Nonlinear control system of car motion (Dubins Car)

Markov, A. A. (1889). Some examples of the solution of a special kind of problem on greatest and least quantities, Soobacenija Charkovskogo matematiceskogo obsesetva, Vol. 2-1 (No. 5,6), 250–276 (in Russian).

$$\dot x = \cos\theta, \qquad |u| \le 1,$$
$$\dot y = \sin\theta, \qquad u \in [b, 1],$$
$$\dot\theta = u; \qquad b = \text{const} \in [-1, 0).$$

Isaacs, R. (1951). Games of pursuit, Scientific report of the RAND Corporation, Santa Monica.
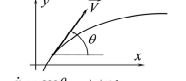
Reachable set at the instant T:
$$G(T) = \bigcup_{u(\cdot)} \begin{pmatrix} x(T) \\ y(T) \\ \theta(T) \end{pmatrix},$$

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Volume LXXIX, Number 3 JULY, 1957, pp 497–516
ON CURVES OF MINIMAL LENGTH WITH A CONSTRAINT ON AVERAGE CURVATURE, AND WITH PRESCRIBED INITIAL AND TERMINAL POSITIONS AND TANGENTS.*
By L. E. DUBINS.
* Received April 29, 1956; revised January 3, 1957.

Reachable set till the instant T:
$$G^*(T) = \bigcup_{0 \le t \le T} G(t).$$

Reachable Sets for Dubins Car in Control Problems: Physical Visualization

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References

1 Cockayne E. J., Hall G. W. C. Plane motion of a particle subject to curvature constraints. SIAM J.Control. Vol. 13, No. 1, pp. 197–220, 1975.

2 Laumond J.-P. (ed.) Robot Motion Planning and Control. Lect. Notes in Contr. and Inform. Sci. Vol. 229. Springer, New York, 1998.

3 Patsko V. S., Pyatko S. G., Fedotov A. A. Three-dimensional reachability set for a nonlinear control system. Journal of Computer and Systems Sciences International. Vol. 42, No. 3, pp. 320–328, 2003.

4 Fedotov A. A., Patsko V. S., Turova V. L. Reachable sets for simple models of car motion. Ed.by A. V. Topalov. Rijeka: InTech Open Access Publisher, pp. 147–172, 2011.

5 Laumond J.-P., Mansard N., Lasserre J.-B. Optimality in robot motion: Optimal versus optimized motion. Communications of the ACM. Vol. 57, No. 9, pp. 82–89, 2014.

6 Horvath J. Mastering 3D Printing (1st edition). Berkely, CA, USA: Apress, 2014.

7 Caner G. Modelisation en 3D de la boule de Reeds& Shepp. Rapport de stage d'option, Mines Paristech, LAAS-CNRS, 2015.
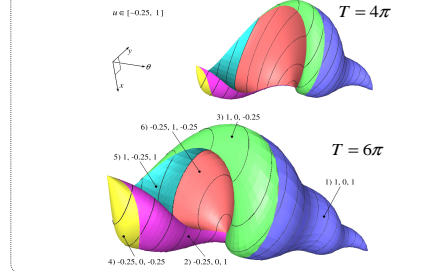

The reachable set at the instant from two vision angles

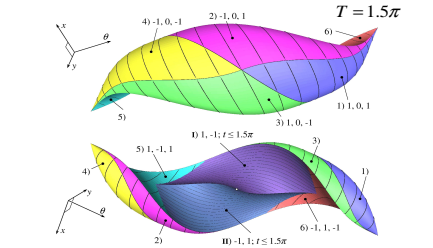control collections and corresponding surfaces on the boundary

$T = 1.5\pi$
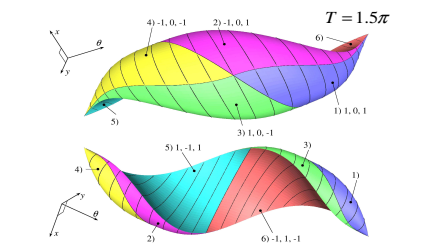
Dubins Car. Growth of the reachable set at the instant

$T = \pi$, $T = 2\pi$, $T = 3\pi$, $T = 4\pi$
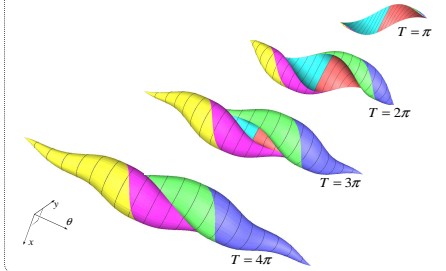
Reachable sets at the instant for nonsymmetric constraint

$u \in [-0.25, 1]$, $T = 4\pi$, $T = 6\pi$

The reachable set till the instant from two vision angles

control collections and corresponding surfaces on the boundary
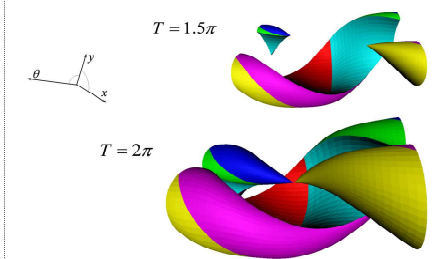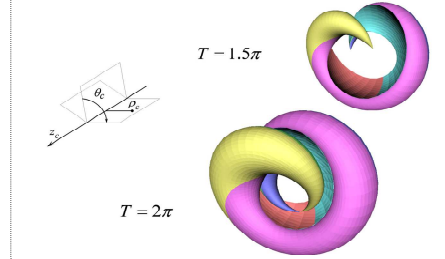

$T = 1.5\pi$

Reachable sets at the instant with θ computed modulo 2π

$T = 1.5\pi$, $T = 2\pi$

Reachable sets at the instant in cylindrical coordinates

$T = 1.5\pi$, $T = 2\pi$

3D printing parameters

| Material for printing	ABS-plastic PLA-plastic diameter of the filament 1.75 mm
Size of work space	120 mm x 120 mm x 120 mm
Thickness of the layer	**0.20**/ 0.25/ 0.30/ 0.35 mm
Dimensions	240 mm (width) 340 mm (length) 355 mm (height)
Weight	6 kg
Requirements for power	100-240 V, 50-60 Hz, 220W

Some problems of a model

- Correct orientation of normals .
- Holes in the grid, "water-impermeability" .
- Internal facets.
- Overlapping facets.
- Common edges.
- Geometry of zero thickness.

Peculiarities of the FDM-technology of printing

- Wall's thickness.
- Orientation of the object.
- Overhanging elements.
- Accuracy.
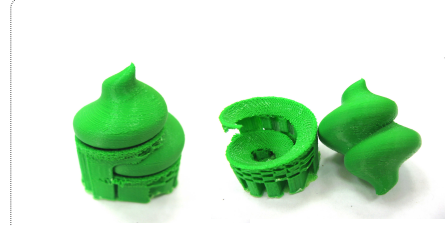- Termo-changing the geometry.

Overhanging elements

- For each overhanging element a supporting construction (the support) is necessary.
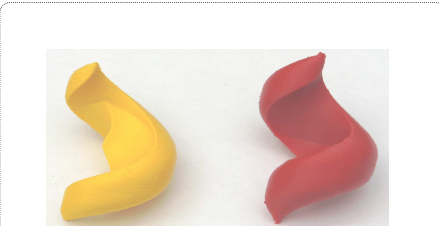- The support deforms the surface touching it.
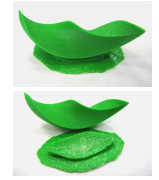
Example of stable support during printing for the reachable set till the instant $T = \pi$
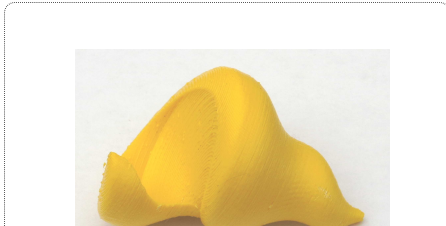

Reachable sets at the instant for $T = 2\pi$, 3π, 4π

Example of stable support during printing for the reachable set at the instant $T = 4\pi$

Reachable sets till the instant (left) and at the instant (right) for $T = 1.5\pi$

Reachable set at the instant $T = 5\pi$ in the case $u \in [-0.25, 1]$

New year tree with 3D reachable sets