

Description of the poster

The work is devoted to 3D-printing of reachable sets *at* a fixed instant and ones *till* an instant (or maybe in English it is better to say “up to an instant”) for the simplest “mathematical car” that is known as the “Dubins car”.

The coordinates x and y correspond to the geometric position of the object in the plane, the coordinate θ is the angle of the velocity vector direction. In mathematical investigations, it is useful to consider this angular coordinate in the interval $(-\infty, \infty)$, but not in the interval $[-\pi, \pi)$. The magnitude of the velocity is constant. We assume it to be equal to 1 in the normalized variables. We define admissible controls as piecewise-continuous functions $t \rightarrow u(t)$ where the magnitude of $u(t)$ is restricted by the constraint $|u(t)| \leq 1$ (the symmetric case), or by the constraint $u(t) \in [b, 1]$ (the nonsymmetric case). In the second variant, the fixed parameter b is taken from the interval $[-1, 0)$. The initial instant is equal to zero. The initial phase position is at the origin. The reachable set $G(T)$ *at* an instant T is the union of all three-dimensional phase positions (x, y, θ) such that it is possible to guide the system to any of them at the instant T by means of some admissible control. The reachable set $G^*(T)$ *till* an instant T is the union of the sets $G(t)$ for $t \in [0, T]$.

In Fig. 1, the works by A.A. Markov (1989), R. Isaacs (1951), and L.E. Dubins (1967) are mentioned. There, some problems with the simplest car are considered. In Fig. 3, reference [1] is the paper where the reachable sets at an instant are investigated in the projection to the plane of the geometrical coordinates. Book [2] is a collection of papers on problems with dynamics of the simplest car and also on problems for objects with some more complex dynamics. The reachable sets at an instant and ones till an instant have been analyzed for the Dubins car in papers [3, 4]. The optimal time problems for the Dubins car and for the Reads-Shepp one, and connected with them reachable sets till an instant are shortly explained in paper [5]. In the Reads-Shepp model, it is additionally admissible to use a control of the reverse motion. Book [6] is devoted to the principles and capabilities of the 3D-printing. Work [7] is a report on the 3D-printing of three-dimensional reachable sets till an instant for the Dubins car and Reads-Shepp one. The printing was elaborated at LAAS-CNRS by the student G. Caner under supervision of J.-P. Laumond. This work is made at the same time as ours. It uses another (in comparison with ours) mathematical description of boundaries of three-dimensional sets till an instant.

Representations of three-dimensional reachable sets at an instant and ones till an instant taken from papers [3, 4] are given in Figs. 4–9. In Fig. 6, the reachable sets at the instants $T = 4\pi$ and $T = 6\pi$ are made for the nonsymmetric constraint on control. The reachable sets at the instants $T = 1.5\pi$ and $T = 2\pi$ shown in Fig. 8 correspond to the case when the angle θ is considered modulo 2π , *i.e.*, in the interval $[-\pi, \pi)$. In Fig. 9, the reachable sets at the instants $T = 1.5\pi$ and $T = 2\pi$ are shown in the cylindrical coordinates. The colors of pieces of the boundary of the reachable sets correspond to different collections of the piece-constant controls that lead to the boundary from the initial position. The knowledge of the boundary structure is useful to form a very economic data for the 3D-printing. The reachable set till an instant differs from the one at an instant by some ridge with a cusp at the origin $x = 0, y = 0, \theta = 0$ (this ridge is shown by the blue color in Fig. 7). Representations from papers [3, 4] shown here are made with the help of VRML programming language.

We used a one-color 3D-printer (Fig. 10). Some serious circumstances, which influence the 3D-printing process, are mentioned in Figs. 11–13.

Photos of 3D-printed reachable sets at an instant and ones till an instant are shown in Figs. 14–18. In Fig. 14, the reachable sets at an instant are shown for three instants. During construction of the reachable sets, because of its complicated shape, it is necessary to use some special supports (Fig. 15). The distinction of reachable sets at an instant and ones till an instant is evidently seen in Fig. 16. With increasing T , the distinction is disappeared: for sufficiently large T , the reachable set at an instant coincides with the one till the corresponding instant. Fig. 17 gives a photo of a nonsymmetric reachable set at the instant $T = 5\pi$. During mastering the process of the reachable sets 3D-printing, many test samples were made. These reachable sets can be used as new-year toys. The photo of a new year tree with such toys is shown in Fig. 18.