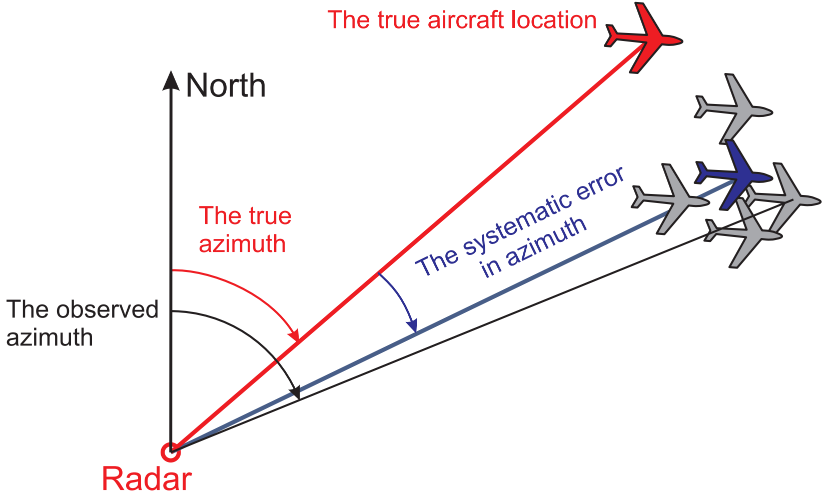
Radar measurement of an aircraft location



Radar measurement of an aircraft location



Radar measurement of an aircraft location



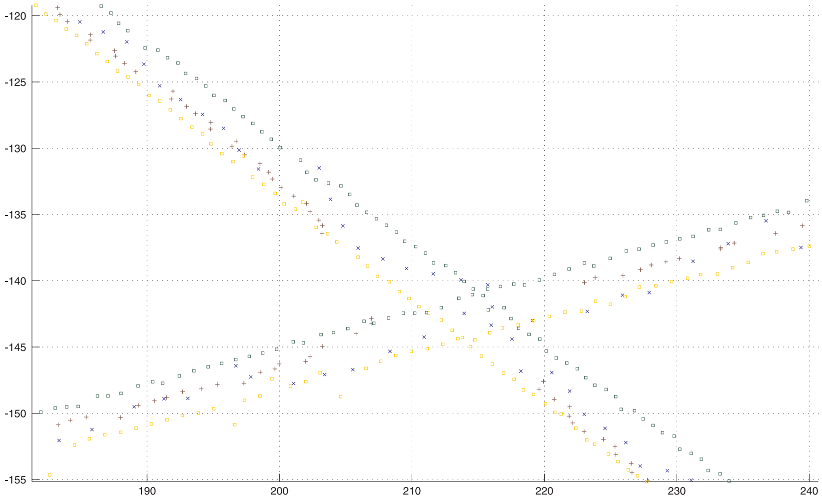
Words of speech

Any radar measures the slant range to the object and the azimuth. The type of radar determines additional available information like altitude.

Radar measurements have errors. It is usual to distinguish the random and systematic errors. The random errors affect measurements like «noise» and are described well by random values and probabilities. The nature of the systematic errors is not random.

In this slide, the simplest type of the systematic error is presented. It is the so-called the «constant» azimuth systematic error. It consists of rotation of observations by some angle and this angle does not depend on geographic location of the target.

Radar observations



Words of speech

In this slide, an example of radar observations corrupted by the systematic errors is shown. There are two aircrafts. One of them flies in 3 hours after another has flown. 4 radars observe their flights. Their measurements are shown by marks of different types and color. It's clearly seen that the yellow and green square marks are shifted significantly from one another. The reason of this behaviour is the systematic errors.

Estimation of the radar systematic errors

For reconstruction of the aircraft location, it is necessary to estimate and compensate the shift vectors s of the radar systematic errors

$$z(t) = x(t) + s(\chi(t)) + w(t)$$

$$\Downarrow$$

$$\begin{aligned}\tilde{z}(t) = z(t) - \hat{s}(\hat{\chi}(t)) = \\ = x(t) + s(\chi(t)) - \hat{s}(\hat{\chi}(t)) + w(t) \approx x(t) + w(t)\,.\end{aligned}$$

It is very useful, if there exists a formula for the dependence $s(\chi)$. Such type of models has name «parametric models».

$$s(\chi) = f(\chi, p)\,.$$

Here, f is a known function but p is an unknown parameter.

Words of speech

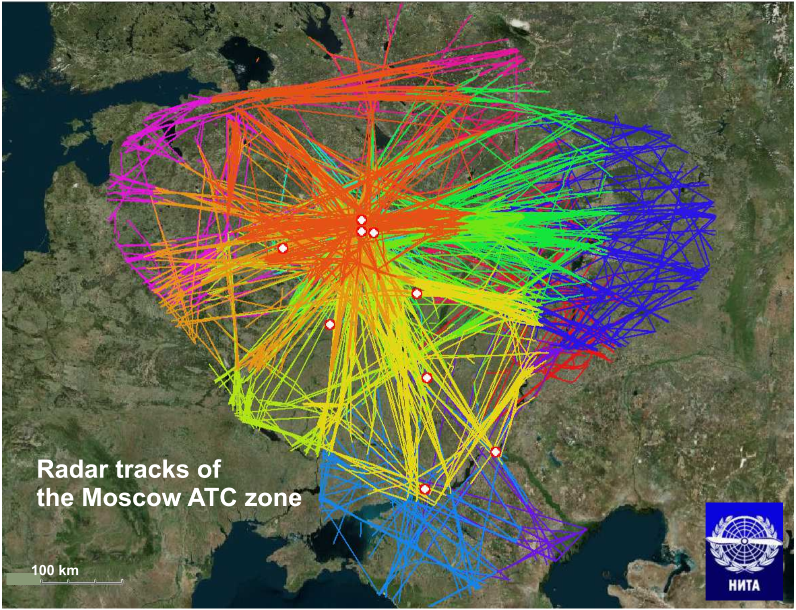
Some authors write that the presence of bias in the measurements can lead to wrong work of maneuver detection algorithms in multi-radar tracking systems. And even in a situation without maneuver, they can corrupt performance of tracking algorithms.

A radar measurement expands to sum of the true location x, the random error w, and the shift s due to the systematic error. For correct work of target tracking algorithms, it is necessary to estimate and compensate values $\hat{s}$ of the shifts s.

For purposes of estimating, it is very useful to know how the function s is organized as the function of the location x, the velocity v, and other parameters. I collect these parameters to the vector χ. Such a knowledge names as «model of funcion». The most useful type of models is parametric models where a formula exists for the reconstructed function.

There are in literature some works about the model of the radar systematic errors. And I decided to investigate their correctness on real data.

Big data set



Words of speech

In my study, I use the ADS-B measurements as a reference to reconstruct residual vectors between the radar measurements and the aircraft locations. For evaluating the residual vectors I use big data sets from the Moscow Air Traffic Control zone. I use only trajectories with measurements by both the radar and ADS-B system. The data set in this slide consists of observations of air traffic in the Moscow air traffic control zone during 8 hours.

ADS-B as a reference

Consider the ADS-B measurements as a reference to reconstruct residual vectors dz that include the systematic and random errors

$$dz(t) = z(t) - \hat{x}(t)\,, \qquad dz(t) = D(x(t)) \begin{bmatrix} dr(t) \\ d\alpha(t) \end{bmatrix}\,.$$

The matrix $D(x)$ describes a transformation from the Cartesian to local radial-transverse coordinates.
The vector $\hat{x}(t)$ is an estimate of the aircraft location at the instant t (when the radar makes a measurement $z(t)$) using the ADS-B track. We imply that

$$\begin{aligned} dz(t) = z(t) - \hat{x}(t) = \\ &= x(t) - \hat{x}(t) + s(\chi(t)) + w(t) \approx s(\chi(t)) + w(t)\,. \end{aligned}$$

Words of speech

Denote as $\hat{x}(t)$ the estimation of the true location x of the aircraft.

The residual vector $dz(t)$ includes the systematic and random error of radar at instance t. In some coordinate system, it can be presented as the residual in range and in azimuth.

To make the estimaton $\hat{x}(t)$, it is neceessary to interpolate ADS-B measurements. Of course, in dz and dr, $d\alpha$ there exists an error of approximation of x by $\hat{x}$. But I mean that this error is small.

Local approximation

An interpolation is necessary to make a vector $\hat{x}(t)$ for any t.

Hypothesis: ADS-B measurements z^{ads} and the airctaft motion have such a structure for τ that close to t:

$$z^{\text{ads}}(\tau) \approx x_0(t) + v_0(t)\,(\tau - t) + a_0(t)\,(\tau - t)^2 + w^{\text{ads}}(\tau)\,.$$

Here, $x_0(t)$, $v_0(t)$, $a_0(t)$ are the location, velocity, and acceleration of the aircraft motion at the instant t.

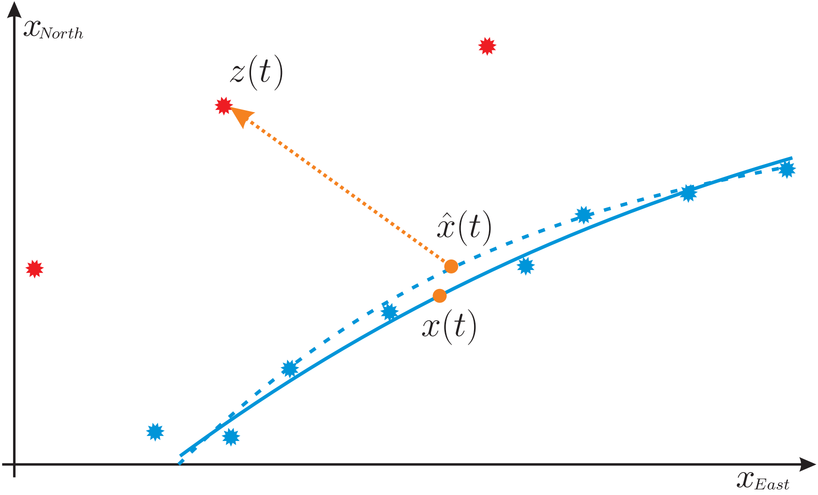
The interpolation is built by the least squares method. The estimation of $x_0(t)$ is used as $\hat{x}(t)$.

Words of speech

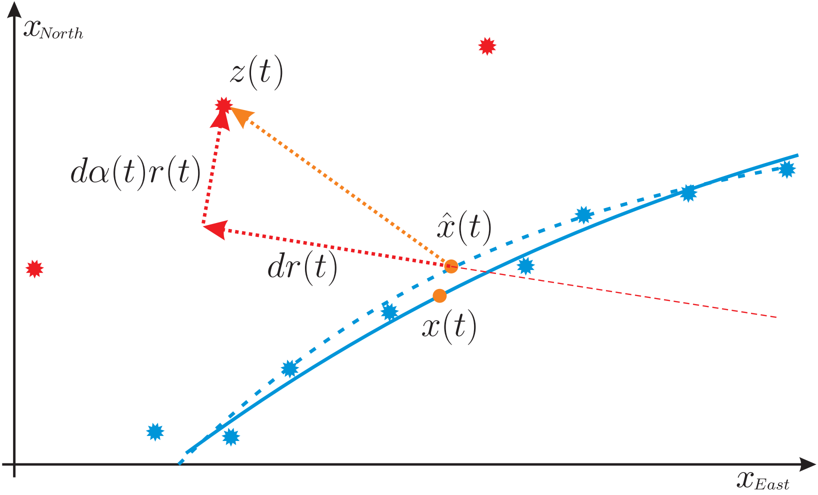
To make the estimaton $\hat{x}(t)$, I use the local approximation technique that requires only measurements of ADS-B whose instants are close to t (the instant of radar measurement). Usually, 5-6 measurements before and after t.

Using these measurements, I estimate $x(t)$ by means of a local curve with this simple structure. This simple model of the motion has small approximation error on short time intervals. The reconstruction of the curve parameters is carried out by the least squares method. And I take an estimation of $x_0(t)$ as the estimate $\hat{x}(t)$.

Making a residual vector



Making a residual vector



Words of speech

In this slide, I show you a process of making a residual vector at an instant t.

The solid blue line is the true trajectory of an aircraft. The dashed blue line is an approximation of the trajectory by the simple dynamics. The location of the aircraft at instant t is $x(t)$. And the measurement at this instant is $z(t)$. The estimate of $x(t)$ is denoted as $\hat{x}(t)$. The blue stars are the measurements of ADS-B and the red stars are the measurements of the radar.

The red dashed line shows the direction to the radar. So the orange residual vector expands to the sum of the radial residual vector with length dr and the transverse residual vector with length $d\alpha$ multiplied by r.

The variables that can affect the systematic errors

- The time t.
- The radar and aircraft. Each specific target and sensor can has its own characteristics.
- The range r and the azimuth α.
- The altitude h and the elevation angle β.
- The velocity v and its projections v_r, ω_α onto the transverse (azimuth) and radial (range) directions of the radar coordinate system.
- Another characteristics of the aircraft trajectoty.

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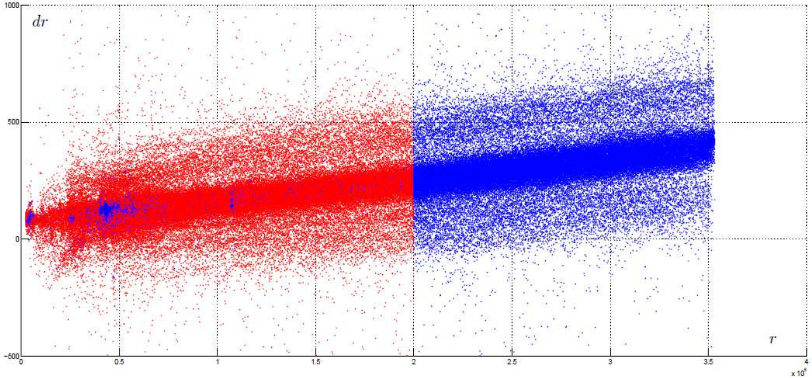
- Another characteristics of the aircraft trajectoty.

Words of speech

The possible variants of variables that can affect the residuals and systematic errors are emphasized in this slide. It can be the time, the specific identifier of sensor or target, the range and azimuth of the observed target, its altitude, its velocity or, may be, the projection of this velocity onto the transverse and radial direction of the radar coordinate system, and so on.

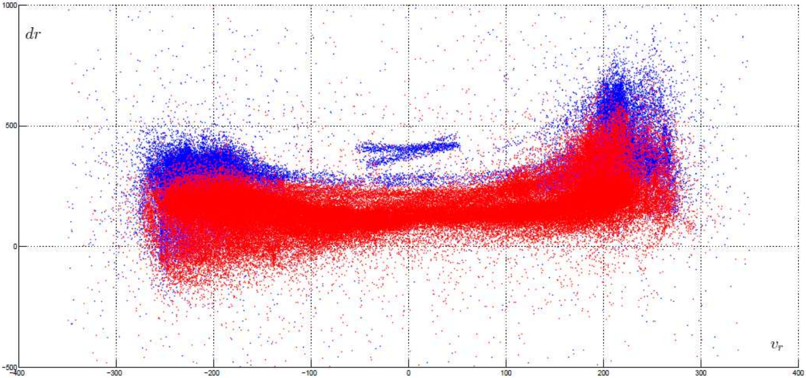
In experiments it has been shown significant influence on the residual values only by the range and azimuth, the radial velocity and the specific identifiers of the target and radar. The main tools for this analysis is the analysis of covariances of the residuals, the visual analysis of shapes of «clouds» of residual points in some coordinates.

The «clouds» of the range residuals dr



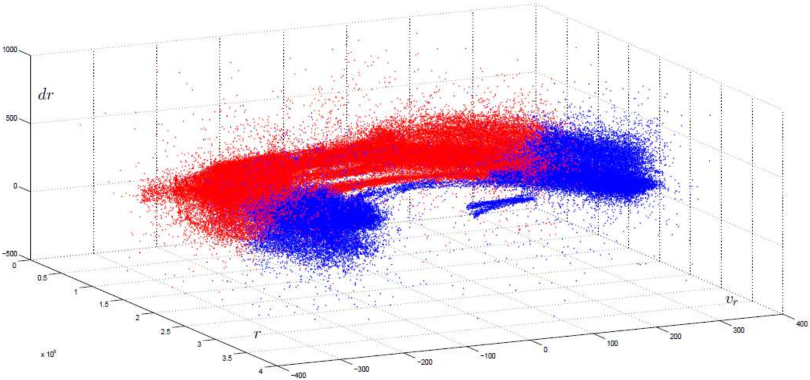
The dependence of dr on the range r is close to linear.

The «clouds» of the range residuals dr



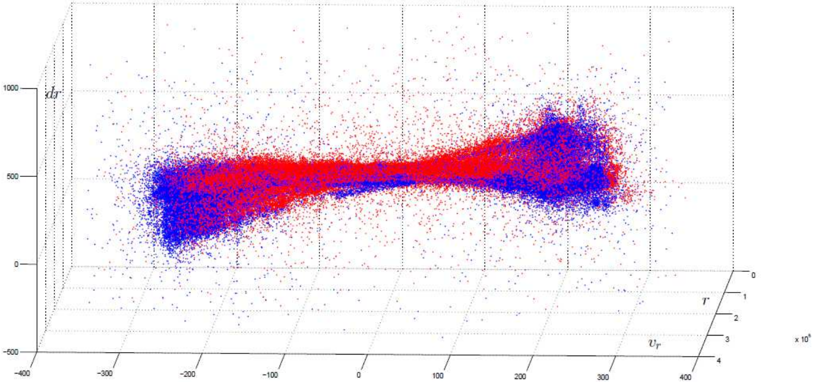
The dependence of dr on the range velocity v_r is linear for each trajectory.

The «clouds» of the range residuals dr



The dependence of dr on the range r and the range velocity v_r.

The «clouds» of the range residuals dr



The dependence of dr on the range r and the range velocity v_r.

The «clouds» of the range residuals dr

1. The range residuals dr depend strongly on the range r and the aircraft radial velocity v_r. There is no any dependence on the azimuth α or angular velocity ω_α.
2. The dependency on v_r is close to linear one. Each trajectory has its own slope coefficient.
3. The dependency on r is very close to affine. The slope coefficients are approximately equal for almost all radars.
4. A level of the random errors in the range residuals dr differs significally for distinct trajectories and radars. The standard deviation σ_r varies from 50 to 300 m.
5. There are no differences in the range residual dr properties for distinct radar devices (Primary, Secondary, and Combined Radars) that are placed to one radar site and one building.

Words of speech

In this slide you can see the «cloud» of residuals dr in the dr–r axes. All measurements are received from radars at one site and one building in Moscow ATC zone. The residuals dr for secondary radar have blue color. Red color marks the measurements of the radar of the combined (primary-secondary) type.

The pattern of measurements in the figure is like that these two devices works as one device.

Additionally, the affine type of the dependency of dr on the range r is clearly shown: the graph looks like it has an intersection with the dr axis at some level and linear growth in r. There are additional «clouds» of measurements above and below the graph. They go parallely to the main trend line.

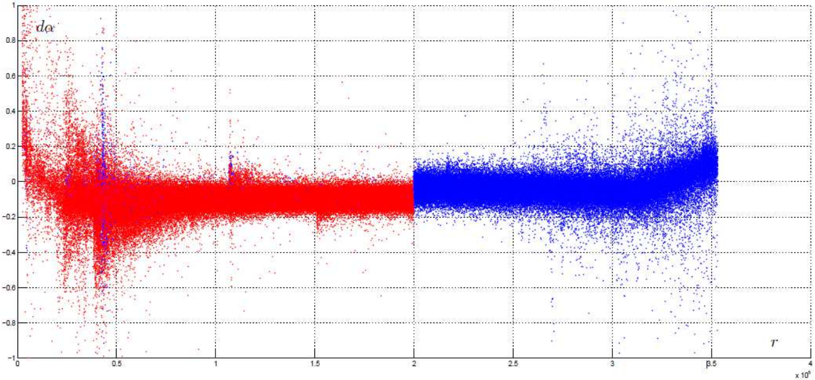
Words of speech

This figure shows a graph of the dependency of dr on the radial velocity v_r. And it explains the phenomenon of additional «clouds» in the previous figure. It is shown that there are measurements with different «reaction» on the same v_r level. The «cross» in the middle of the figure consists of measurements of two distinct trajectories: each trajectory has different slope of the graph with respect to v_r.

In this figures three-dimensional graphs of the dependency of dr on r and v_r are shown. They emphasize the deviation in dr as a funcion of v_r. The graph looks like a bundle.

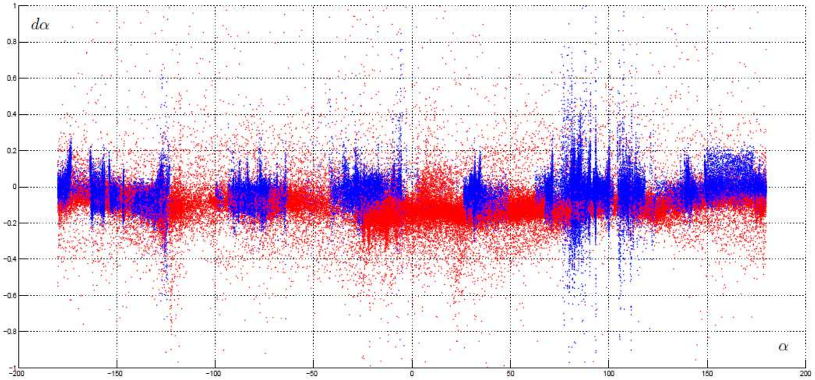
In this slide the short summary is shown. ...

The «clouds» of the azimuth residuals $d\alpha$



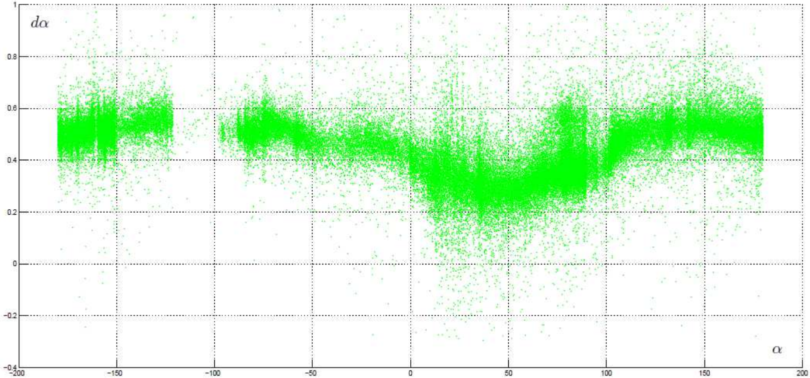
The typical dependency of $d\alpha$ on range r.

The «clouds» of the azimuth residuals $d\alpha$



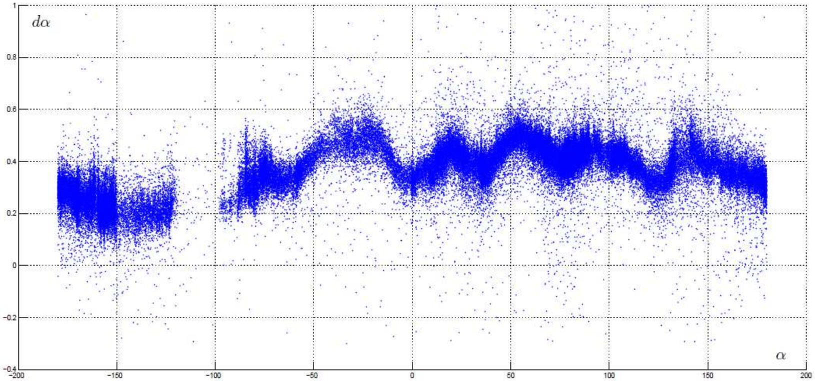
The dependency of $d\alpha$ on azimuth α at the same radar.

The «clouds» of the azimuth residuals $d\alpha$



A complicated dependency of $d\alpha$ on azimuth α at other primary radar.

The «clouds» of the azimuth residuals $d\alpha$



A complicated dependency of $d\alpha$ on azimuth α at other secondary radar.

The «clouds» of the azimuth residuals $d\alpha$

1. The azimuth residuals $d\alpha$ depend on the range r weakly. The slope is not significant for distances up to 320 k. For greater distances there exists significant slope for a number of radars.
2. The absence of dependency of $d\alpha$ on azimuth α is the proper model for many radars. But there exist a number of radars with a very complicated dependence. The shape of the function $d\alpha(\alpha)$ is not close to sine funcion.
3. For a number of radars, there exist some differences in the properties of the azimuth residual $d\alpha$ for distinct radar devices (Primary, Secondary, and Combined Radars) that are placed to one radar site and one building.

Words of speech

In figure, the graph of dependency of $d\alpha$ on the range r is shown for radar devices at one site and building in Moscow ATC zone. In this figure, colors of points mean the same as for graphs for dr.

The measurements of both SSR and combined radars have approximately constant (flat) type of dependency of $d\alpha$ on the range r. But we can see the jump in the level of $d\alpha$ at the point where the measurements of the combined radar change to the SSR measurements near at 200 km. At distance 320 km, the SSR measurements change their character — the residuals in $d\alpha$ have almost linear growth farther than 320 km.

The radar devices at this site do not show the dependency of $d\alpha$ on α. But the radar devices at another site in the next figures do show. The green color denotes the PSR measurements and blue color is used for the SSR measurements. There is a significant difference in behavior of these two devices. Both graphs show a complicated pattern of the dependency on α.

Again, short summary is shown. . . .

Besada model

Consider a model suggested by Besada et al. in article

Besada, J., de Miguel, G., Tarrio, P., Bernardos, A., and Garcia, J., Bias estimation for evaluation of ATC surveillance systems, *Information Fusion, 2009. FUSION '09. 12th International Conference on*, IEEE, 2009, pp. 2020 – 2027.

Besada model

$$\Delta_i^r = \Delta_{i0}^r + r\,c_i^r + v_{rj}\Delta_{ij}^r\,,$$
$$\Delta_i^\alpha = \Delta_{i0}^\alpha + c_i^{\mathrm{ecc}}\cos\left(\alpha - \alpha_{i0}\right)\,.$$

Here,

- Δ_i^r and Δ_i^α are the range and the azimuth components of the systematic errors of the radar i;
- Δ_{i0}^r and Δ_{i0}^α are the constant levels of these systematic errors;
- c_i^r is the linear part of the range systematic error;
- v_r, ω_α are the range velocity and the azimuthal angular velocity of the jth target;
- c_i^{ecc} is the coefficient of radar eccentricity, α_{i0} is the azimuth of maximal azimuth error;
- Δ_{ij}^r is the specific bias of jth aircraft, different for each aircraft. It can be induced by transponder.

Words of speech

This is a generally accepted model suggested by Besada et alii in their article.

In this model there are some basic constant levels Δ_0^r and Δ_0^α of the systematic errors in range and azimuth. The range component has additionally a linear slope. The coefficient of this slope is c_i^r.

The azimuthal component Δ^α has additionally an eccentricity part that describes radar distortion of sine shape. Some researchers as, for example, Pchelintsev consider that the reason of this eccentricity is the Azimuth Change Pulse encoder errors.

The value Δ_{ij}^r is the specific bias of jth aircraft, different for each aircraft. As Besada writes, it can be induced by transponder.

Residual model

$$dr_i = \Delta_{i0}^r + r\,c_i^r + v_{rj}\Delta_{ij}^r + v_{rj}\Delta_j^t\,,$$
$$d\alpha_i = \Delta_{i0}^\alpha + c_i^{\mathrm{ecc}}\cos\left(\alpha - \alpha_{i0}\right) + \omega_{\alpha j}\Delta_j^t\,.$$

Here,

- Δ_{ij}^r is the specific bias of the jth aircraft, different for each aircraft. It can be induced by transponders.
- $\Delta_j^t \geqslant 0$ is the time bias of the ADS-B measurements.

Residual model

$$dr_i = \Delta_{i0}^r + r\, c_i^r + v_{rj}\Delta_{ij}^r + v_{rj}\Delta_j^t - v_{rj}\Delta_i^t\,,$$
$$d\alpha_i = \Delta_{i0}^\alpha + c_i^{\text{ecc}} \cos\left(\alpha - \alpha_{i0}\right) + \omega_{\alpha j}\Delta_j^t - \omega_{\alpha j}\Delta_i^t\,.$$

Here,

- Δ_{ij}^r is the specific bias of the jth aircraft, different for each aircraft. It can be induced by transponders.
- $\Delta_j^t \geqslant 0$ is the time bias of the ADS-B measurements.
- Δ_i^t is the time bias of the radar.

Residual model

$$
\begin{aligned}
dr_i &= \Delta^r_{i0} + r\, c^r_i + v_{rj}\Delta^r_{ij} + v_{rj}\Delta^t_j - v_{rj}\Delta^t_i\,,\\
d\alpha_i &= \Delta^\alpha_{i0} + c^{\mathrm{ecc}}_i \cos\left(\alpha - \alpha_{i0}\right) + \omega_{\alpha j}\Delta^t_j - \omega_{\alpha j}\Delta^t_i\,.
\end{aligned}
$$

Here,

- Δ^r_{ij} is the specific bias of the jth aircraft, different for each aircraft. It can be induced by transponders.
- $\Delta^t_j \geqslant 0$ is the time bias of the ADS-B measurements.
- Δ^t_i is the time bias of the radar.

Residual model

$$dr_i = \Delta_{i0}^{r} + r\, c_i^{r} + \widetilde{\Delta}_{ij}\ ,$$
$$d\alpha_i = \Delta_{i0}^{\alpha} + c_i^{\mathrm{ecc}} \cos\left(\alpha - \alpha_{i0}\right)\ .$$

Here,

- $\widetilde{\Delta}_{ij}$ is the term that reflects bias specific to the ith radar and jth aircraft.

Residual model

$$dr_i = \Delta_{i0}^{r} + r\, c_i^{r} + v_{rj}\Delta_j^{t} - v_{rj}\Delta_i^{t}\,,$$
$$d\alpha_i = \Delta_{i0}^{\alpha} + c_i^{\mathrm{ecc}}\cos\left(\alpha - \alpha_{i0}\right)\,.$$

Here,

- $\Delta_j^t \geqslant 0$ is the term that reflects specific bias of the jth aircraft, different for each aircraft. It can be induced by the transponder or time bias of the ADS-B measurements.
- Δ_i^t is the term that reflects specific bias of the ith radar and may be connected with its time bias.

Residual model

$$dr_i = \Delta^r_{i0} + r\,c^r_i + v_{rj}\Delta^t_j - v_{rj}\Delta^t_i\,,$$
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Here,

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- Δ^t_i is the term that reflects specific bias of the ith radar and may be connected with its time bias.

Residual model

$$dr_i = \Delta_{i0}^r + r\, c_i^r + v_{rj}\Delta_j^t - v_{rj}\Delta_i^t\,,$$
$$d\alpha_i = L_i^\alpha(r) + c_i^{\mathrm{ecc}} \cos\left(\alpha - \alpha_{i0}\right)\,.$$

Here,

- $\Delta_j^t \geqslant 0$ is the term that reflects specific bias of the jth aircraft, different for each aircraft. It can be induced by the transponder or time bias of the ADS-B measurements.
- Δ_i^t is the term that reflects specific bias of the ith radar and may be connected with its time bias.
- A complicated dependency function (r in km):

$$L(r) = \begin{cases} (1-c)\Delta_{\alpha 0} + c\Delta_{\alpha 320}\,, & r \in [0, 320]\,, \quad c = \frac{r}{320}\,, \\ (1-c)\Delta_{\alpha 320} + c\Delta_{\alpha 400}\,, & r > 320\,, \quad c = \frac{r-320}{400-320} \end{cases}$$

Residual model

$$dr_i = \Delta_{i0}^r + r\,c_i^r + v_{rj}\Delta_j^t - v_{rj}\Delta_i^t\,,$$
$$d\alpha_i = L_i^\alpha(r) + c_i^{\mathrm{ecc}}\cos\left(\alpha - \alpha_{i0}\right).$$

Here,

- $\Delta_j^t \geqslant 0$ is the term that reflects specific bias of the jth aircraft, different for each aircraft. It can be induced by the transponder or time bias of the ADS-B measurements.
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- A complicated dependency function (r in km):

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Residual model

$$
\begin{aligned}
dr_i &= \Delta_{i0}^r + r\, c_i^r + v_{rj}\Delta_j^t - v_{rj}\Delta_i^t\,,\\
d\alpha_i &= L_i^\alpha(r) + f_i(\alpha)\,.
\end{aligned}
$$

Here,

- $\Delta_j^t \geqslant 0$ is the term that reflects specific bias of the jth aircraft, different for each aircraft. It can be induced by the transponder or time bias of the ADS-B measurements.
- Δ_i^t is the term that reflects specific bias of the ith radar and may be connected with its time bias.
- A complicated dependency on r (r in km):

$$
L(r) = \begin{cases} (1-c)\Delta_{\alpha 0} + c\Delta_{\alpha 320}\,, & r \in [0, 320]\,, \quad c = \frac{r}{320}\,,\\ (1-c)\Delta_{\alpha 320} + c\Delta_{\alpha 400}\,, & r > 320\,, \quad c = \frac{r-320}{400-320} \end{cases}
$$

- A function $f_i(\alpha)$ reflects a complicated dependency on α for some radars.

Words of speech

If we consider a residual between the radar measurement and the interpolated ADS-B measurement, it consists of all parts of the radar errors minus errors that are specific to ADS-B. The latter consists of a spatial bias that reflects influence of the time bias Δ^t of ADS-B. This time bias Δ^t_j can be specific for each aircraft.

Such a term can exist for radar time biases too. In this slide, it denotes as Δ^t_i.

The «blue» terms Δ^r_{ij}, Δ^t_i, Δ^t_j influence the residual in the same way and their contribution to the output are mixed. The influence of the red terms on the azimuth residuals $d\alpha$ are negligeable so they can be removed from the formula. The influence of the blue terms on the range residuals dr can be described as the influence of one combined parameter $\widetilde{\Delta}_{ij}$.

Words of speech

In the experiments, it has been established that the coefficient $\widetilde{\Delta}_{ij}$, nevertheless, can be devided into two coefficents Δ_i^t, Δ_j^t without losses of the fitting quality.

The coefficient Δ_j^t is the term that reflects specific bias of the jth aircraft, different for each aircraft. It can be induced by the transponder or time bias of the ADS-B measurements. And nobody can distinguish its true source. The coefficient Δ_i^t is the term that reflects specific bias of the ith radar and may be connected with its time bias.

Words of speech

As we can see in slides with «clouds» for $d\alpha$, the dependence of $d\alpha$ on the range r can be more complicated than a constant Δ_0^α. A function $L(r)$ provides a better fitting for some radars. It is the continous piecewise linear function with a knot at 320 km.

The last peculiarity that I want to consider in the model is the dependence of $d\alpha$ on α. For a number of radars, this dependence is complicated and it is hardly can be described. It is not the sine function. It seems to be like a sawtooth and cannot be presented in a parametric way. I think, if this type of dependence is founded out for a radar, it will be applicable to use semiparametric algorithms to recover it.

So, I can say that my main contribution to the problem of the model selecion for the radar systematic error is general confirmation of the correctness of the model suggested by Besada and finding out some peculiarities of radar distortions.

Words of speech

That's all