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ALGORITHM USING THE KALMAN FILTER FOR IDENTIFICATION OF RADAR SYSTEMATIC ERRORS IN AZIMUTH MEASUREMENTS *

D.A. Bedin[1]
Institute of Mathematics and Mechanics, Ural Branch of RAS
620990, Russia, Ekaterinburg, S.Kovalevskaya str., 16,
Tel.: +7 (343) 3753444. Fax: +7 (343) 3752581. E-mail: Jango.Urals@gmail.com

Abstract

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A problem which has to be solved in air traffic control of civil aircrafts is considered. For a system of several observing radars, it is necessary to identify the systematic azimuth errors in measurements of each radar. Such an error leads to a shift in the observed track of the aircraft. A mathematical formulation of the problem is suggested as a problem of estimation of a random vector on the basis of a nonlinear equation of observation. An analytical solution is presented as a law of posterior distribution. The analytical solution is difficult to implement in practice because of its complexity. As a result, a simplified solution is introduced that uses computational formulas on the basis of the Kalman filtering. The algorithm of simplified solution has been tested on model and real radar data.

1. Introduction

The case is considered when several radars observe an aircraft and, at discrete instants, measure the slant range to the aircraft and its azimuth. It is assumed that the radar system is such that the Earth surface can be presented by a plane (denote it as Π) in a zone of its operation. The results of measuring are marks that are drawn in the plane Π by the measured range and azimuth from the point of the radar.

The range and azimuth measurements have errors. These errors are classified as systematic and random. Distribution laws of random errors can be assumed as normal with zero means. Systematic errors are stipulated by corruptions of other nature. The error in azimuth can be of significant value. If the radar system has several (more than two) radars, an opportunity of determination of their systematic errors appears, since directions of shifts in measurements are different for different radars [1].

2. Radar observation model

Denote by x_i the vector of the true aircraft position in the plane Π at the instant t_i. Assume that at each instant t_i only one radar implements its measuring; the radar number is given by the function $k(i)$ and the radar is placed at the point $l_{k(i)}$ of the plane Π. Introduce symbols (Fig. 1) e_{i1}, e_{i2}, which are the unit vectors showing the direction of acting the errors in the range and azimuth, correspondingly, r_i is the true range between the radar and aircraft, σ_r is the mean-square deviation of the random error in range, σ_φ is the mean-square deviation of the random error in azimuth. Take these deviations to be equal for different radars; but the systematic errors denoted by λ_k are assumed to be different.

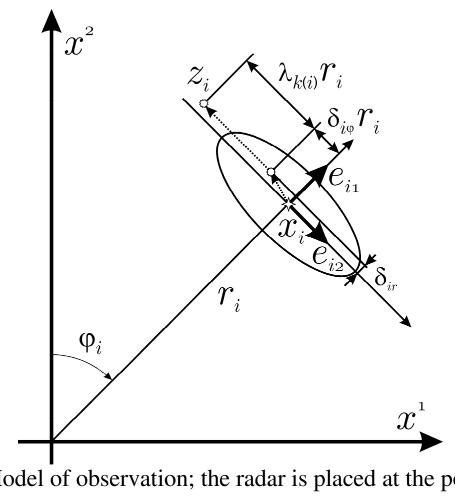

Fig. 1: Model of observation; the radar is placed at the point (0, 0)

[1] Researcher

The equation of observation has the form

$$z_i = x_i + e_{i1} \cdot \sigma_r w_i^r + e_{i2} \cdot \sigma_\varphi w_i^\varphi r_i + e_{i2} \cdot \lambda_{k(i)} r_i . \tag{1}$$

Here, w_i^r, w_i^φ are scalar, independent, normally distributed values with zero means and unit variance.

Let us assume that the observed aircraft has straight-line and steady speed motion. In this case, the aircraft position in the plane Π at the instant t_i is a linear function of the initial position and velocity. Denote by $y \in R^m$ the column vector including both these parameters and all unknown systematic errors λ_k. Relation between x_i and y has the form

$$x_i = A_i y , \tag{2}$$

where A_i is a matrix depending only on the time instants t_i. Since parameters e_{i1}, e_{i2}, r_i depend on x_i, equation (1) can be presented as follows:

$$z_i = C_i(x_i) y + D_i(x_i) w_i = C_i(A_i y) y + D_i(A_i y) w_i . \tag{3}$$

Here, $C_i(\cdot)$ and $D_i(\cdot)$ are matrix functions of the variable x_i; w_i contains the random values w_i^r, w_i^φ.

3. Nonlinear filtration

Using the nonlinear observation equation (3), the problem is reduced to estimation of unknown random value Y, which values are all possible vectors y. The *a priori* distribution of Y is assumed to be given, for example, as the density distribution function ρ_0. The density ρ' (without normalization) of the posterior distribution on the basis of measurements $\{z_1, \ldots, z_k\}$ is described [2] as follows:

$$\rho'_{Y|\{z_1,\ldots,z_k\}}(y) = \prod_{i=1}^{k} \left(g_i(y) e^{f_i(y)} \right) \rho_0(y) , \tag{4}$$

$$g_i(y) = \frac{1}{2\pi \left| \det D_i(A_i y) \right|}, \quad f_i(y) = -\frac{1}{2} \left(z_i - C_i(A_i y) y \right)^T \left(D_i^{-1}(A_i y) \right)^T D_i^{-1}(A_i y) \left(z_i - C_i(A_i y) y \right), \quad y \in R^m .$$

The final posterior density distribution is calculated by normalization of expression (4).

Formula (4) completely characterizes the posterior distribution of the random value Y obtained on the basis of measurements $\{z_1, \ldots, z_k\}$. But significant difficulties arise with its application. Consider a simplified version of relation (4) based on substitution of x_i by z_i:

$$C_i(A_i y) = C_i(x_i) \approx C_i(z_i) , \; D(A_i y) = D(x_i) \approx D_i(z_i) . \tag{5}$$

The approximate distribution function (without normalization) has the form

$$\tilde{\rho}'(y) = \prod_{i=1}^{k} \left(\tilde{g}_i e^{\tilde{f}_i(y)} \right) \rho_0(y) , \tag{6}$$

$$\tilde{g}_i = \frac{1}{2\pi \left| \det D_i(z_i) \right|}, \qquad \tilde{f}_i(y) = -\frac{1}{2} \left(z_i - C_i(z_i) y \right)^T \left(D_i^{-1}(z_i) \right)^T D_i^{-1}(z_i) \left(z_i - C_i(z_i) y \right), \qquad y \in R^m .$$

In (6), functions $\tilde{f}_i$ are positive defined quadratic forms of y. If the *a priori* density ρ_0 has the normal distribution, the $\tilde{\rho}'$ will correspond to the normally distributed random value $\tilde{Y}_k$ that is an approximation to Y on the results of k measurements. The value $\tilde{Y}_k$ is completely characterized by its mean m_k and covariation matrix P_k that can be recurrently calculated by the following formulas (for simplicity, the arguments in $C_k(z_k)$ and $D_k(z_k)$ are omitted):

$$\begin{aligned} \beta_k &= P_{k-1} C_k^T \left(C_k P_{k-1} C_k^T + D_k D_k{}^T \right)^{-1} , \\ m_k &= m_{k-1} + \beta_k \left(z_k - C_k m_{k-1} \right), \\ P_k &= \left(I - \beta_k C_k \right) P_{k-1} . \end{aligned} \tag{7}$$

Formulas (7) correspond to the observation equation (3), in which substitution (5) is performed:

$$z_i = C_i(z_i) y + D_i(z_i) w_i \; . \tag{8}$$

Relations (7) coincide with expressions of the Kalman filter [2] that could be written out for (8) in the case of absence of dependence on z_i in the right-hand side.

4. Program implementation

Algorithm (7) was implemented in a research program and has demonstrated good results in its applications to model and real radar data [1]. Fig. 2 shows a sample of three radar tracks corresponding to motion of the same aircraft. One track with large systematic error in azimuth outstanding far from other tracks is seen. In Fig. 3, the tracks turned by their found systematic errors in azimuth are presented.

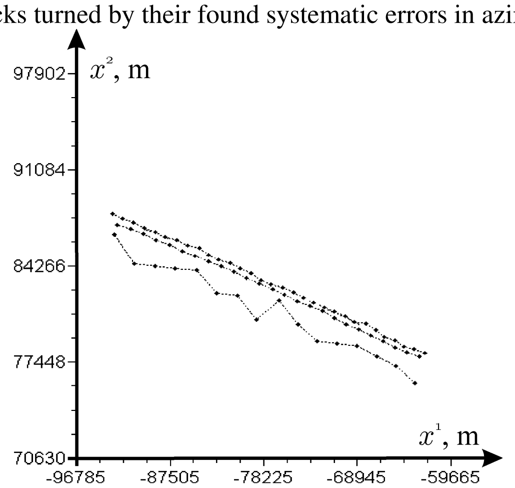

Fig. 2: Initial tracks from three radars

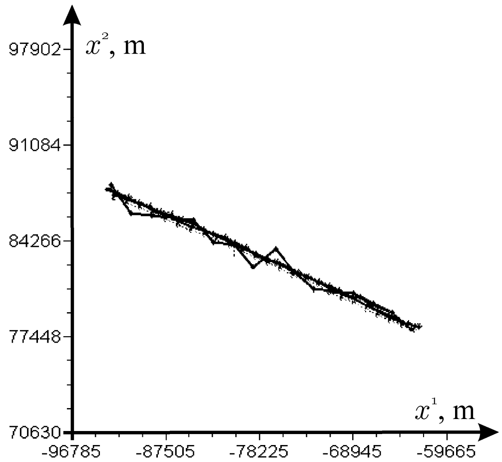

Fig. 3: Three radar tracks after processing

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