Problem of Vertical Passing an Obstacle by an Aircraft under Wind Disturbances

S.A. Ganebny[1], A.I. Krasov[2], V.S. Patsko[1], M.A. Smolnikova[3]

Abstract— **Methods of the differential game theory are applied to the problem of overpassing a vertical obstacle by an aircraft. An algorithm is suggested for constructing the aircraft control by the thrust and elevator that is adaptive with respect to the unknown level of a wind disturbance. The state constraint on the pitch angle is implicitly taken into account. Simulation results are presented.**

Keywords: adaptive control, differential games, aircraft control, wind disturbance.

I. Introduction

Aircraft taking-off and landing stages are the most sensitive to wind disturbances. Application of modern methods of the mathematical control theory and differential game theory to the taking-off, landing, and abort landing problems in presence of wind disturbances was stimulated by papers [16], [17], [18] of A. Miele and his collaborators, and also by works [10], [11] of V.M. Kein. Among the investigations on this topic, let us note the articles [1], [2], [3], [4], [12], [15], [19], [20], [21].

The main peculiarity of the mentioned works is an attempt to refuse traditional schemes for constructing autopilots on the basis of PID-regulators. New methods use non-linear control laws and take into account constraints for control mechanisms more adequately.

For testing new algorithms, wind disturbance was taken quite hard: it was generated by a wind microburst [5], [7], [16]. For numerical experiments, either the field of wind velocity is given, or limits for wind velocity components are prescribed only. In the first case, the results obtained are treated as potentially possible. In the second one, a problem of guaranteed control is considered.

The problem of overpassing a vertical obstacle is close to the taking-off and abort landing problems. The peculiarity of our formulation is that no constraint for the wind disturbance is specified in advance. Suggested feedback control automatically tunes its level to the current disturbance level, and, therefore, is adaptive. This control is constructed on the basis of the antagonistic differential game theory [13], [14]. The facilitating thing is the possibility to pass (after linearization of the original nonlinear dynamics) to a linear differential game with one-dimensional phase variable.

In simulations, we suppose that all state variables are measured exactly.

We use variable denotations, which are usual in Russian literature on aviation.

II. Problem formulation

The problem of preventing collision of an aircraft with an earth surface obstacle is illustrated in Fig. 1. Avoidance is implemented by a vertical maneuver, so the aircraft motion is considered in the vertical plane. Sudden wind blasts are possible.

Simulations are made in the framework of nonlinear ordinary differential system of the eighth order. The system is taken from [19] with the variables corresponding to the lateral channel set to zero. The state variables of the vertical channel:

- x_g, y_g are the longitudinal and vertical coordinates of the aircraft mass center, m;
- V_{xg}, V_{yg} are the longitudinal and vertical velocities, m/sec;
- ϑ, ω_z are the pitch angle and its angular velocity, rad and rad/sec;
- P is the thrust force, N;
- δ_e is the elevator deflection, rad.

The control is implemented by the command (setting) signals δ_{ps} and δ_{es} of the thrust force and elevator, respectively. The disturbances affecting the system are the longitudinal W_{xg} and vertical W_{yg} components of the wind velocity.

The crucial requirement is in taking into account the pitch angle state constraint. But this angle is a state coordinate and its direct limitation is impossible in the framework of the using approach. Thus, we introduce a new fictive control, namely, the "prescribed pitch angle", on which the necessary

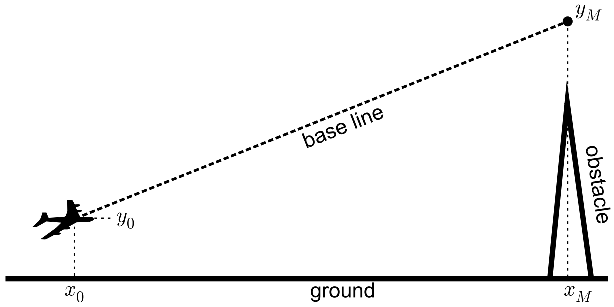

Fig. 1. Obstacle overpassing problem

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constraint is put. The elevator control is defined by the linear dependence

$$\delta_{es} = k_1(\vartheta - u_\vartheta) + k_2(\widehat{V} - \widehat{V}_0) + k_3\omega_z, \qquad (1)$$

where k_1, k_2, and k_3 are nonnegative coefficients; $\widehat{V}$, $\widehat{V}_0$ are the current and nominal aerial (relative) velocities; u_ϑ is the new control, which can be treated as the prescribed pitch angle.

Control law (1) directs the real pitch angle to the prescribed one.

It is necessary to construct the thrust force and prescribed pitch angle controls guaranteeing the passage over the obstacle. With that, the controls must "softly" react to the wind disturbance, i.e., if the wind disturbance is weak, then the parrying controls must be weak also. We call *adaptive* the control that adjusts to the actual disturbance level.

We assume that the obstacle is located at the point x_M on the horizontal axis x_g and the aircraft altitude at the obstacle overpassing instant must be not smaller than the given value y_M, which takes into account the obstacle height and the requirement of the minimal reserve excess.

III. Nonlinear dynamics

Nonlinear dynamics of the aircraft vertical channel is described as follows:

$$\begin{aligned}
\dot{x}_g &= V_{xg}, \\
\dot{V}_{xg} &= [(P\cos\sigma - qSc_x)\cos\vartheta - \\
&\quad (P\sin\sigma + qSc_y)\sin\vartheta)]/m, \\
\dot{y}_g &= V_{yg}, \\
\dot{V}_{yg} &= [(P\cos\sigma - qSc_x)\sin\vartheta + \\
&\quad (P\sin\sigma + qSc_y)\cos\vartheta]/m - g, \\
\dot{\vartheta} &= \omega_z, \\
\dot{\omega}_z &= M_z/I_z, \\
\dot{P} &= k_p(\delta_{ps} - P), \\
\dot{\delta}_e &= k_e(\delta_{es} - \delta_e).
\end{aligned} \qquad (2)$$

Here,

- $q = \rho\widehat{V}^2/2$ is the dynamic pressure, $\mathrm{kg\,m^{-1}sec^{-2}}$;
- $\rho = 1.207$ is the air density, $\mathrm{kg\,m^{-3}}$;
- $\widehat{V} = (\widehat{V}_{xg}^2 + \widehat{V}_{yg}^2)^{1/2}$ is the relative velocity, m/sec;
- $\widehat{V}_{xg} = V_{xg} - W_{xg}$, $\widehat{V}_{yg} = V_{yg} - W_{yg}$ are components of the relative velocity vector, m/sec;
- $I_z = 6.5 \cdot 10^6$ is the inertia moment, $\mathrm{kg\,m^2}$;
- $M_z = qSbm_z$ is the aerodynamic moment, $\mathrm{N\,m}$;
- m_z is the aerodynamic moment coefficient;
- c_x, c_y are the aerodynamic force coefficients;
- $S = 201$ is the reference surface, $\mathrm{m^2}$;
- $b = 5.285$ is the mean aerodynamic chord, m;
- $m = 75 \cdot 10^3$ is the aircraft mass, kg;
- $g = 9.81$ is the acceleration of gravity, $\mathrm{m/sec^2}$;
- $\sigma = 1.72$ is the thrust inclination, grad;
- $k_p = 1$, $k_e = 4$ are time constants, 1/sec.

Aerodynamic coefficients of forces and moments are specified by the following relations [19]:

$$\begin{aligned}
c_x &= \tilde{c}_x\cos\alpha - \tilde{c}_y\sin\alpha, \\
c_y &= \tilde{c}_y\cos\alpha + \tilde{c}_x\sin\alpha, \\
\tilde{c}_x &= 0.21 + 0.004\alpha + 0.47\cdot 10^{-3}\alpha^2, \\
\tilde{c}_y &= 0.65 + 0.09\alpha + 0.003\delta_e, \\
m_z &= 0.033 - 0.017\alpha - 0.013\delta_e + \\
&\quad 0.047\delta_{st} - 1.29\omega_z/\widehat{V}.
\end{aligned}$$

Here, δ_{st} is the tailplane position, α is the attack angle. In these formulas, all angular values must be taken in degrees.

The attack angle is computed as follows:

$$\alpha = \arcsin[(\widehat{V}_{xg}\sin\vartheta - \widehat{V}_{yg}\cos\vartheta)/\widehat{V}].$$

Numerical data is given for Tupolev Tu-154 aircraft, but the system can be used for any midsize transport aircraft.

IV. Linearization with respect to the base line

At the beginning instant of the avoidance maneuver, the base line is built in the geometric coordinates x_g, y_g. It connects the initial aircraft position (x_{g0}, y_{g0}) and a point (x_M, y_M) over the obstacle.

Further, the nominal motion parameters are computed; this motion corresponds to ascent along the prescribed base line. The vector of difference coordinates x is introduced; it is the deviation of the current position of system (2) from the nominal vector on the base line at the corresponding instant.

Linearization of the original nonlinear system is carried out with respect to the motion along the prescribed line:

$$\dot{x} = Ax + Bu + Cw. \qquad (3)$$

In the linear system obtained, the two-dimensional control vector u is composed of the following components: u_p is the command deviation of the thrust value from the nominal one ($u_p = \delta_{ps} - P_0$); u^* is the prescribed pitch angle deviation from the nominal one ($u^* = u_\vartheta - \vartheta_0$). The vector w of the wind disturbance is composed of components ΔW_{xg} and ΔW_{yg} that are counted from their nominal values.

Note that the linear system is used for the adaptive control construction, but the aircraft motion simulation is implemented in the original nonlinear system.

V. One-dimensional differential game

Consider an auxiliary linear differential game with dynamics (3) and fixed terminal time that can be regarded equal to zero, for convenience. The objective of the first player governing the control u is to make the component Δy_g of the state vector x nonnegative at the termination instant. The upper constraint for the component Δy_g desirable for the first player will be introduced later.

The prescribed pitch angle u_ϑ in the nonlinear system is bounded by $0° \le u_\vartheta \le 20°$, and, respectively, the control u^* in the linear system (since this system is written in deviations) changes in the bounds $\mu_1 = 0° - \vartheta_0 \le u^* \le 20° - \vartheta_0 = \mu_2$.

The command value u_p of the thrust force in the linear system obeys the inequality $0 \le u_p \le \mu_p$, where $\mu_p =$

$1.2m$, m is the mass of the aircraft. Explain these bounds in the following way. The thrust force P_0 corresponds to the nominal motion along the ascending base line. The reserve of the thrust is described by the relation $(P_{\max} - P_0)/m = 1.2$, i.e., $P_{\max} - P_0 = 1.2m$. This magnitude is taken as the bound μ_p. The lower value of u_p, which equals zero, means that the thrust must not be smaller than the nominal value during the obstacle overpassing maneuver.

So, we described the compact constraint for the useful control u in system (3). We do not consider *a priori* any constraint for the disturbance w.

Using the standard transformation [13], [14], let us pass to one-dimensional dynamics

$$\dot{\xi} = D(t)u + E(t)w, \quad t \leqslant 0,$$
$$\xi(t) = \Phi_3(0,t)x(t), \qquad (4)$$
$$D(t) = \Phi_3(0,t)B, \quad E(t) = \Phi_3(0,t)C,$$

where $\Phi_3(0,t)$ is the third row of the fundamental Cauchy matrix of the system $\dot{x} = Ax$. The state variable ξ can be treated as the altitude prognosis at the obstacle overpassing instant.

VI. Constructing the main bridge

Take some reasonable possible constraint $Q_{\max}$ for the wind disturbance w (that is, the second player control).

Let $\tau = -t$ be the backward time. In the space $\tau \times \xi$, consider two families of semipermeable [6] curves (Fig. 2). Lines in each family differ from each other only by the vertical shift. Lines of the family I guarantee that the wind disturbance providing the maximal down shift to system (4) under the constraint $w \in Q_{\max}$ can be parried by the first player's control such that the motion will not cross any chosen line in the downward direction. Respectively, lines of the family II guarantee that for the wind providing the maximal upper shift, a control of the first player can be found such that it does not allow the motion to cross these curves in the upward direction. Any pair of nonintersecting semipermeable lines from these families generates a weak-invariant set, i.e., an arbitrary wind disturbance with the values from the set $Q_{\max}$ is parried by the first player control in such a way that the motion does not go out the considered invariant set. By terminology of books [13], [14], such a set is a *stable bridge*. A line from the family I establishes the bridge lower boundary, and a line from the family II establishes its upper boundary.

As the main bridge lower boundary, let us take the semipermeable line of the family I that passes through the point $\tau = 0$, $\xi = 0$. Remind that our problem is to construct a control, which provides a guarantee to guide the aircraft above this point. Denote such a line by $\Gamma(\tau)$ and its maximum by Γ_m (Figs. 2 and 3). The main bridge upper boundary is chosen from the family II such that it does not intersect the lower one. The exact way for choosing this upper boundary will be described in the next section.

By virtue of one-dimensionality of the state variable ξ, the main bridge construction is very simple.

Note again, that the constraint $Q_{\max}$ is taken only to describe the main stable bridge. The adaptive feedback control constructed in the next section is oriented to any (weak or strong) disturbance $w(t)$.

VII. Constructing the control

From the sense of the semipermeable curves, it follows that if the current position of system (4) is on the main bridge lower boundary, such a control has to be chosen that gives the maximal upper shift. Respectively, if the current position is on the upper boundary, such a control has to be chosen that provides the maximal lower shift. This gives the guarantee that if the initial position belongs to the bridge and the wind disturbance obeys the constraint $Q_{\max}$, the further motion will go inside the bridge and reach the segment above the aim point $\xi = 0$.

Thus, the control on the main bridge boundaries is described in a certain way and uses its maximal values. But inside the bridge, the control can be chosen in an arbitrary way. So, it is reasonable to use a switch line, on which the control changes its sign.

Take the horizontal switch line: $\tilde{z} = \tilde{z}(\tau) = \Gamma_m + \varepsilon$, where ε is some fixed distance from the switch line to the bridge lower boundary (Fig. 3). Choose the upper boundary $\bar{\Gamma}(\tau)$ such that there is the same distance ε from the switch line.

The horizontal switch line corresponds to the stable bridge with zero control of the first player and zero wind disturbance. Use the method [8], [9] of adaptive control based on

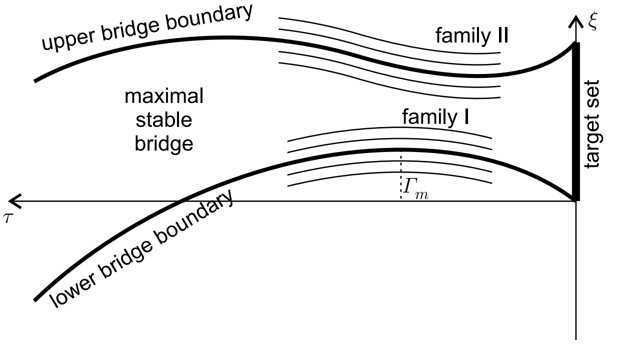

Fig. 2. Two families of the semipermeable lines; constructing the main bridge

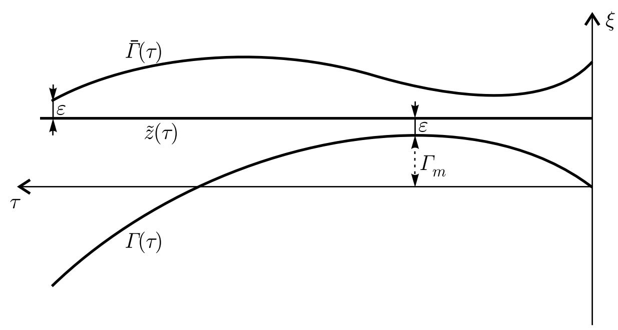

Fig. 3. Curves used for constructing the control

constructing stable bridges with the increasing constraints for the control and disturbance from their zero values and up to the maximal possible ones. With that, the elaborated control level adjusts to the actual disturbance level.

The controls u^* and u_p are chosen as

$$u^* = \begin{cases} \mu_2, & \xi(\tau) \le \Gamma(\tau), \\ \mu_2 \cdot \dfrac{\xi(\tau) - \tilde{z}}{\Gamma(\tau) - \tilde{z}}, & \Gamma(\tau) < \xi(\tau) < \tilde{z}, \\ \mu_1 \cdot \dfrac{\xi(\tau) - \tilde{z}}{\bar{\Gamma}(\tau) - \tilde{z}}, & \tilde{z} \le \xi(\tau) < \bar{\Gamma}(\tau), \\ \mu_1, & \xi(\tau) \ge \bar{\Gamma}(\tau), \end{cases} \quad (5)$$

$$u_p = \begin{cases} \mu_p, & \xi(\tau) \le \Gamma(\tau), \\ \mu_p \cdot \dfrac{\xi(\tau) - \tilde{z}}{\Gamma(\tau) - \tilde{z}}, & \Gamma(\tau) < \xi(\tau) < \tilde{z}, \\ 0, & \xi(\tau) \ge \tilde{z}. \end{cases} \quad (6)$$

These controls take their extremal values at the main bridge boundaries and change proportionally inside the bridge. Since one of the extremal values of the thrust force control is equal to zero, there is no changing the thrust force above the switch line, and the control maintains to be zero.

VIII. Numerical data and the problem formulation peculiarities

The coefficients k_1, k_2, and k_3 in control law (1) for the elevator setting are taken 1, 0.0075, and 0.2, respectively.

Let the aircraft move horizontally until the obstacle is detected. Its longitudinal velocity is 70 m/sec, the pitch angle, thrust force, and the elevator are in the nominal positions corresponding to the horizontal flight, and the wind disturbance is absent.

Suppose that the distance to the obstacle at the detection instant is 1400 m. The initial altitude is taken 30 m, the target altitude is 130 m.

We construct the base line, for which the new nominal longitudinal and vertical velocities are computed, on the basis of the condition that the modulus of the velocity is 70 m/sec. The new nominal pitch angle and thrust force are also computed. In computations, we suppose the nominal wind disturbance to be zero. The obstacle detection instant is supposed to be the beginning of simulation.

Despite the aircraft geometric coordinates at the obstacle detection instant are on the base line (i.e., in their nominal positions) the initial point in system (4) is not zero, since the pitch angle and the thrust force correspond to the previous values of the horizontal flight, which differ from the values of the motion along the base line. The thrust force changes to its new nominal value gradually, accordingly to its differential equation.

Since the initial distance to the obstacle is 1400 m and the nominal horizontal velocity is approximately 70 m/sec, the nominal time τ_0 to the encounter with the obstacle is about 20 sec.

The supposed bounds of the wind values are the following: for the longitudinal component, the constraint is $|\Delta W_{xg}| \le 10$ m/sec, for the vertical component, the constraint is $|\Delta W_{yg}| \le 5$ m/sec. These bounds define the constraint $Q_{\max}$ for the adaptive control method.

The time step of the discrete scheme for control construction and for simulation is 0.1 sec. The value ε is taken 3 m.

In control construction, we compute the prognosis backward time $\tilde{\tau} = \big(x_M - x_g(t)\big)/V_{xg}(t)$ and choose (at the discrete scheme instants) the controls u^* and u_p by formulas (5) and (6) taking $\tau = \tilde{\tau}$. Thus, we abandon the idealized assumption of fixation of the termination instant; we made this assumption in the consideration of auxiliary linear differential games (3) and (4).

IX. Wind microburst model

We shall suppose that during the obstacle overpassing the aircraft crosses a wind microburst zone. The wind microburst is a nature phenomenon appearing when an air down-flow strikes the ground and fluxes horizontally with creating the whirls [5]. When reaching the microburst zone, at first, the aircraft meets a headwind, which changes quite fast (within ten seconds) to a down-flow and further to a tail wind. Such a change is a very complex phenomenon from the point of view of the aircraft aerodynamics. The headwind increases the aircraft aerial velocity and, therefore, the lifting force; vice versa, the tail wind and the down-flow decrease it. So, such a change of the wind direction from the headwind to the tail one leads to a quick decrease of the lifting force.

For simulation, we use the wind microbust model from paper [7]. In the space, a torus is determined (Fig. 4). Out of the torus, the turbulence is created, but inside it the proportional decrease of the wind velocity takes place as closing to the torus main ring. The microburst parameters are the following: $\mathcal{V}$ is the wind velocity at the central point (this value is not maximal, the maximal value can be up to twice larger in the torus boundary layers); h is the height of the torus main ring over the ground; R is the radius of the central line of the torus ring; $R_C = 0.8h$ is the torus main ring radius; $(\tilde{x}_0, \tilde{z}_0)$ is the torus center position in the ground plane.

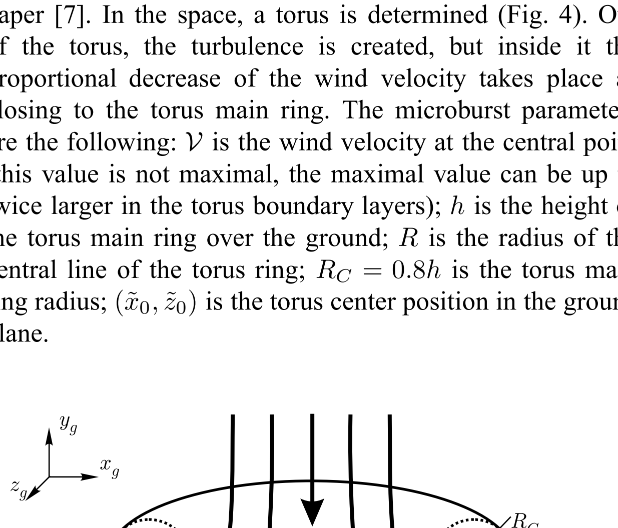

Fig. 4. Wind microburst model

X. Simulation results

Now consider simulation results under the wind microburst. The microburst parameters were: the height of the central point is 400 m; the torus main ring radius is 800 m; the central point is placed on the axis x_g at the distance of 200 m from the initial position of the aircraft. Two variants of the microburst force, i.e., of the wind velocity at the central point were taken: 8 and 4 m/sec. We assume that the microburst is symmetrically placed with respect to the projection of the base line onto the ground plane. Thus, the wind disturbance acts only in the vertical channel. In Fig. 5, the thick solid curves mark the trajectories and graphs corresponding to the stronger microburst, the thick dotted curves correspond to the weaker one.

In the plane $\tau \times \xi$ of game (4) (the first picture from the top), the thin solid lines denote the lower and upper boundaries of the main bridge, and also the horizontal switch line. The dashed line shows the horizontal line coming out from zero. In the altitude graph represented in the plane $x_g \times y_g$ of the geometric coordinates (the second picture), the base line is given in dashes.

The horizontal axis corresponds to the absolute time in other graphs. Since the simulation is stopped at the obstacle overpassing instant (and such an instant depends on acting the wind disturbance), the graphs cease at different instants. The dashed lines mark the levels of the minimal, nominal, and maximal values in the graph of the prescribed pitch angle u_ϑ. As for the graph of the thrust force P, there are only two auxiliary lines, since its minimal value coincides with the nominal one. The initial value of the thrust is smaller than the nominal one, since at the initial instant the aircraft moved horizontally. In the graphs of wind velocity components, dashed lines denote the maximal expected wind level and zero nominal value.

One can see the wind came out of the assumed bounds in the case of the strong wind microburst. Nevertheless, the suggested adaptive control has successfully parried such a wind.

XI. Conclusion

Application of the traditional antagonistic differential games to the problems of aircraft control under wind disturbances requires exact description of the constraint for the wind. In the paper for the problem of overpassing a vertical obstacle by an aircraft, the described control adjusts to the actual level of the disturbance. Usage of the control extremal values appears only under rather strong and “sophisticated” disturbance.

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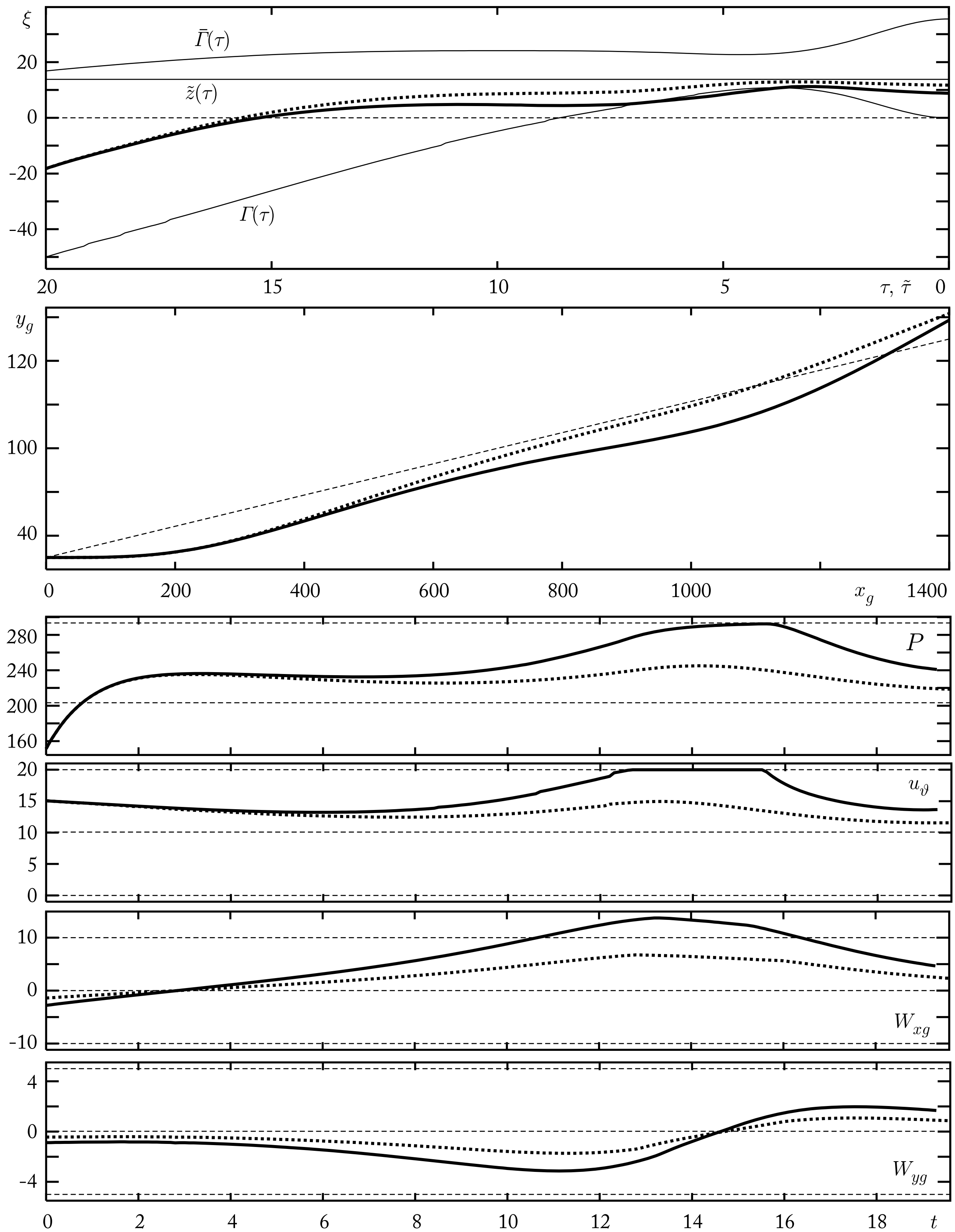

Fig. 5. Simulation results; the plane of auxiliary game (4) in the upper fragment; below that fragment, the graph of the altitude; the graphs of the thrust force P ($\times 1000$ N); the prescribed pitch angle u_ϑ (grad); the longitudinal W_{xg} (m/sec) and vertical W_{yg} (m/sec) components of the wind velocity