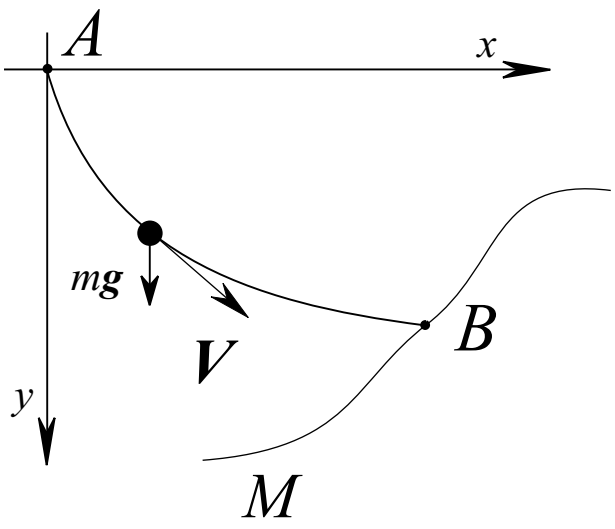
The classical brachistochrone problem



Velocity of the mass point:

$$V(x,y)=\sqrt{2gy}$$

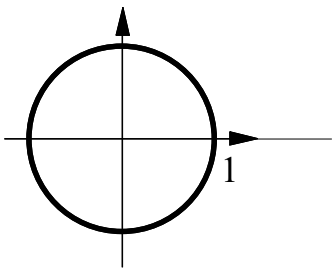
Controlled system:

$$\dot{x}=\sqrt{2gy}\,u_1$$

$$\dot{y}=\sqrt{2gy}\,u_2$$

Admissible controls :

$$u=(u_1,u_2)\in P=$$



Space of states:

$$N=\{(x,y)\in \mathrm{R}^2 : y\geq 0\}$$

t_f is the time of attaining the terminal set M

Aim of control: $t_f \to \min$