Aircraft Landing Control under Wind Disturbances

S.A. Ganebny, S.S. Kumkov, V.S. Patsko
Institute of Mathematics and Mechanics,
Ekaterinburg, Russia

A.I. Krasov
Academy of Civil Aviation
Saint-Petersburg, Russia

The 12th International Symposium on Dynamic Games and Applications,
July, 3-6, 2006, Sophia-Antipolis, France

IMM

References

Aircraft problems:

A. Miele, T. Wang, H. Wang, W.W. Melvin, C.Y. Tzeng

G. Leitmann, S. Pandey

R. Bulirsch, F. Montrone, H.J. Pesch

Microburst models:

M. Ivan, S. Zhu, B. Etkin, A. Miele

Aircraft problems (Russia):

V.M. Kein, A.I. Krasov, S.M. Fedorov, A.N. Parikov, M. Yu. Smurov

I.N. Titovskii, A.A. Melikyan, V.A. Korneev

Ekaterinburg-Sverdlovsk works on aircraft problems:

V.S. Patsko, N.D. Botkin, V.L. Turova, M.A. Zarkh, A.G. Ivanov

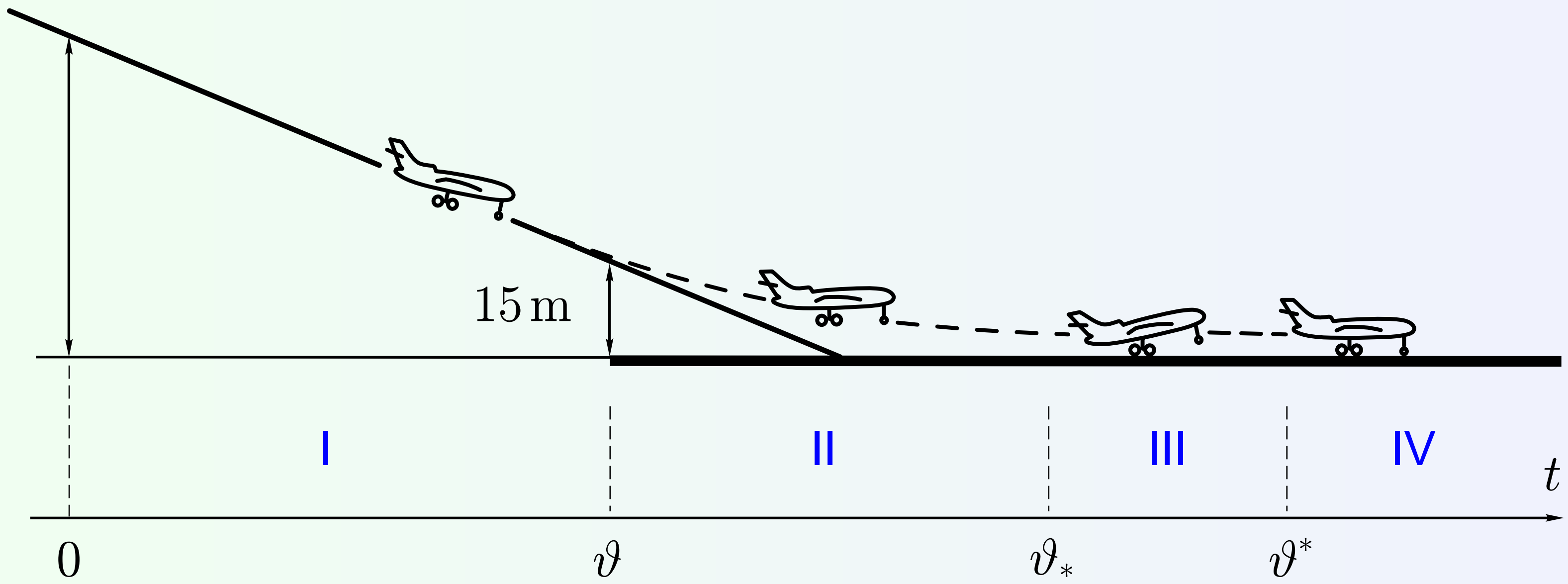
Nowadays, there are a lot of works concerning application of modern methods of the control theory to problems of aircraft landing and take-off under wind disturbances.

Among Russian scientists, V. Kein was the first who investigated such problems.

Our work is devoted to application of differential game methods developed in the Ekaterinburg school to a landing problem.

IMM

Aircraft landing stages



I. Descent till passing the runway threshold – the problem under investigation

II. Levelling till contact with the runway (the stage of flare)

III. Running on the main wheels

IV. Running on all wheels

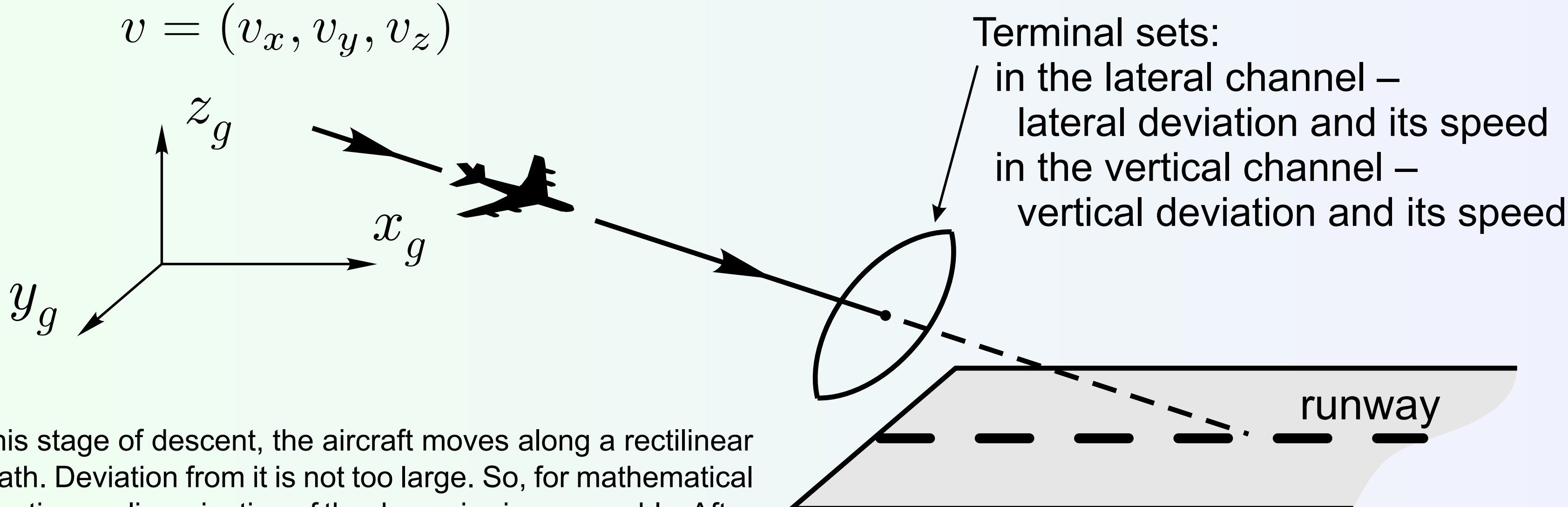
In this work, we consider the time interval of descent from the altitude 400 m until passing the runway threshold at the altitude 15 m. Duration of this stage for a midsize aircraft is approximately 120 sec.

IMM

Aircraft landing problem

$$\dot{\xi} = f(\xi, u, v),$$
$$t \in T, \quad \xi \in \mathbb{R}^{16},$$
$$u = (u_p, u_e, u_r, u_a),$$
$$v = (v_x, v_y, v_z)$$

After linearization, the original nonlinear system actually disjoins into two independent subsystems: of the lateral and vertical motions



At this stage of descent, the aircraft moves along a rectilinear glide path. Deviation from it is not too large. So, for mathematical investigations, a linearization of the dynamics is reasonable. After linearization of the original nonlinear system, the resulting dynamics is disjoined into two independent subsystems of the lateral and vertical motions. For both of them, we consider auxiliary linear differential games with fixed terminal time. A terminal set for the lateral channel is defined as a convex set in the plane of two phase coordinates, namely, lateral deviation and its velocity. A terminal set in the vertical channel is constructed similarly. The controls for the lateral channel are the scheduled aileron and rudder angles. For the vertical channel, the controls are the scheduled thrust deviation from the nominal value and the scheduled elevator angle. Some geometric constraints on the controls are given *a priori*.

Control laws obtained in the framework of the linearized dynamics are tested further in the original nonlinear system.

Conceptual difference between this presentation and works made in Ekaterinburg twenty years ago is that earlier there was a requirement for the geometric constraint on the wind disturbance in the formulation of the problem. In engineering practice, defining such a constraint is quite problematic. Formulation of this work does not include such a demand.

IMM

References

Differential games (Ekaterinburg-Sverdlovsk school):

N.N. Krasovskii, A.I. Subbotin

H^{∞}-problems from a differential game point of view:

T. Basar, P. Bernhard

L^{1}-optimization:

A.E. Barabanov, M.A. Dahleh, J.B. Pearson, V.F. Sokolov

Close concept of robustness:

V. Turetsky, V.Y. Glizer

So, we shall speak about a robust control, which should work satisfactorily in a wide interval of wind disturbance.

Our concept of robustness is close to the one in works by V.Turetsky and V.Glizer.

Robust formulation

$$\dot{\mathbf{x}} = \mathbf{A}(t)\mathbf{x} + \mathbf{B}(t)u + \mathbf{C}(t)v,$$
$$\mathbf{x} \in \mathbb{R}^m, \quad t \in T = [\vartheta_0, \vartheta], \quad M \subset \mathbb{R}^n,$$
$$u \in P \subset \mathbb{R}^p, \quad v \in \mathbb{R}^q$$

We call a feedback control $U(t, \mathbf{x})$ *robust* in the following case:

1) if the second player's control (that is a disturbance) is weak, then the first player (that is our useful control) should lead the system to the terminal set as close to its center as possible and, additionally, the realization of its control should be weak too;
2) if the second player's control is stronger, then the first player should still provide successful termination of the process by means of its stronger or even extremal control;
3) if the second player's control is too strong to be successfully parried within the first player's constraint, then some miss from the terminal set is allowed. This miss should be minimized by the first player.

We clarify the concept of the robust control in the following way.

IMM

Equivalent system

By means of the standard transformation, we pass to the system

$$\dot{x} = B(t)u + C(t)v,$$

$$x \in \mathbb{R}^n, \quad t \in T, \quad M \subset \mathbb{R}^n,$$

$$u \in P \subset \mathbb{R}^p, \quad v \in \mathbb{R}^q$$

The transformation is defined as follows:

$$x(t) = X_{n,m}(\vartheta, t)\mathbf{x}(t),$$

$$B(t) = X_{n,m}(\vartheta, t)\mathbf{B}(t),$$

$$C(t) = X_{n,m}(\vartheta, t)\mathbf{C}(t)$$

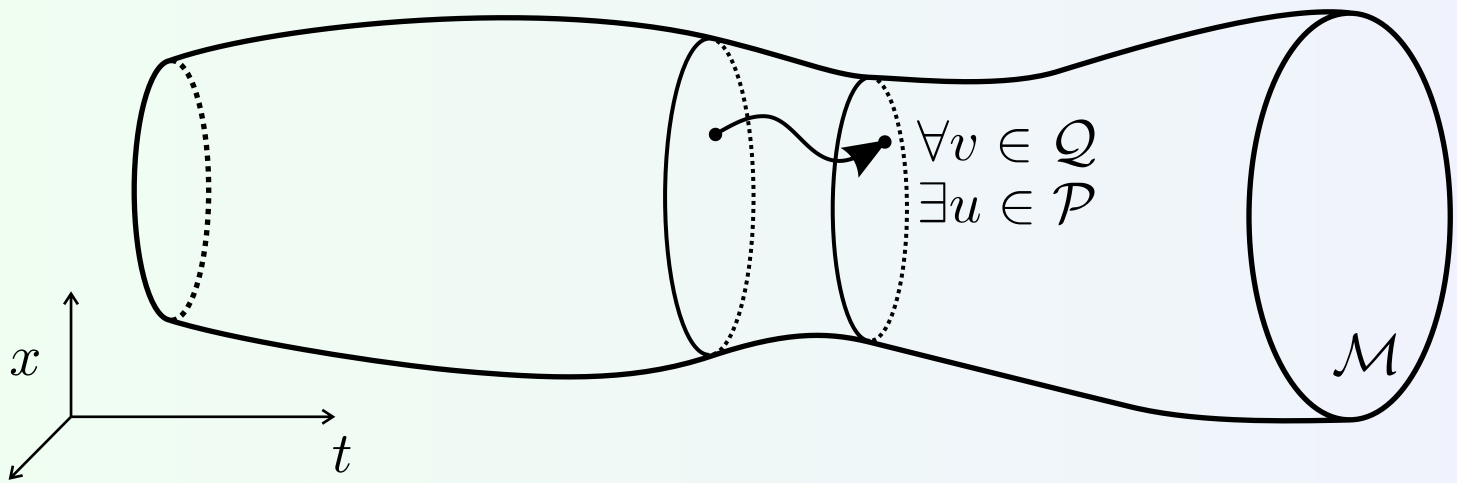
Here, $X_{n,m}(\vartheta, t)$ is the matrix combined of n selected rows (corresponding to the terminal set M subspace) of the fundamental Cauchy matrix.

Because of linearity of the dynamic system and fixed time of termination, we can pass to a system without the state variable in the right-hand side. This standard transformation is made by means of corresponding rows of the fundamental Cauchy matrix. In our case, the state vector of the new system is two-dimensional.

Stable bridges

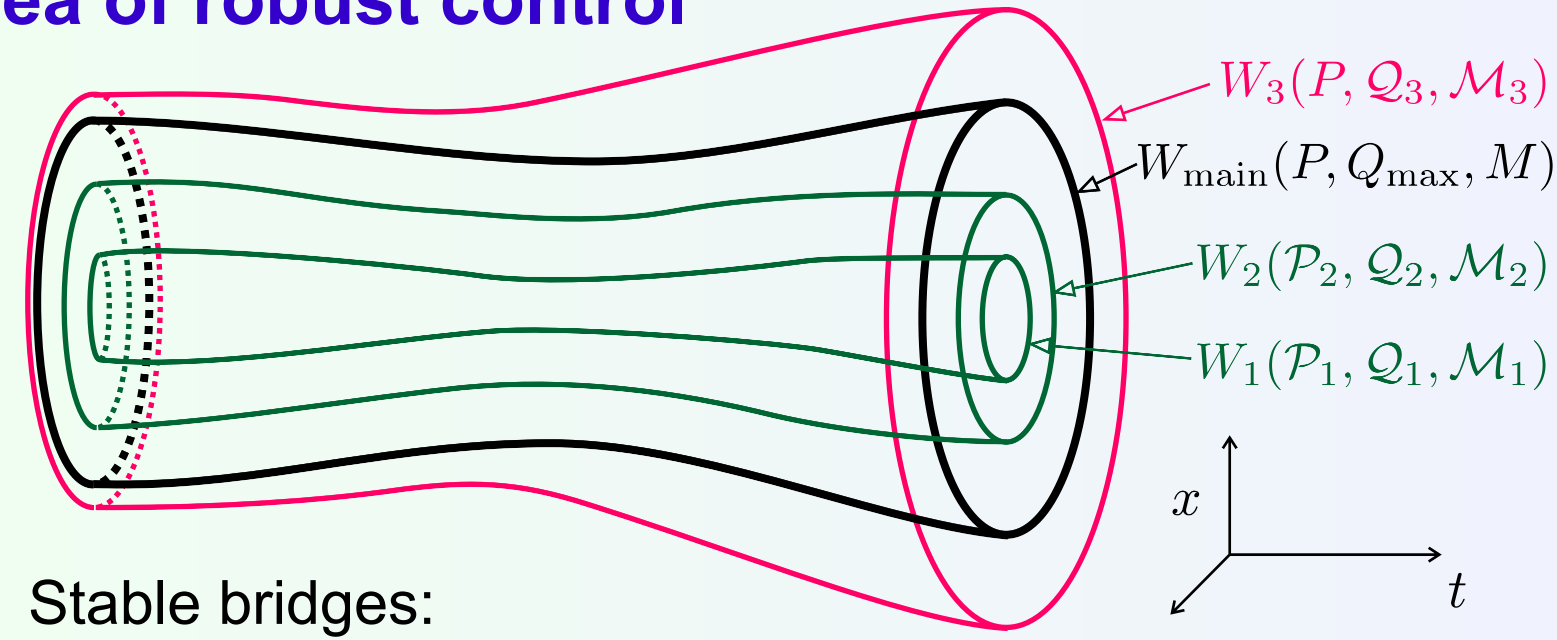
$$\dot{x} = B(t)u + C(t)v,$$
$$x \in \mathbb{R}^n, \quad t \in T, \quad \mathcal{M},$$
$$u \in \mathcal{P}, \quad v \in \mathcal{Q}$$

$(\mathcal{P},\ \mathcal{Q},\ \mathcal{M})$ — game parameters



This system is very attractive because we have a computer algorithm for fast construction of maximal stable bridges for the case when geometric constraints are given on both players controls. The maximal stable bridge (in the space of time and state vector) is the discriminating kernel by the viability terminology. A bridge and tube are the same notions in our talk.

Idea of robust control



Stable bridges:

$$W_1 \subset W_2 \subset \ldots \subset W_{\rm main} \subset \ldots \subset W_3 \subset \ldots$$

Bridge parameters:

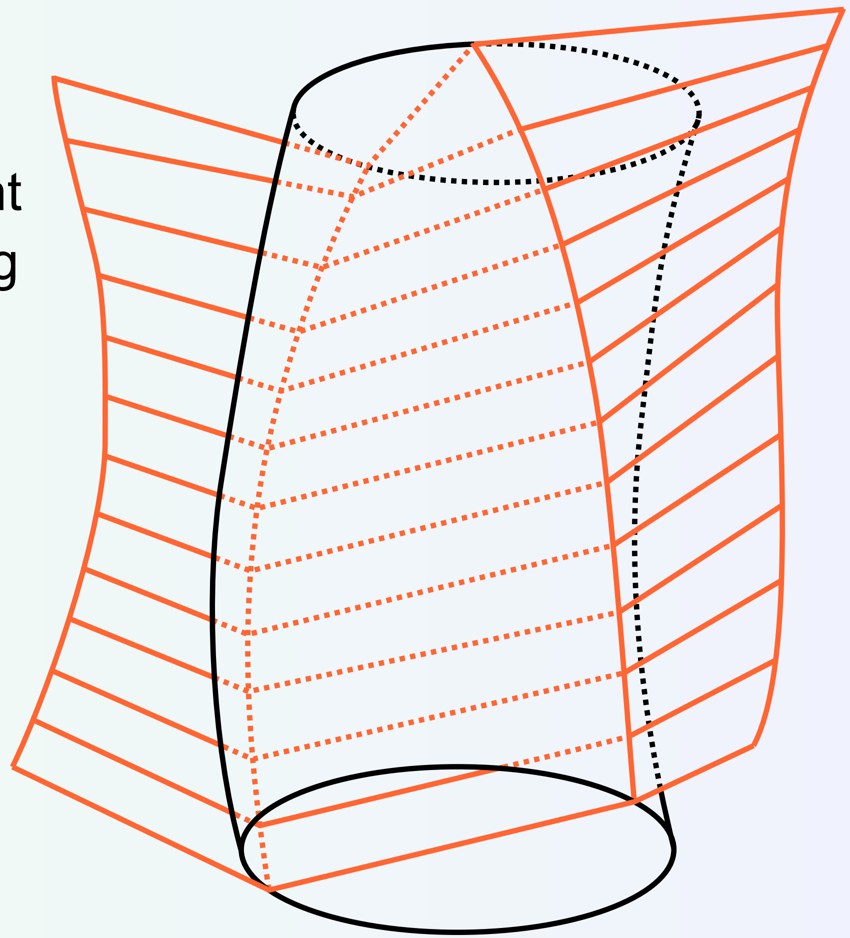
$$\mathcal{P}_1 \subset \mathcal{P}_2 \subset \ldots \subset P$$
$$\mathcal{Q}_1 \subset \mathcal{Q}_2 \subset \ldots \subset Q_{\rm max} \subset \ldots \subset \mathcal{Q}_3 \subset \ldots$$
$$\mathcal{M}_1 \subset \mathcal{M}_2 \subset \ldots \subset M \subset \ldots \subset \mathcal{M}_3 \subset \ldots$$

The main idea is the following. Let us consider a family of differential games, where the constraints for the players' controls, the terminal set and, consequently, the maximal stable tube are parametrized by a positive number. This family of tubes is ordered by inclusion with increasing the parameter. The first player guarantees keeping the system inside any tube if a realization of the second player's control is of the corresponding level. With that, the control applied by the first player belongs to the constraint corresponding to the stable tube. This family of tubes can be considered as a definition of a Lyapunov function in the game space. It allows to construct a first player's feedback control and to evaluate the guaranteed result provided by this control.

In our case of linear dynamics, we can store only one stable bridge. We call it the main bridge. This bridge corresponds to the maximal constraint for the first player's control. All the tubes inside are calculated by means of homothety. One additional bridge gives the basis for the construction of stable tubes outside the main bridge.

Switching surfaces

Using specifics of the equivalent game we compute the switching surfaces *a priori* on the basis of the main bridge and some additional stable bridge

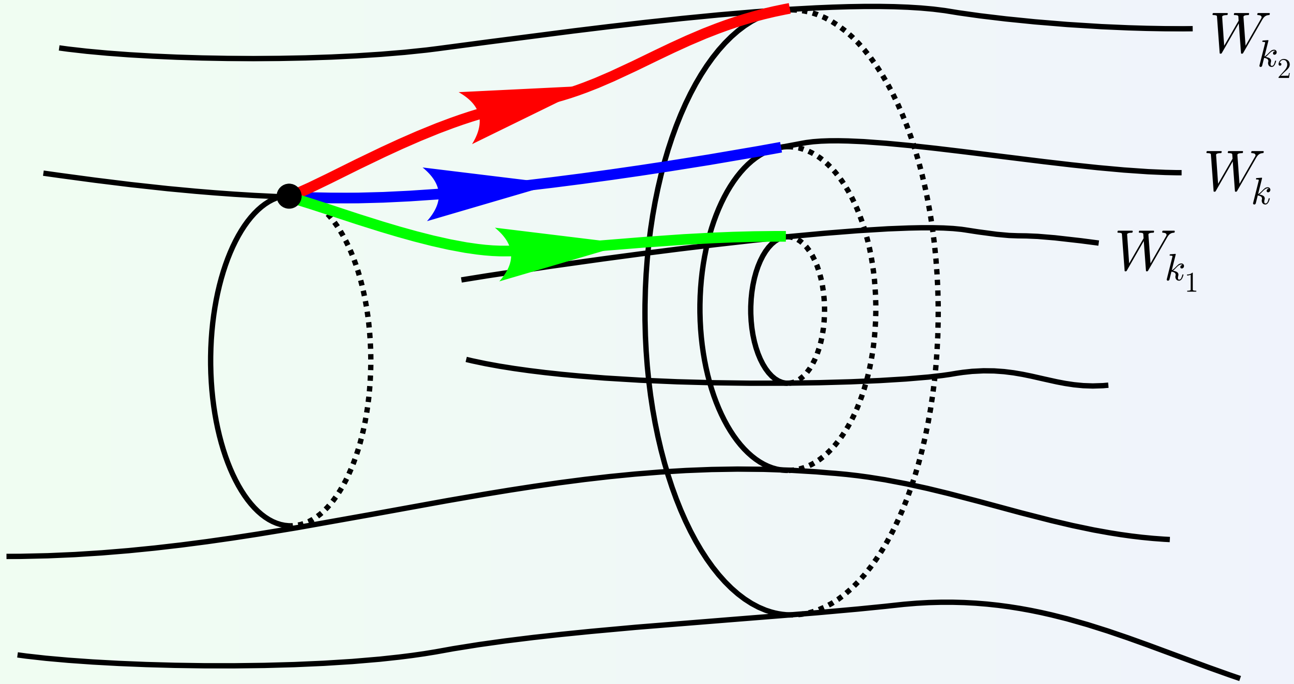


Objects required to calculate control:

1. The main stable bridge
2. Switching surfaces

Since the components of the control vector are independently constrained in our problem, we can compute switching surface for each component. So, we need to store the main bridge and two switching surfaces for two control components for the lateral channel. Here, we show one of this surfaces. For the vertical channel, we need to store one main bridge and two switching surfaces also.

Behavior of trajectories



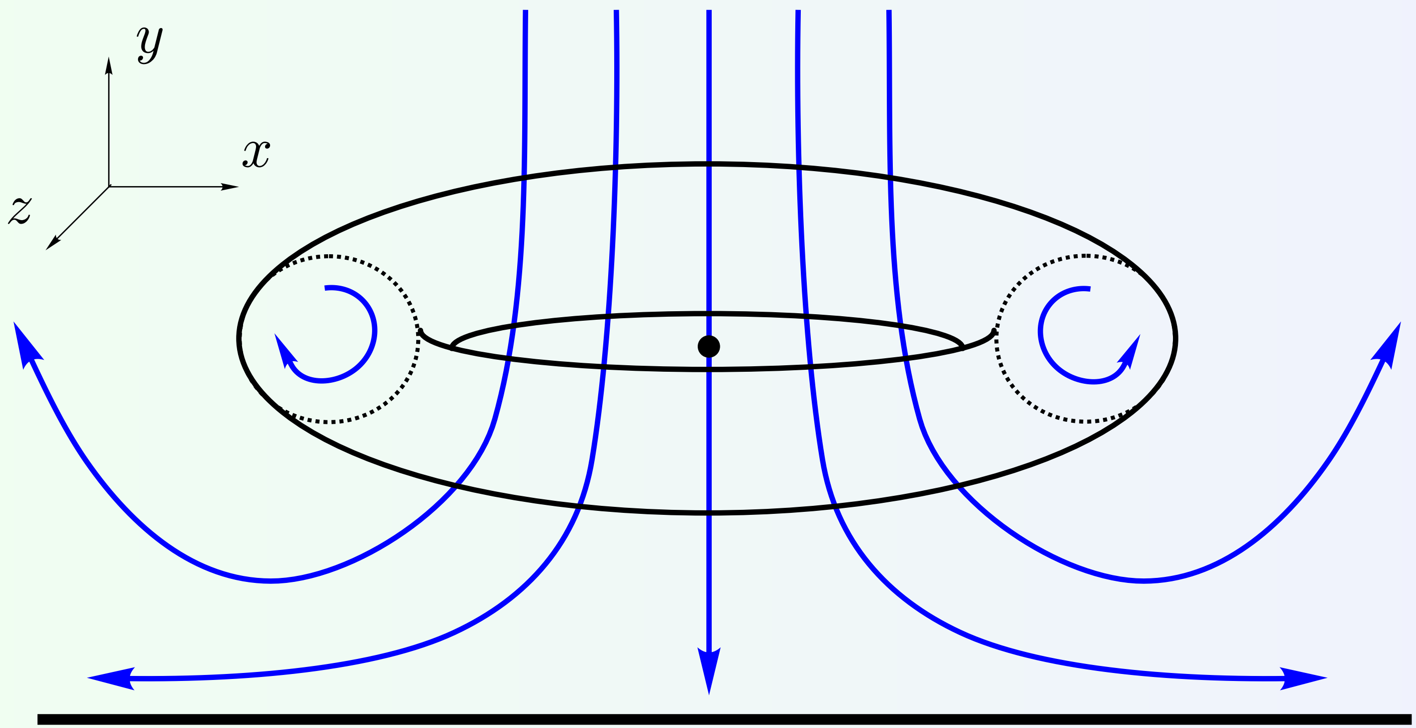
– if $v(t)$ is stronger than $kQ_{\max}$

– if $v(t)$ corresponds to $kQ_{\max}$

– if $v(t)$ is weaker than $kQ_{\max}$

Let our robust feedback control is used. In the case when the wind disturbance is weaker than actually applied useful control, the motion goes to lower level of our tubes. In the case of stronger disturbance, the motion goes to some greater level. Since the disturbance is bounded, the level of its realization stops to grow along the motion.

Microburst model



M. Ivan. A ring-vortex downburst model for real-time flight simulation of severe windshears // AIAA Flight Simulation Technology Conference, St. Louis, Missouri, pp. 57–61, 1985.

When testing the robust control we suggested, we use several variants of wind disturbance model. One of them is definition of the disturbance by means of a wind microburst model from the paper by M.Ivan.

IMM

Publications of authors

1. **Patsko V.S., Botkin N.D., Kein V.M., Turova V.L., and Zarkh M.A.** *Control of an aircraft landing in windshear.* In: Journal of Optimization Theory and Applications, Vol.83, No.2, pp.237–267, 1994.
2. **Botkin N.D., Kein V.M., and Patsko V.S.** *The model problem of controlling the lateral motion of an aircraft during landing.* In: Journal of Applied Mathematics and Mechanics, Vol.48, No.4, pp.395–400, 1984.
3. **Ganebny, S.A., Kumkov S.S., Patsko V.S., and Pyatko S.G.** *Robust Control in Game Problems with Linear Dynamics.* Preprint. Institute of Mathematics and Mechanics, Ekaterinburg, Russia, 2005.
4. **Ganebny, S.A., Kumkov S.S., and Patsko V.S.** *Constructing robust control in game problems with linear dynamics.* In: Game Theory and Applications, Vol.11, 2006.
5. **Ganebny S.A., Kumkov S.S., Patsko V.S., and Pyatko S.G.** *Constructing robust control in differential games. Application to aircraft control during landing.* In: Annals of the International Society of Dynamic Games, Advances in Dynamic Game Theory and Applications, Vol.9, S.Jorgensen, T.Vincent, M.Quincampoix (Eds.), Birkhauser, 2006.