Homicidal chauffeur

becomes more dangerous

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Homicidal chauffeur becomes more dangerous

Album of illustrations to the presentation made
at the 12-th Symposium on Dynamic Games and Applications
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It is known that time-optimal control problems with dynamics that describe a realistic inertial car possessing variable magnitude of the velocity are very complicated to study. In the work, a relaxed case where instantaneous variations of the velocity magnitude are allowed is considered.

A pursuit-evasion problem that is analogous to the famous “homicidal chauffeur” game by R.Isaacs is studied. Note that the classical statement of this problem assumes a constant magnitude of the linear velocity of the car. In the case of the variable magnitude of the linear velocity, the pursuer becomes “more dangerous”.

Numerical algorithms are applied to the computation of level sets of the value function (of solvability sets of the pursuit-evasion problem). The dependence of these sets on a parameter that defines the bounds on the magnitude of the velocity is investigated.

Also, families of smooth semipermeable curves related to the dynamics under consideration are explored. The knowledge of these families allows to verify the correctness of the numerical construction of lines on which the value function is discontinuous.

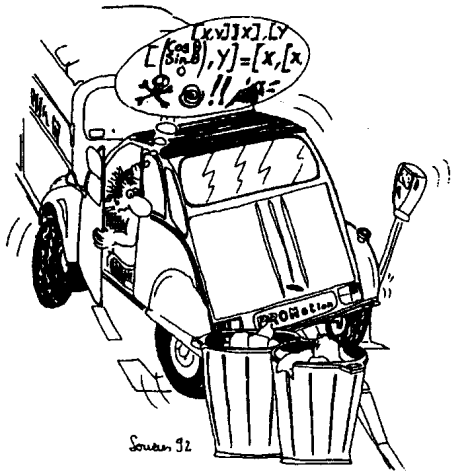
Together with the pursuit-evasion game, control problem corresponding to the immovable evader is considered. Both time-limited reachable sets and reachable sets at a given time are computed.

The results presented in this work are based on the methods being developed by the Ekaterinburg scientific school on optimal control and differential games.

Collection of survey papers

J.-P. Laumond (Ed.)

Robot Motion Planning and Control

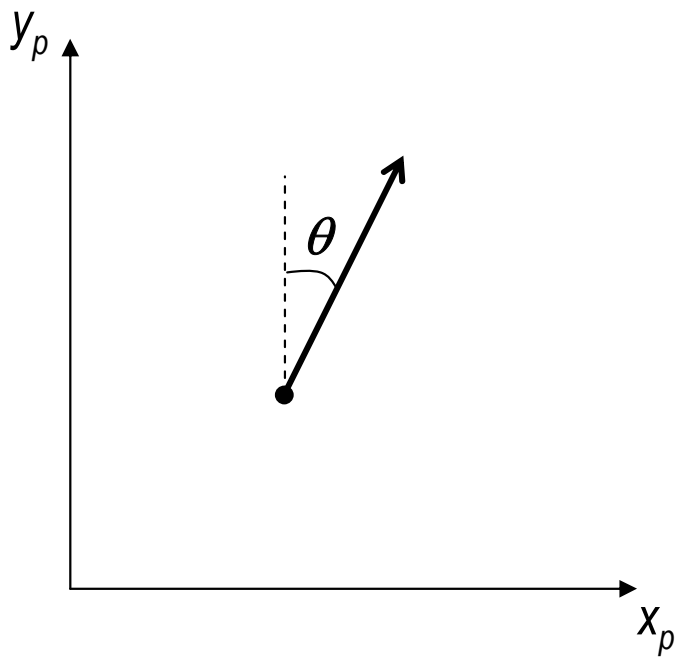


Springer

This book edited by J.-P. Laumond from LAAS laboratory in Tolouse was published in 1998. It contains many models of robot car dynamics.

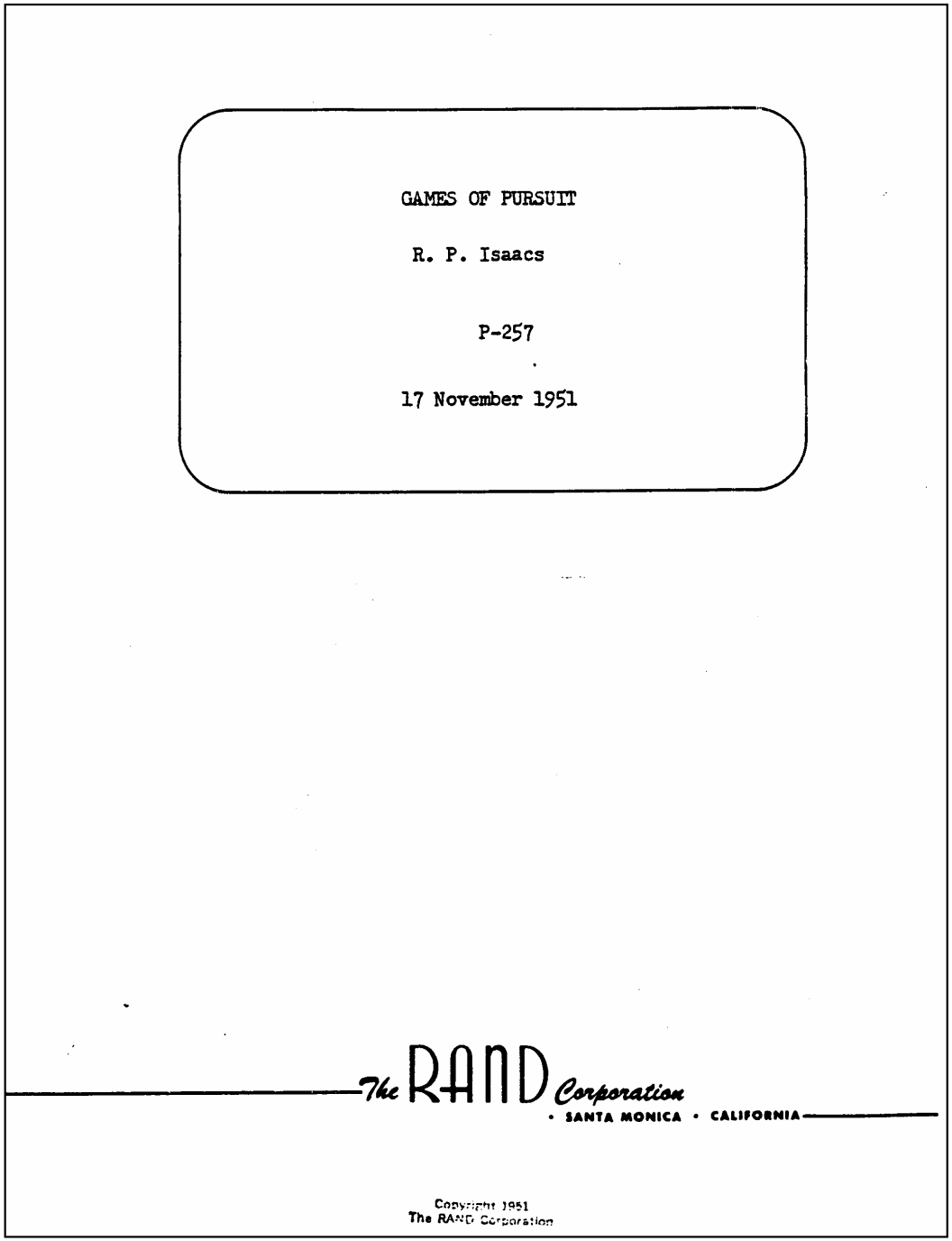
Simplest car model

$$\dot{x}_p = \sin\theta$$
$$\dot{y}_p = \cos\theta$$
$$\dot{\theta} = u$$
$$|u| \leq 1$$



On this slide, the simplest model is shown. Here, x_p, y_p are geometric coordinates of the car, θ is the angle between the velocity vector and the vertical axis. The magnitude of the velocity vector is constant. R. Isaacs used intensively such a model in his works on differential games.

Report by R. Isaacs, 1951



GAMES OF PURSUIT

R. P. Isaacs

P-257

17 November 1951

The RAND Corporation

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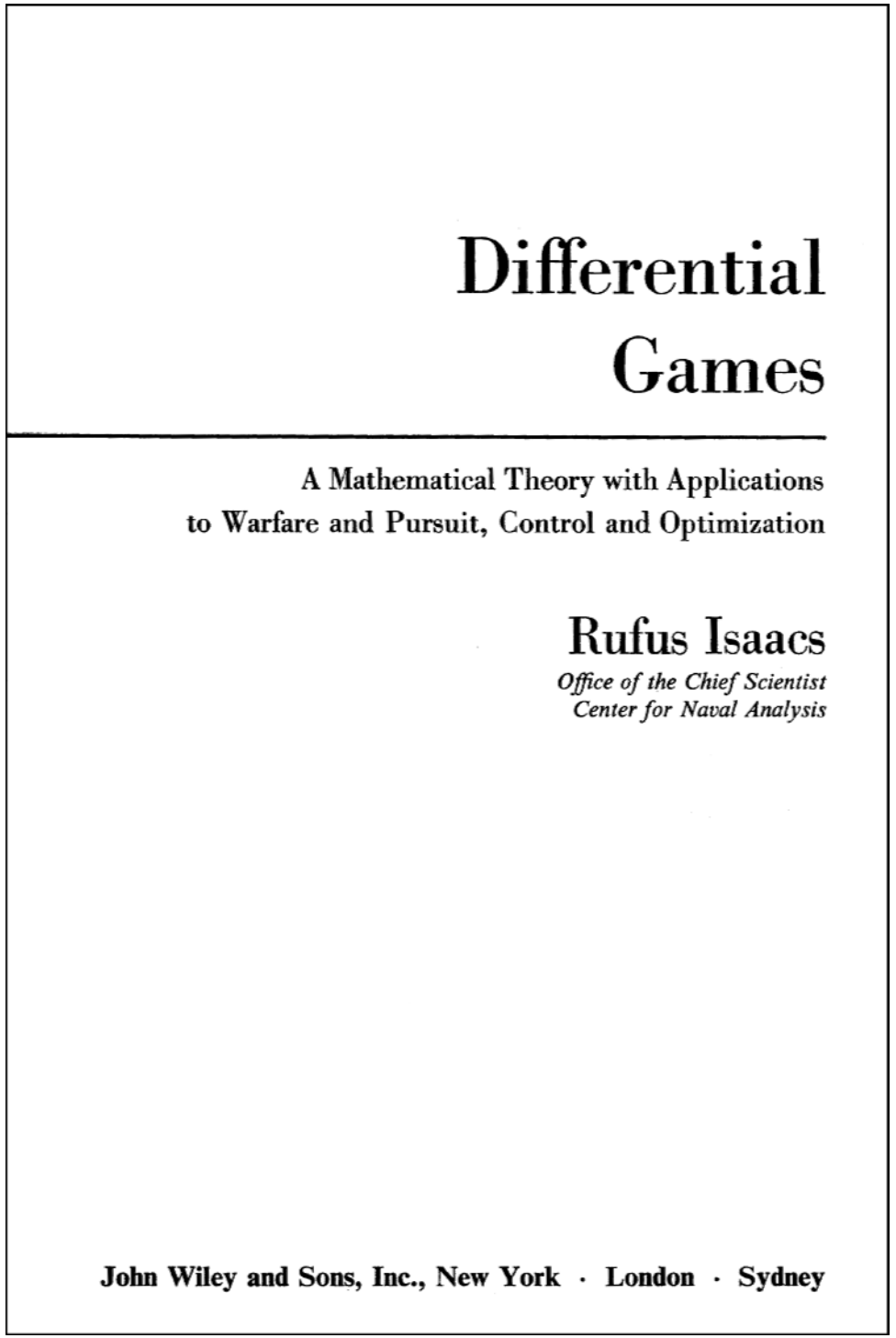
The RAND Corporation

This is the title page of Isaacs´ report on pursuit games issued in 1951.

Book by R.Isaacs, 1965



Differential Games

A Mathematical Theory with Applications to Warfare and Pursuit, Control and Optimization

Rufus Isaacs

Office of the Chief Scientist
Center for Naval Analysis

John Wiley and Sons, Inc., New York · London · Sydney

The cover of the famous book by R. Isaacs, 1965.

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JOURNAL OF MATHEMATICS
FOUNDED BY THE JOHNS HOPKINS UNIVERSITY

Volume LXXIX, Number 3
JULY, 1957

ON CURVES OF MINIMAL LENGTH WITH A CONSTRAINT ON AVERAGE CURVATURE, AND WITH PRESCRIBED INITIAL AND TERMINAL POSITIONS AND TANGENTS.* [1]

By L. E. DUBINS.

We have now established our main result:

THEOREM I. *Every planar R-geodesic is necessarily a continuously differentiable curve which is either* (1) *an arc of a circle of radius R, followed by a line segment, followed by an arc of a circle of radius R; or* (2) *a sequence of three arcs of circles of radius R; or* (3) *a subpath of a path of type* (1) *or* (2).

In papers on theoretical robotics, the car model with constant magnitude of the linear velocity is often referred as Dubins´ car. The name is due to L.E.Dubins whose paper of 1957 contains a theorem on the number and type of switches of an open-loop control that brings the object from a given state with a specified direction of the velocity vector to a terminal state for which the direction of the velocity vector is also prescribed.

The paper by A.A.Markov

Нѣсколько примѣровъ рѣшенія особаго рода задачъ о наибольшихъ и наименьшихъ величинахъ.

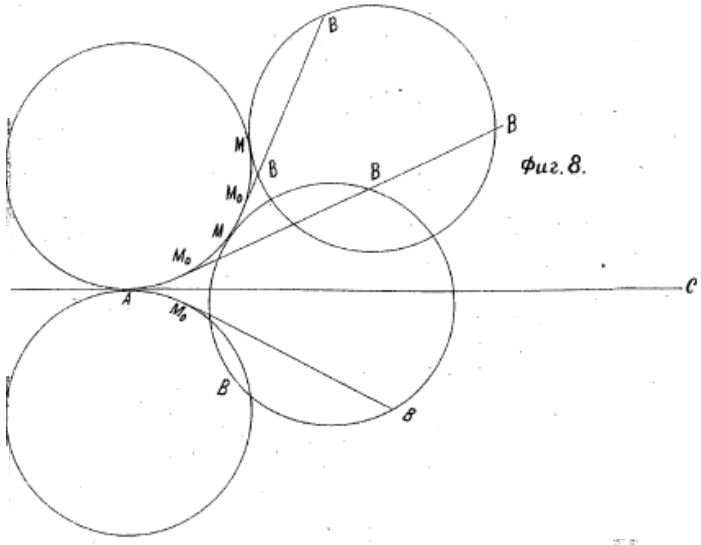
А. А. Маркова.

ЗАДАЧА 1.

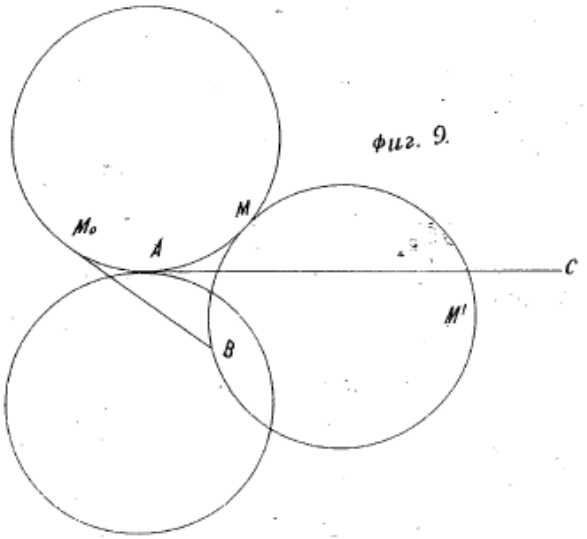
Между данными точками A и B (см. фиг. 1-ю) провести кратчайшую кривую линію при слѣдующихъ двухъ условіяхъ: 1) радіусъ кривизны нашей кривой повсюду долженъ быть не меньше данной величины ϱ, 2) въ точкѣ A касательная къ нашей кривой должна имѣть данное направленіе AC.

РѢШЕНІЕ.

Пусть M одна изъ точекъ нашей кривой, а прямая NMT соотвѣтствующая касательная.



Фиг. 8.



Фиг. 9.

Four problems related to R.Isaacs and L.E.Dubins' car have been already considered by A.A.Markov in the paper "Some examples of the solution of a special kind of problem on greatest and least quantities"published in "Soobscenija Charkovskogo matematiceskogo obscestva", Ser.2,1, NN5,6 (1889) pp. 250-276 (in Russian).

Reeds and Shepp´s car

$$\dot{x}_p = w \sin\theta$$
$$\dot{y}_p = w \cos\theta$$
$$\dot{\theta} = u$$
$$|u| \leq 1, \quad |w| \leq 1$$

The next, more complicated, model described in the book “Robot Motion Planning and Control” is called Reeds and Shepp´s car. The new factor w is a control which can change instantaneously the magnitude of the linear velocity. If w switches from a positive value to a negative one, then the direction of the velocity vector changes as well. In this model, θ is the angle between the vertical axis and the direction of forward motion that corresponds to $w = 1$.

Intermediate car model

$$\dot{x}_p = w \sin\theta$$
$$\dot{y}_p = w \cos\theta$$
$$\dot{\theta} = u$$
$$|u| \leq 1, \quad a \leq w \leq 1$$

a is a fixed parameter of the problem, $a \in [-1, 1]$

In this paper, an intermediate model between Isaacs-Dubins´ car and Reeds and Shepp´s car will be investigated. The constraint on the value of the angular velocity u does not depend on the magnitude of the linear velocity. The value w of the linear velocity can change instantaneously within the bounds $a \leq w \leq 1$ where a is a fixed parameter that fulfils the condition $a \in [-1, 1]$. If $a = 1$, we have Isaacs-Dubins´ car. For $a = -1$, we arrive at Reeds and Shepp´s car.

Classical homicidal chauffeur game

$$P: \quad \dot{x}_p = w \sin\theta$$
$$\dot{y}_p = w \cos\theta$$
$$\dot{\theta} = w\, u/R, \quad |u| \le 1$$

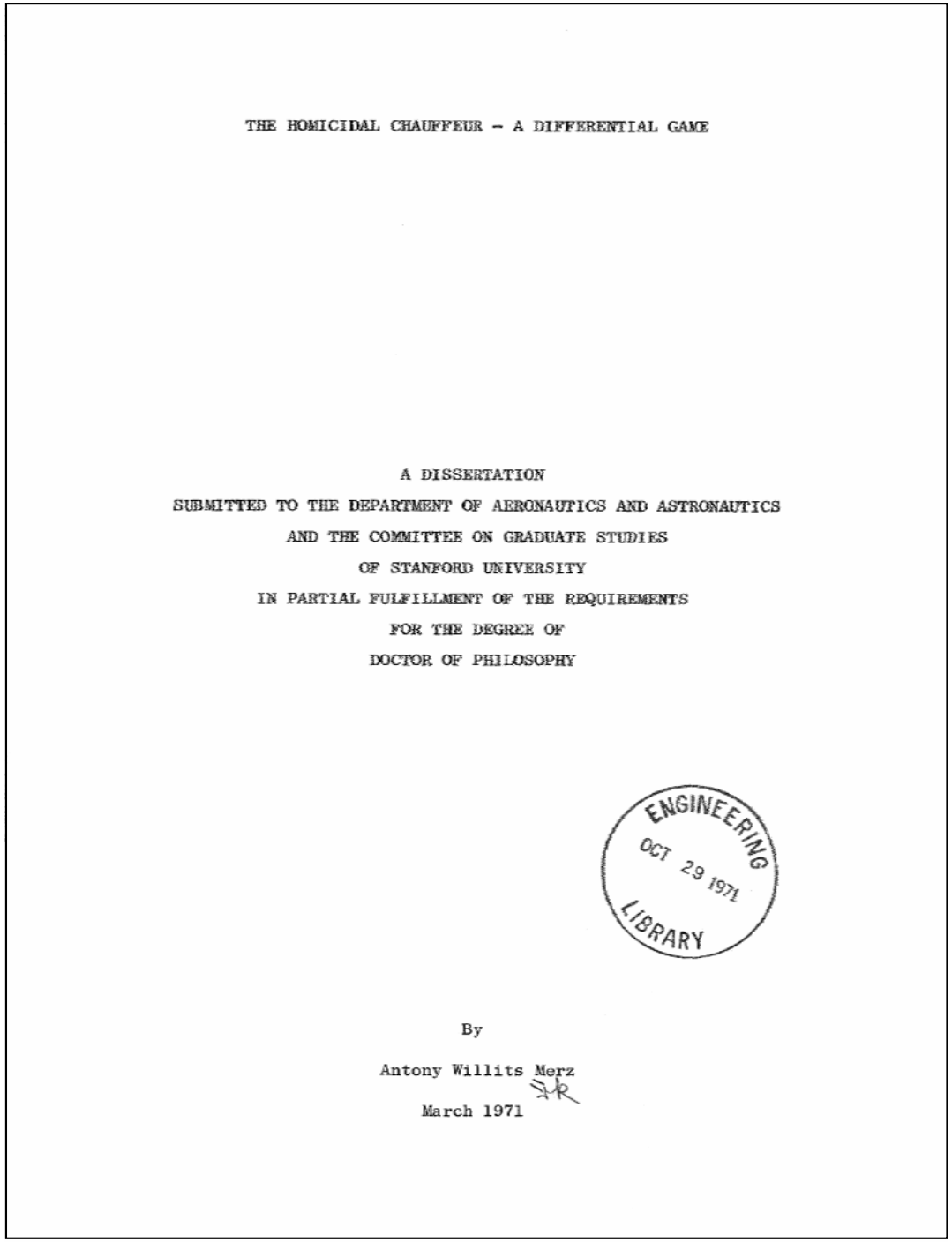
$$E: \quad \dot{x}_e = v_1$$
$$\dot{y}_e = v_2, \quad |v| \le \nu$$

Here, the dynamics of the classical “homicidal chauffeur” problem by R.Isaacs is shown. Pursuer *P* controls the car whose linear velocity has constant magnitude. Evader *E* is a non-inertial object that can change the value and direction of his velocity $v=(v_1,v_2)^T$ instantaneously. The maximal possible value of the velocity is specified. By a given circular neighborhood of his current geometric position, player *P* tries to capture player *E* as soon as possible.

From Isaacs-Dubins´ car to Reeds and Shepp´s car in the homicidal chauffeur problem

Dissertation by A.W. Merz, 1971

A complete solution to the “homicidal chauffeur” problem in the classical statement of R.Isaacs is obtained by A.W.Merz in his PhD thesis supervised by J.V.Breakwell.



THE HOMICIDAL CHAUFFEUR - A DIFFERENTIAL GAME

A DISSERTATION
SUBMITTED TO THE DEPARTMENT OF AERONAUTICS AND ASTRONAUTICS
AND THE COMMITTEE ON GRADUATE STUDIES
OF STANFORD UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

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By

Antony Willits Merz

March 1971

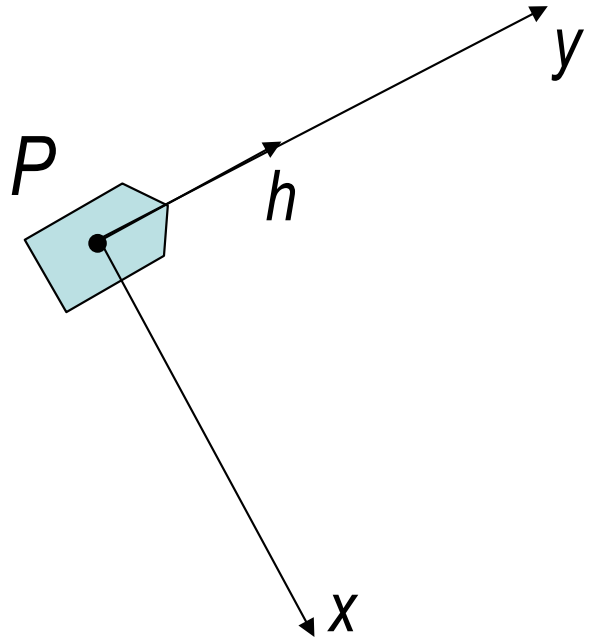
Reinforced homicidal chauffeur

$$P: \quad \dot{x}_p = w \sin\theta$$
$$\dot{y}_p = w \cos\theta$$
$$\dot{\theta} = u, \quad |u| \le 1, \qquad a \le w \le 1$$

$$E: \quad \dot{x}_e = v_1$$
$$\dot{y}_e = v_2, \qquad |v| \le \nu$$

We will investigate the variant of the “homicidal chauffeur” game with the reinforced pursuer.

Isaacs´ transformation



$$\dot{x} = -y\ u + v_x$$

$$\dot{y} = \quad x\ u - w + v_y$$

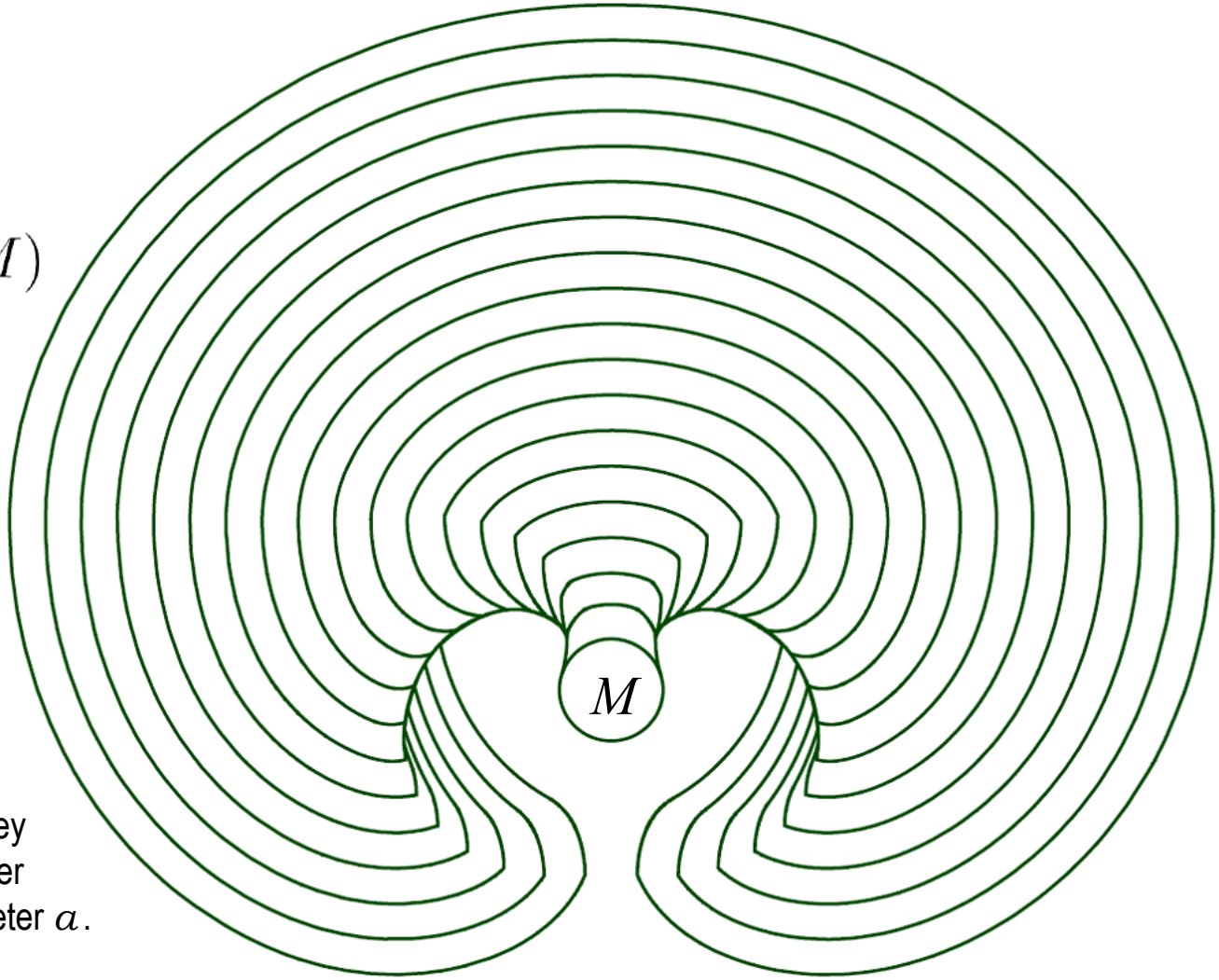
$$|u| \le 1, \quad a \le w \le 1, \quad v = (v_x, v_y)^T, \quad |v| \le \nu$$

Let *h* be the standard direction from the back to the front of the car. Using Isaacs´ transformation, we pass to the above two-dimensional dynamics. So, we deal with time-optimal problem in the plane.

Level sets of the value function

$$W(\tau, M) = \{(x, y) : \ V(x, y) \leq \tau\}$$

$W(\tau, M)$



We have developed an algorithm for the computation of level sets of the value function. In other words, our numerical procedure gives isochrones or wave fronts. The question is how they are looking in the differential game under study and how they depend on parameter a.

Families of semipermeable curves

First of all, for dynamic problems in the plane, it is very useful to investigate families of semipermeable curves. These families depend on system dynamics only. Based on the knowledge of families of semipermeable curves, one can determine lines in the plane on which the value function of time-optimal differential game can be discontinuous.

Roots of the Hamiltonian

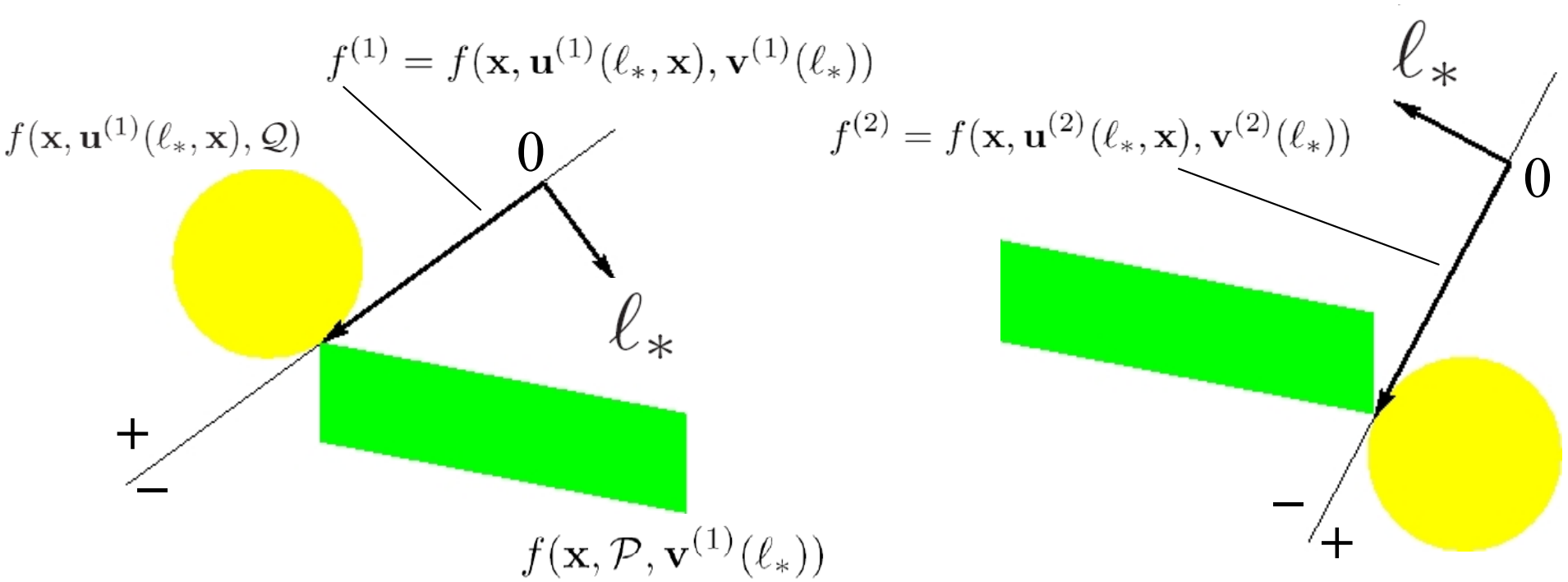
$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}, \mathbf{v}), \ \ \mathbf{x} \in R^2, \ \ \mathbf{u} \in \mathcal{P}, \ \ \mathbf{v} \in \mathcal{Q}$$

$$\min_{\mathbf{u}\in\mathcal{P}} \max_{\mathbf{v}\in\mathcal{Q}} \ell^T f(\mathbf{x}, \mathbf{u}, \mathbf{v}) = \max_{\mathbf{v}\in\mathcal{Q}} \min_{\mathbf{u}\in\mathcal{P}} \ell^T f(\mathbf{x}, \mathbf{u}, \mathbf{v})$$

$$H(\ell, \mathbf{x}) = \min_{\mathbf{u}\in\mathcal{P}} \max_{\mathbf{v}\in\mathcal{Q}} \ell^T f(\mathbf{x}, \mathbf{u}, \mathbf{v}) \ , \quad H(\ell_*, \mathbf{x}) = 0$$

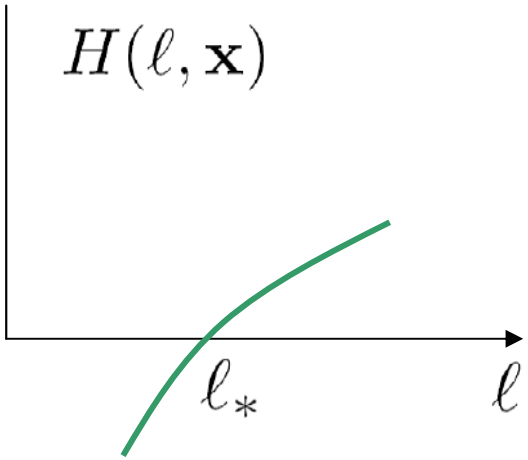
Smooth semipermeable curves are constructed due to analysis of roots of the Hamiltonian. For every fixed $\mathbf{x} \in R^2$, the dependency $\ell \to H(\ell, \mathbf{x})$ is analyzed and roots of the equation $H(\ell, \mathbf{x}) = 0$ are considered. Thereby, one can assume that $|\ell| = 1$.

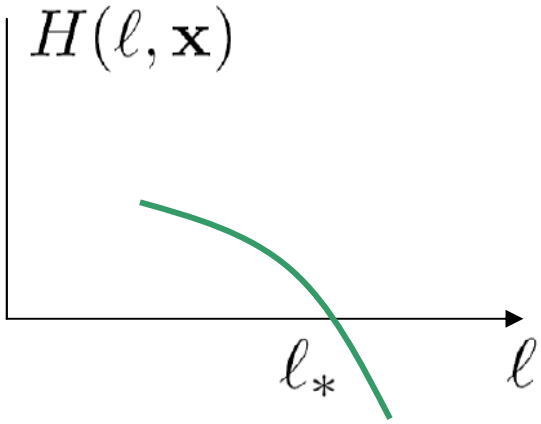
Separation of vectograms

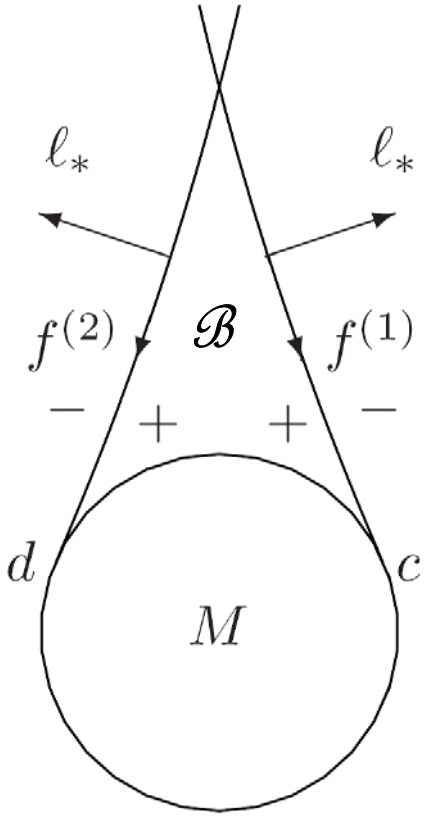


Let ℓ_* be the root of equation $H(\ell, \mathbf{x}) = 0$. The root corresponds to the separation of vectograms of players *P* and *E*. The half-plane which contains the vectogram of player *P* is labeled with "-", the opposite half-plane containing the vectogram of player *E* is provided with sign "+". The semipermeable direction is orthogonal to ℓ_*. We distinct two cases. In the first case, the semipermeable direction is obtained by rotating the vector ℓ_* through 90° clockwise; in the second case, the semipermeable direction is produced using counterclockwise rotation of ℓ_* through 90°.
Smooth curve whose tangent vector at every point coincides with a semipermeable direction is called semipermeable.

Strict roots of the first and second type







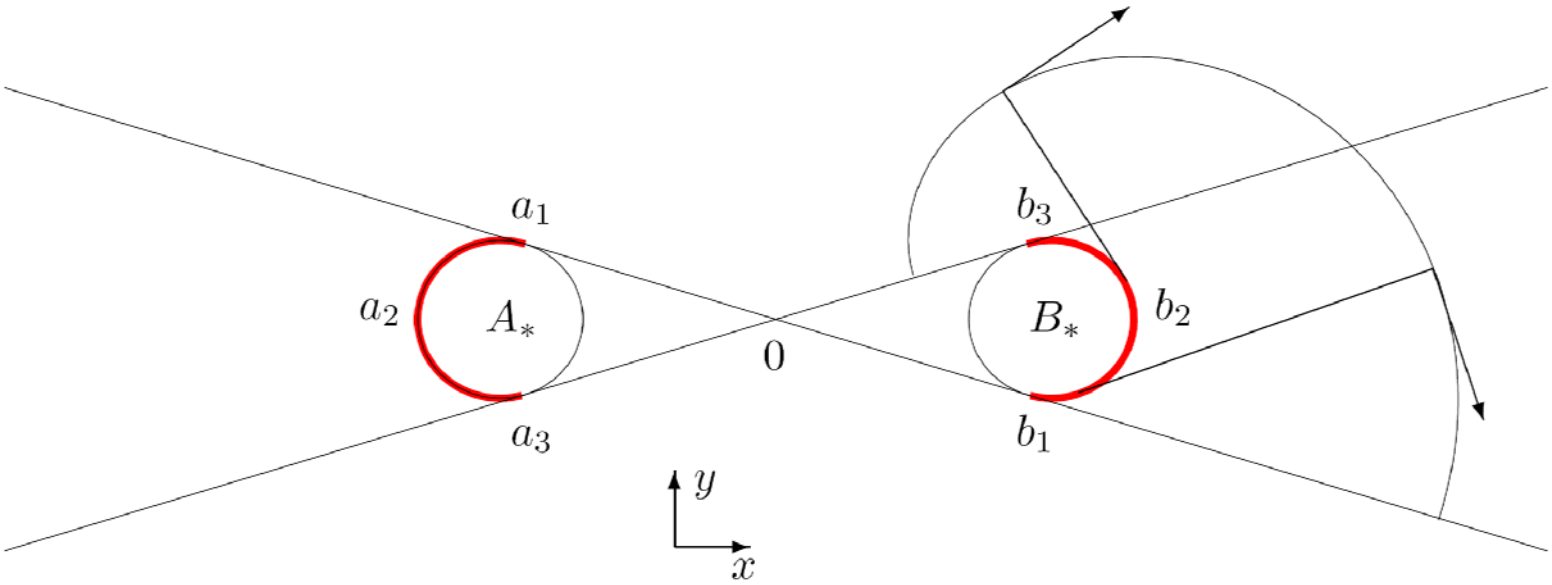
Assuming clockwise traveling around circumference of the unit circle, we have strict root minus to plus in the first case and strict root plus to minus in the second case.The picture on the right explains the role of semipermeable curves in the time-optimal game. The capture set $\mathscr{B}$ is completely defined by semipermeable curves of the first and second type emitted backward in time from, respectively, points *c* and *d* of the usable part.

Families of semipermeable curves in the classical homicidal chauffeur problem

$$a = 1, \quad |v| \le \nu, \quad \nu \in (0,1)$$

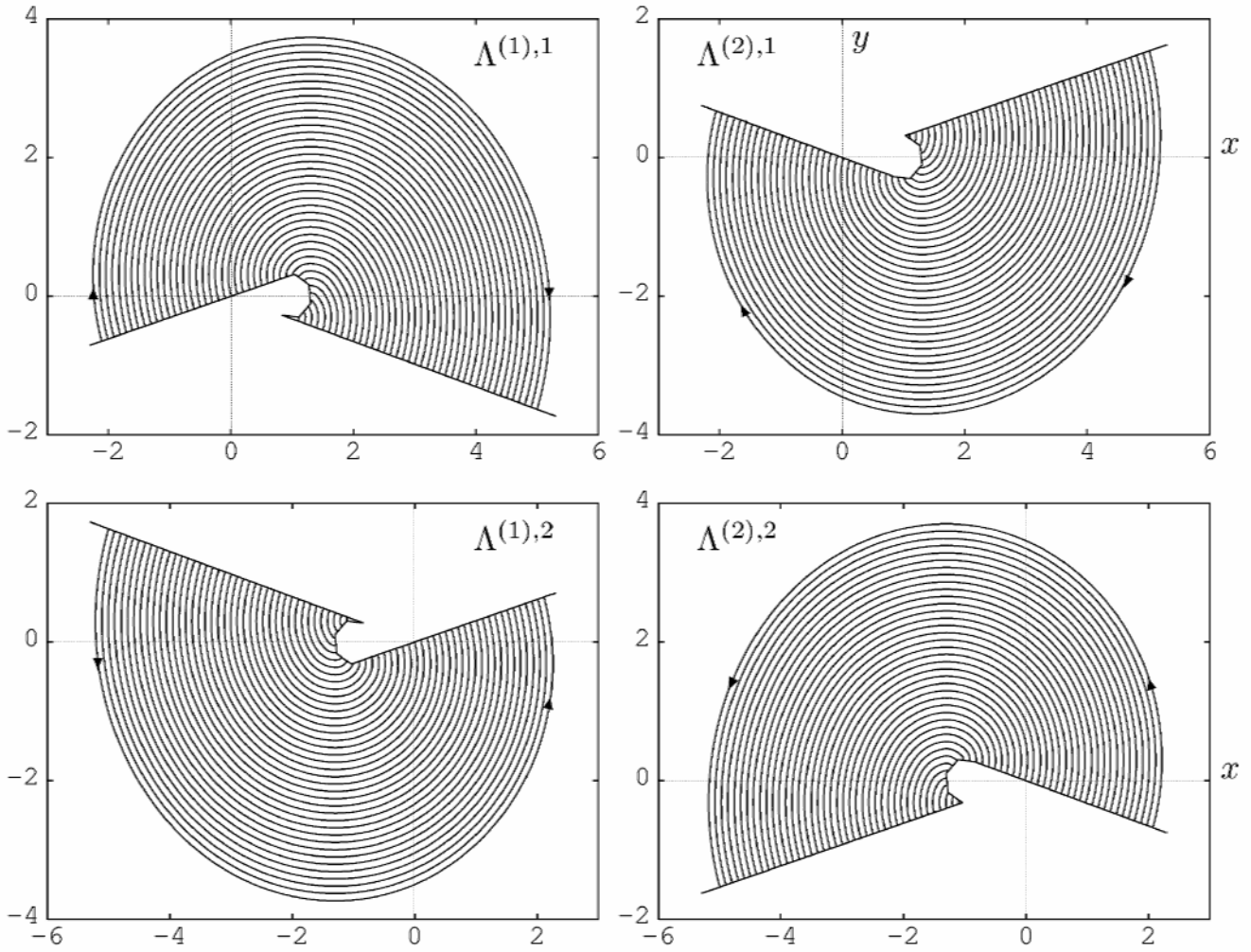
$$B_* = \{(x,y) : \ -y + v_x = 0, \ x - 1 + v_y = 0, |v| \le \nu\}$$

$$A_* = \{(x,y) : \ y + v_x = 0, \ -x - 1 + v_y = 0, |v| \le \nu\}$$



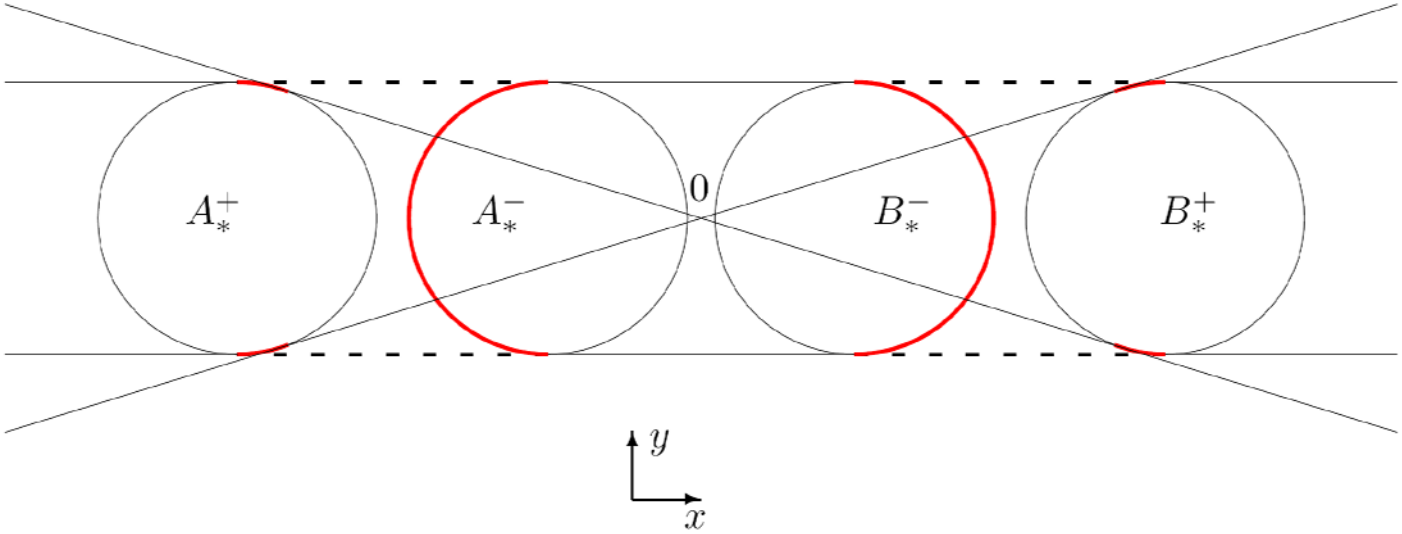
For the classical homicidal chauffeur problem, smooth semipermeable curves are involutes of two thick arcs on the boundary of circles B_* and A_*. The arrows correspond to traveling along semipermeable curves backward in time.

Families of semipermeable curves for homicidal chauffeur dynamics



In the classical problem, we have two families $\Lambda^{(1),1}$ and $\Lambda^{(1),2}$ of smooth semipermeable curves of the first type and two families $\Lambda^{(2),1}$ and $\Lambda^{(2),2}$ of the second type. If player *E* is immovable, then $\nu = 0$ and sets B_*, A_* become points. Thereby, the involutes are replaced by semicircles.

Basis for families of semipermeable curves in reinforced homicidal chauffeur dynamics



$$a = 0.33$$
$$\nu = 0.3$$

$$B_*^+ = \{(x,y):\ -y + v_x = 0,\ x - 1 + v_y = 0,\ |v| \le \nu\}$$

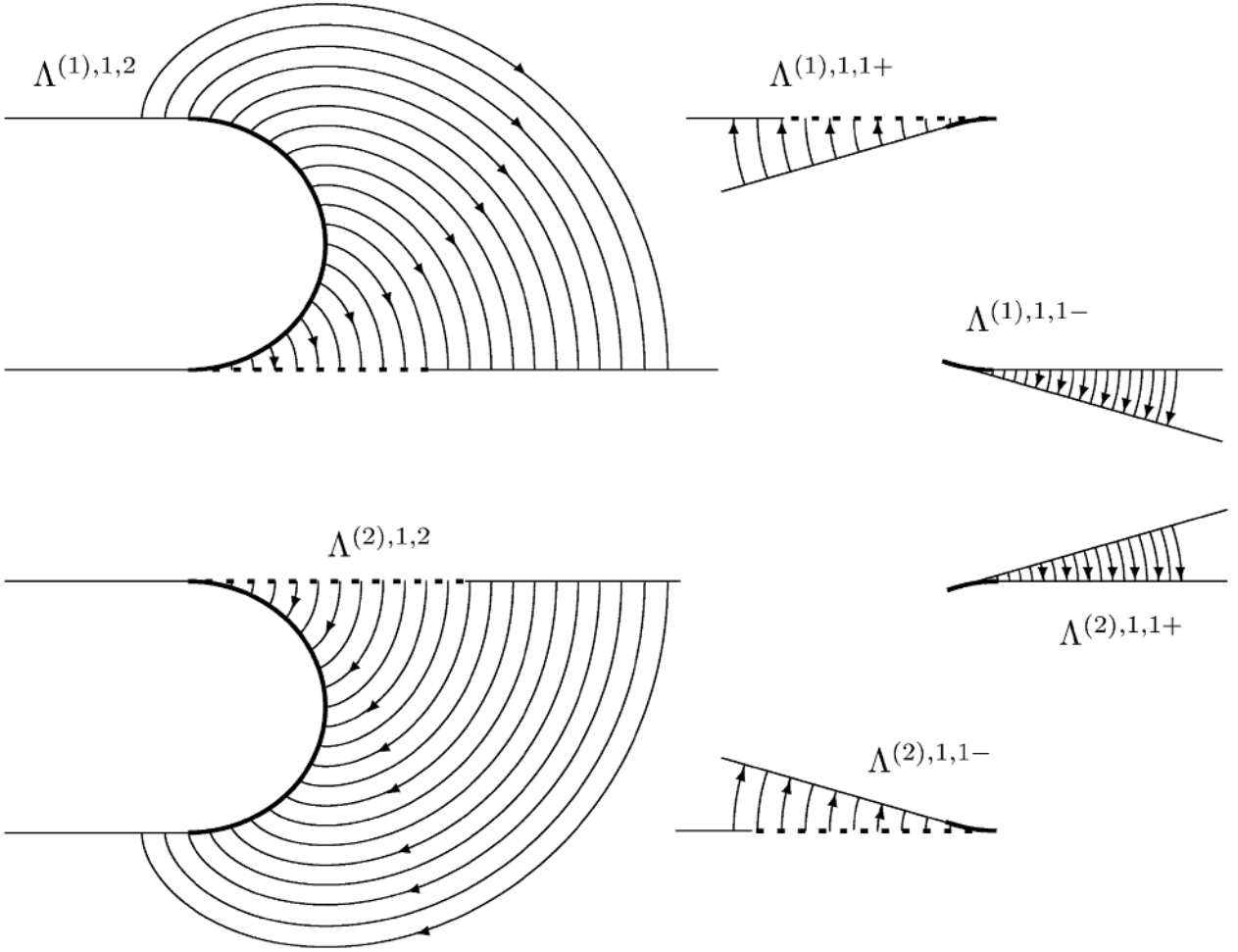
$$A_*^+ = -B_*^+$$

$$B_*^- = \{(x,y):\ -y + v_x = 0,\ x - a + v_y = 0,\ |v| \le \nu\}$$

$$A_*^- = -B_*^-$$

For the reinforced homicidal chauffeur dynamics, the situation is more complicated. However, all families can be constructed. On this slide, those arcs which are used for the construction of smooth semipermeable curves in the case $a \ge -\nu$ are shown in red. Semipermeable curves are involutes of these arcs.

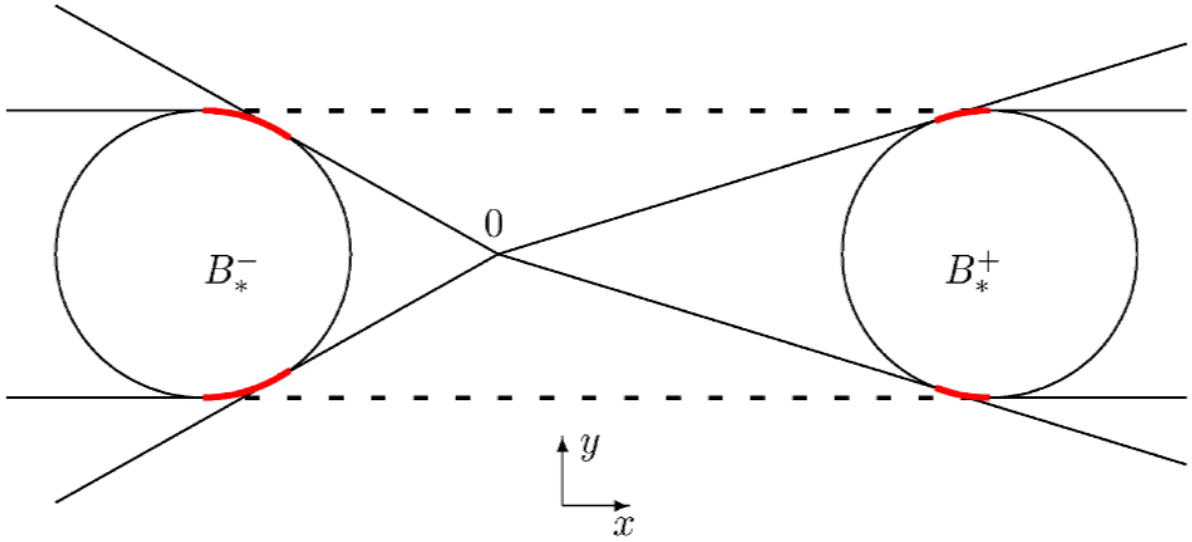
Families of semipermeable curves for reinforced homicidal chauffeur dynamics ($a \geq -\nu$)



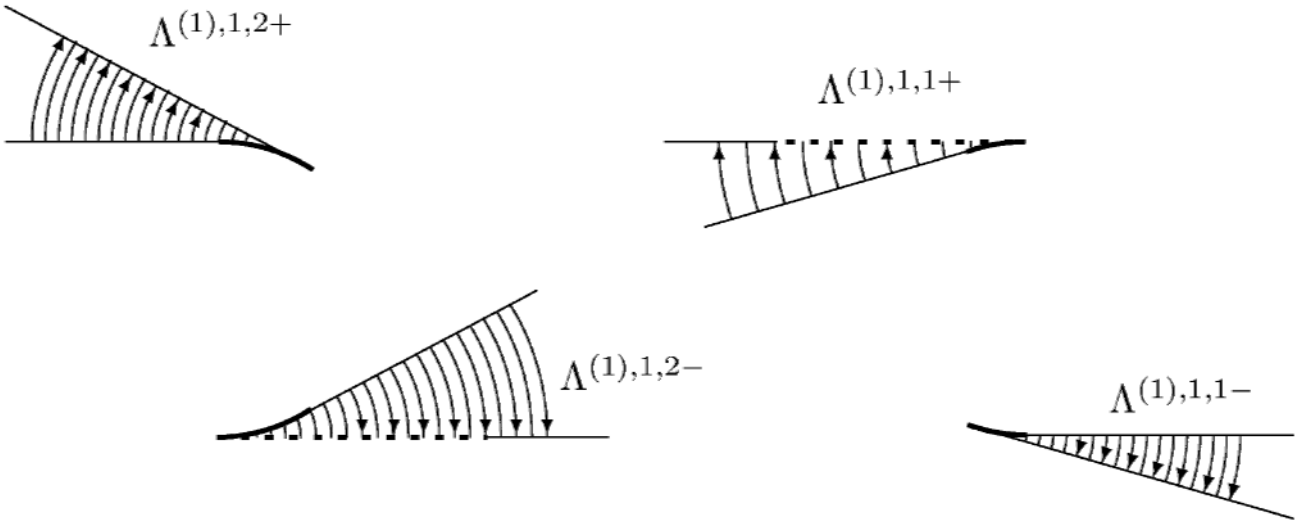
$\Lambda^{(2),2,1+}$, $\Lambda^{(2),2,1-}$, $\Lambda^{(2),2,2}$, $\Lambda^{(1),2,1+}$, $\Lambda^{(1),2,1-}$

Here, the families of smooth semipermeable curves of the first and second type corresponding to the extremal control $u = 1$ are presented for the case $a \geq -\nu$. The families that correspond to $u = -1$ can be obtained using reflection about the vertical axis.

Families of semipermeable curves for reinforced homicidal chauffeur dynamics ($a < -\nu$)



On this slide, those arcs which are used for the construction of smooth semipermeable curves in the case $a < -\nu$ are marked with red color. Below, the families corresponding to the extremal control $u = 1$ are presented. The other families can be obtained by reflections about the horizontal and vertical axes.



Level sets of the value function

and reachable sets

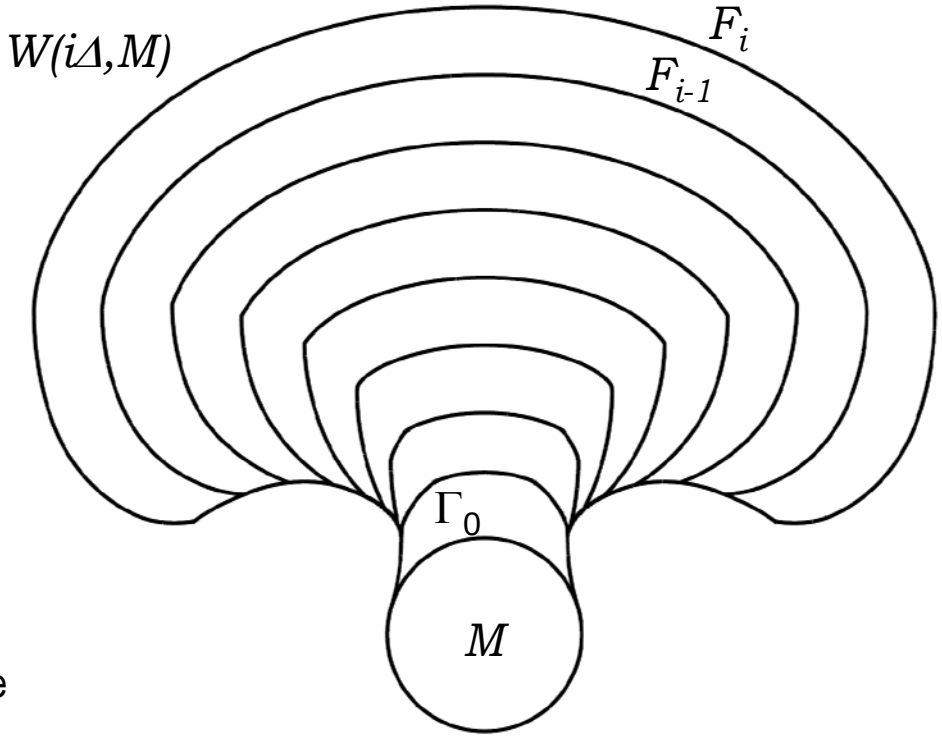
Let us now present results of computation of level sets $W(\tau, M)$ of the value function $V(x, y)$. Note that if $\nu = 0$ (player *E* is immovable), level set $W(\tau, M)$ can be interpreted (for geometric coordinates) as a time-limited reachable set of player *P* corresponding to time τ provided that the initial set is *M* and the initial direction *h* is oriented along axis *y*.

Backward construction of level sets of the value function

$$\Gamma_0 = \mathrm{cl}\{\mathbf{x} \in \partial M : \min_{\mathbf{u}\in\mathcal{P}} \max_{\mathbf{v}\in\mathcal{Q}} \ell^T f(\mathbf{x}, \mathbf{u}, \mathbf{v}) < 0\}$$

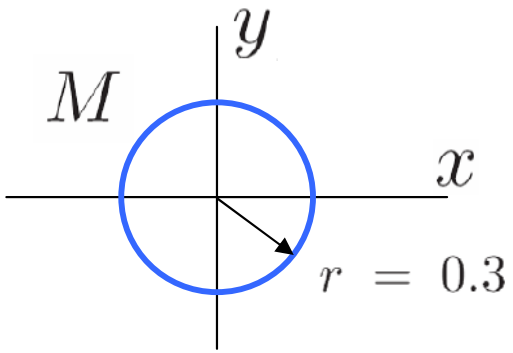
$$W(\Delta, M) \subset W(2\Delta, M) \subset \ldots \subset W(i\Delta, M) \subset \ldots \subset W(\tau_f, M)$$

The collection of all points $(x, y) \in \partial W(\tau, M)$ such that $V(x, y) = \tau$ is called the front corresponding to the reverse time τ. Our computational procedure for the construction of level sets runs backward in time on the interval $[0, \tau_f]$. The construction starts with the computation of usable part Γ_0 on the boundary of the set *M*. We use an automatic adjustment of the step width Δ of the backward procedure. The initial value of Δ which is usually equal to 0.01 can decrease up to ten times in the course of construction. On the next slides, the fronts are depicted with the step being a multiple value of the initial step Δ. For example, the fronts on slide 28 are shown with the step 0.1. The sets *Q* and *M* are approximated by inscribed regular *n*-sided polygons. Usually, *n* = 8 and *n* = 30 for *Q* and *M,* respectively.



Computed variants

$$\dot{x} = -y\ u + v_x \qquad |u| \le 1, \quad a \le w \le 1$$
$$\dot{y} = \quad x\ u - w + v_y \qquad |v| \le \nu$$

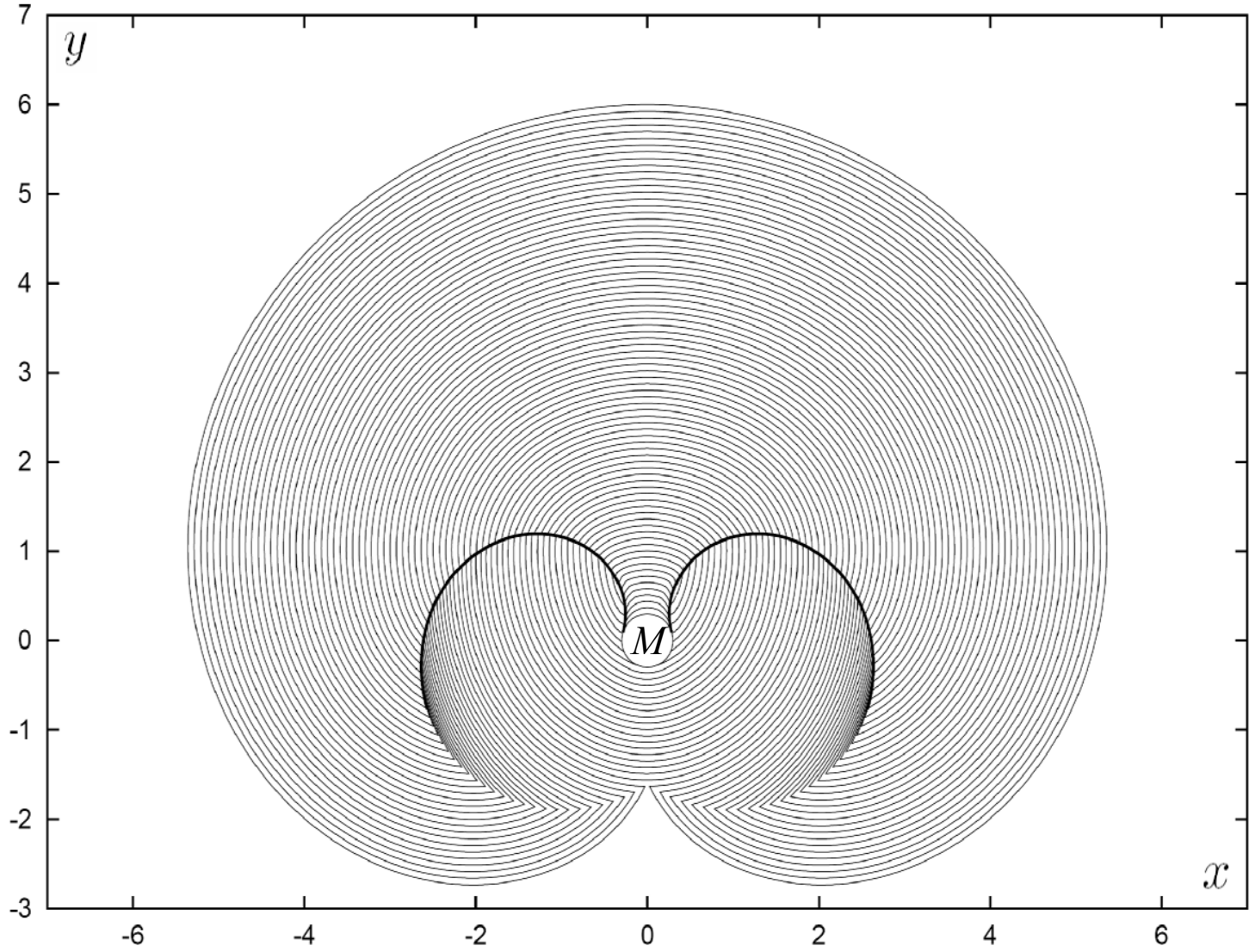


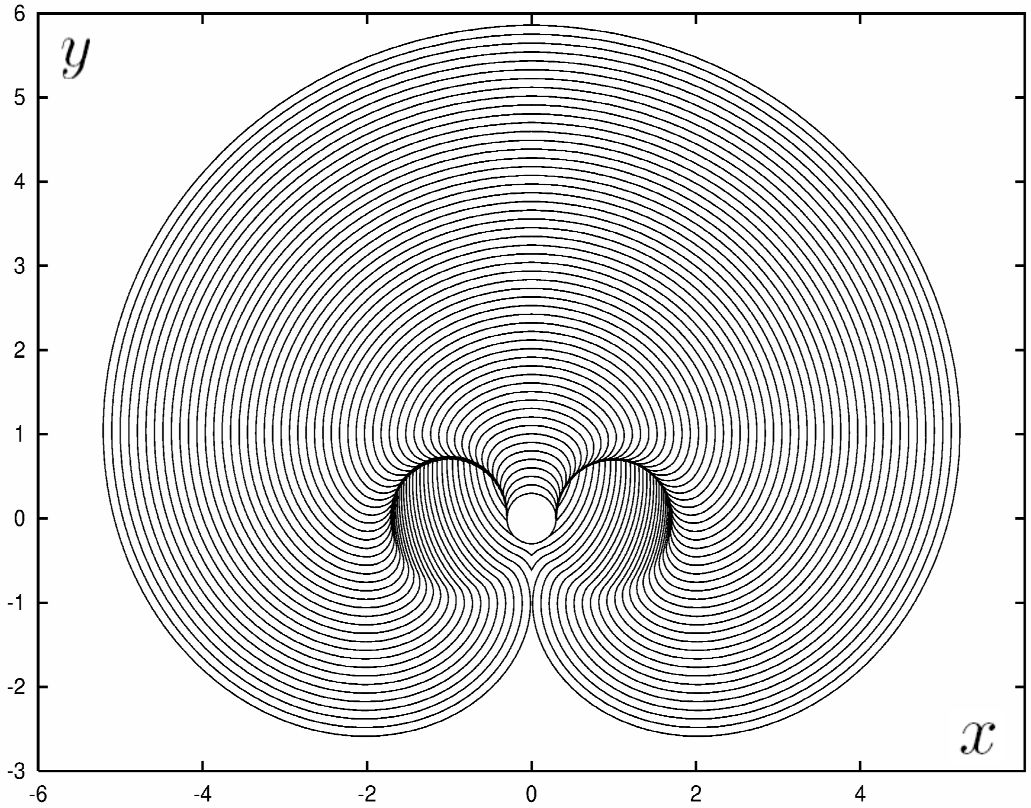
1. $a = 1, \quad |v| \le 0.3;$
2. $a = 1, \quad v \equiv 0;$
3. $a = 0.25, \quad |v| \le 0.3;$
4. $a = 0.25, \quad v \equiv 0;$
5. $a = -0.1, \quad |v| \le 0.3;$
6. $a = -0.1, \quad v \equiv 0;$
7. $a = -0.6, \quad |v| \le 0.3;$
8. $a = -0.6, \quad v \equiv 0;$
9. $a = -1, \quad |v| \le 0.3;$
10. $a = -1, \quad v \equiv 0.$

On the top, the dynamics and bounds on controls of the players are given once more. The terminal set is a circle of radius 0.3. On the bottom, variants for which numerical results will be presented are listed. The variants of the right column correspond to an immovable player *E*.

Classical variant of the homicidal chauffeur problem, $\tau_f = 10.3\,,\ \ \nu = 0.3$

For the first variant (classical homicidal chauffeur problem), the value function is discontinuous on two barrier lines emanating from the right and left endpoints of the usable part. One can verify that these lines are semipermeable curves of families $\Lambda^{(1),1}$ and $\Lambda^{(2),2}$, respectively. After the termination, the barriers are continued by the lines formed of the corner points on the fronts of the level sets. The value function is not differentiable on these lines.
After bending round the right and left barriers, the right and left parts of the front meet on the vertical axis at time $\tau = 7.82$. A united front occurs. Further constructions until the time $\tau_f = 10.3$ are done for the inner part of the united front only.



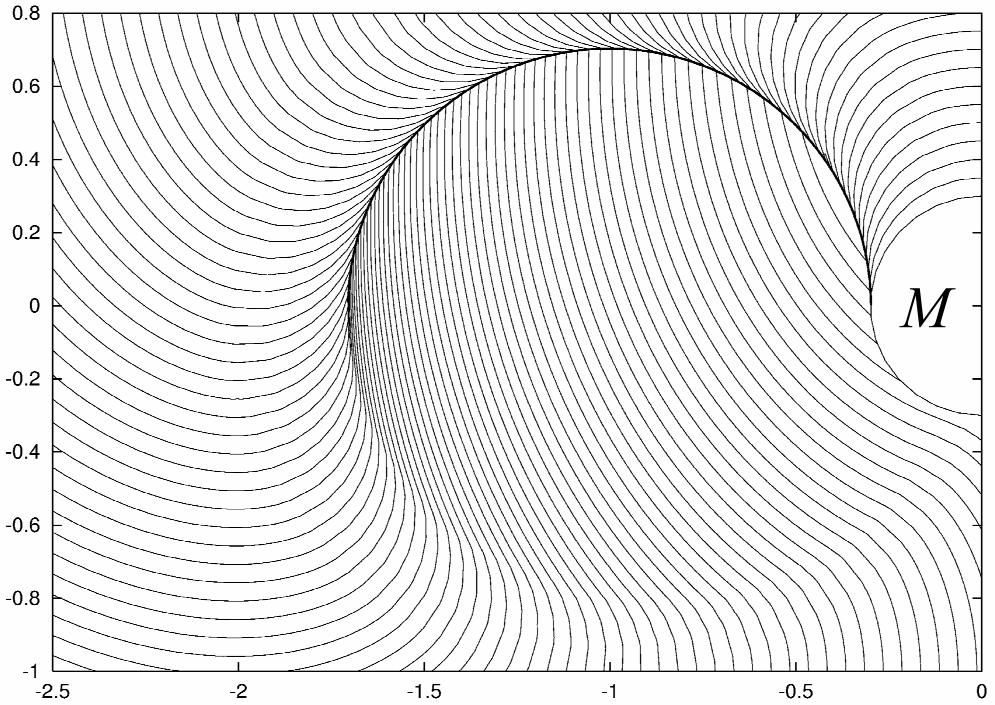


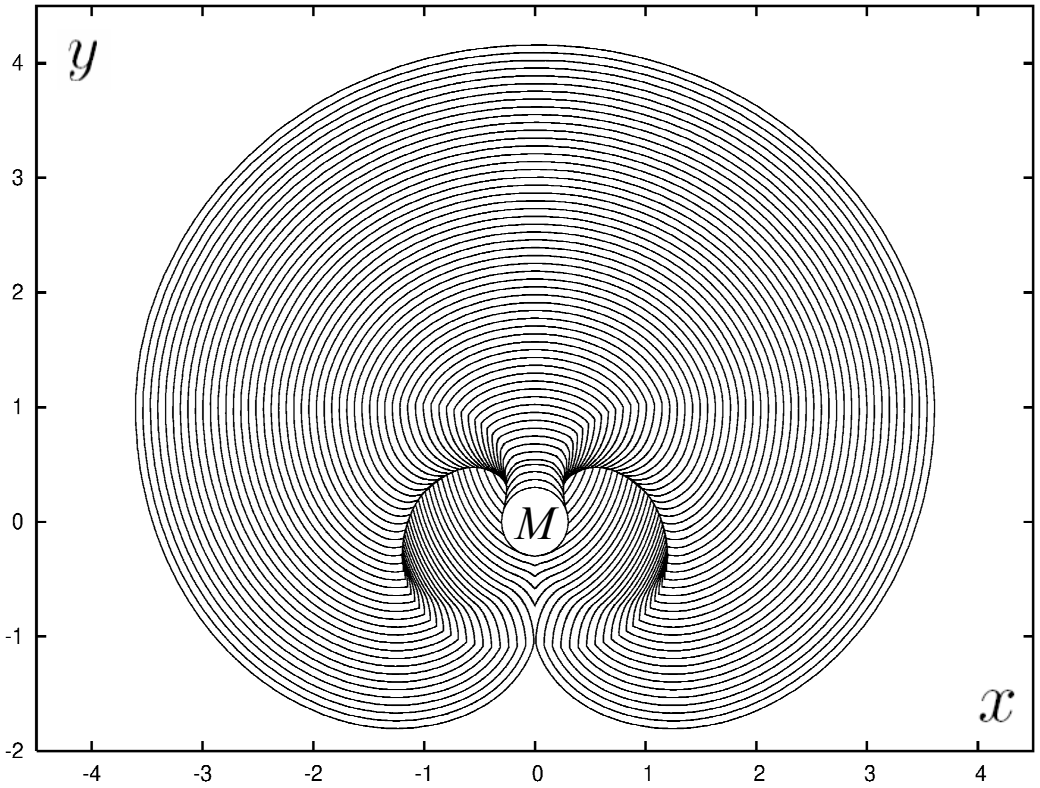
$$a = 1,\ v \equiv 0$$

$$\tau_f = 6.16$$

Isaacs-Dubins´ car

The principal difference between results for variants 1 and 2 is the absence of corner points on the fronts after bending round the barrier lines in variant 2. The barrier lines terminate on the horizontal axis.

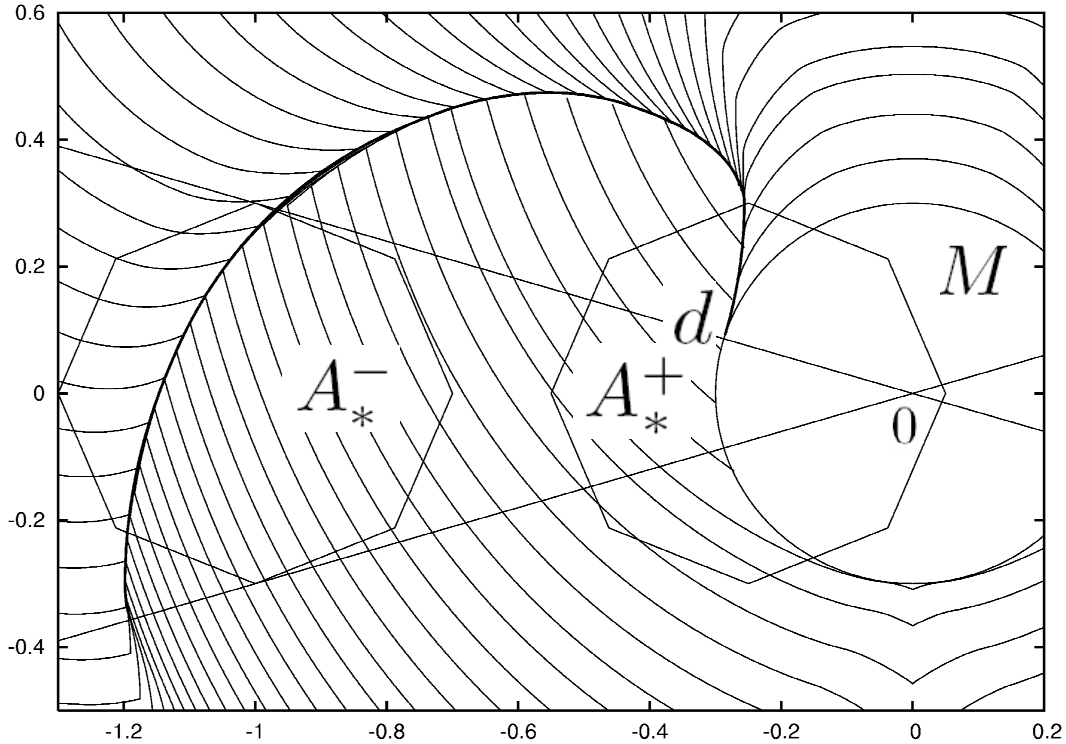




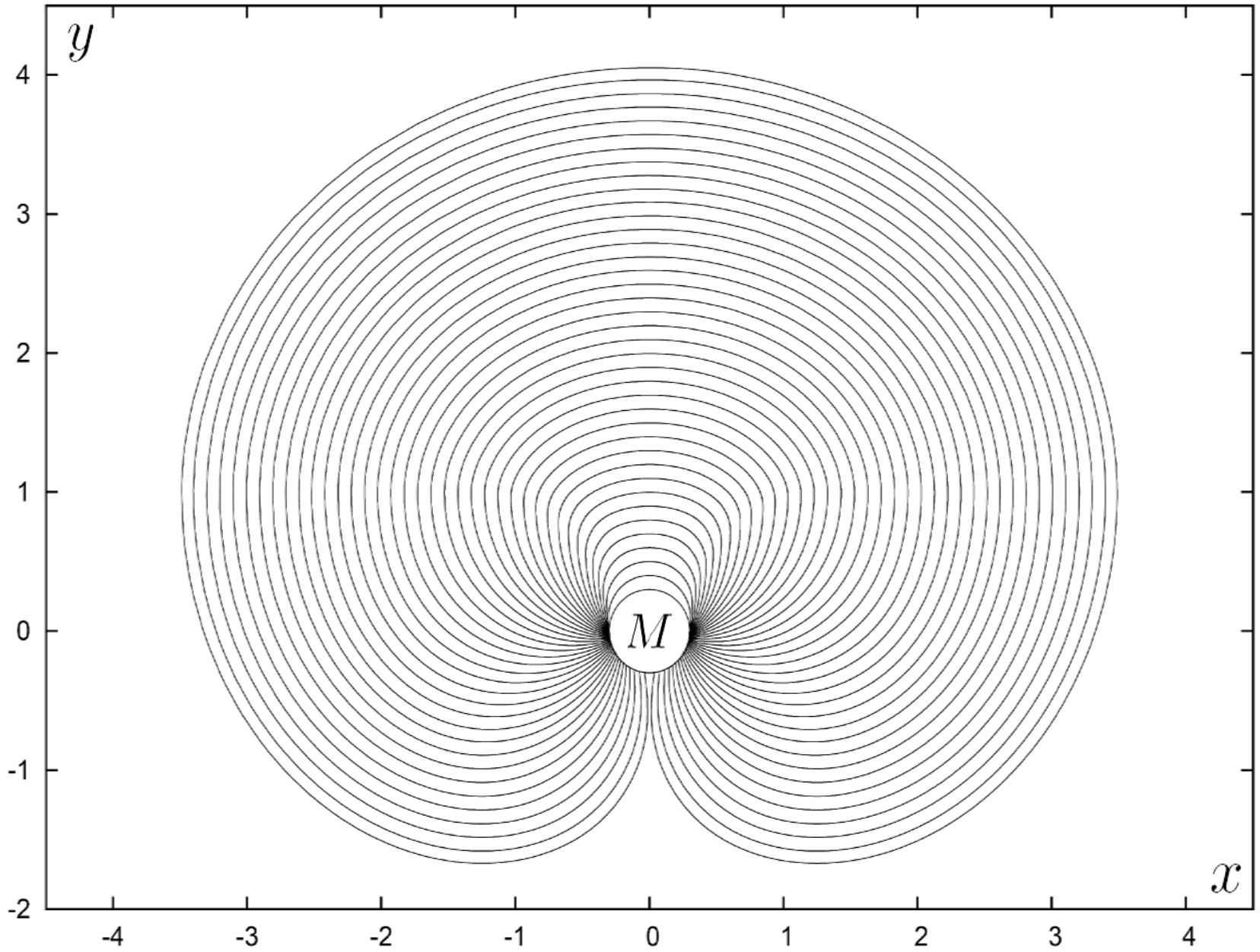
$$a = 0.25, \ \nu = 0.3$$

$$\tau_f = 6.7$$

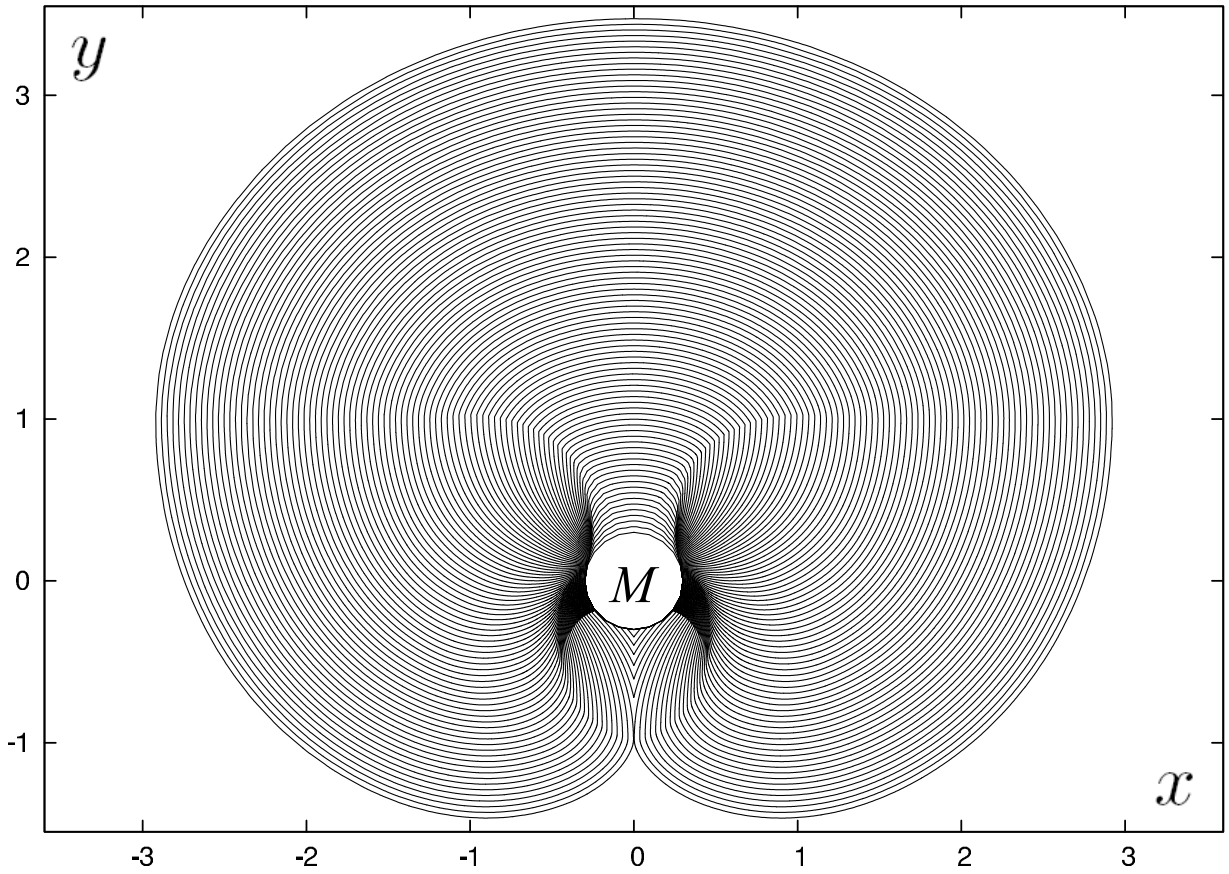
In the computations, the circular constraint *Q* on the control of player *E* is approximated by a regular octagon. This slide presents results for variant 3. An enlarged fragment on the right shows additionally sets A_*^- and A_*^+ . These sets are octagons as well.



$$a = 0.25,\ v \equiv 0,\ \tau_f = 3.8$$



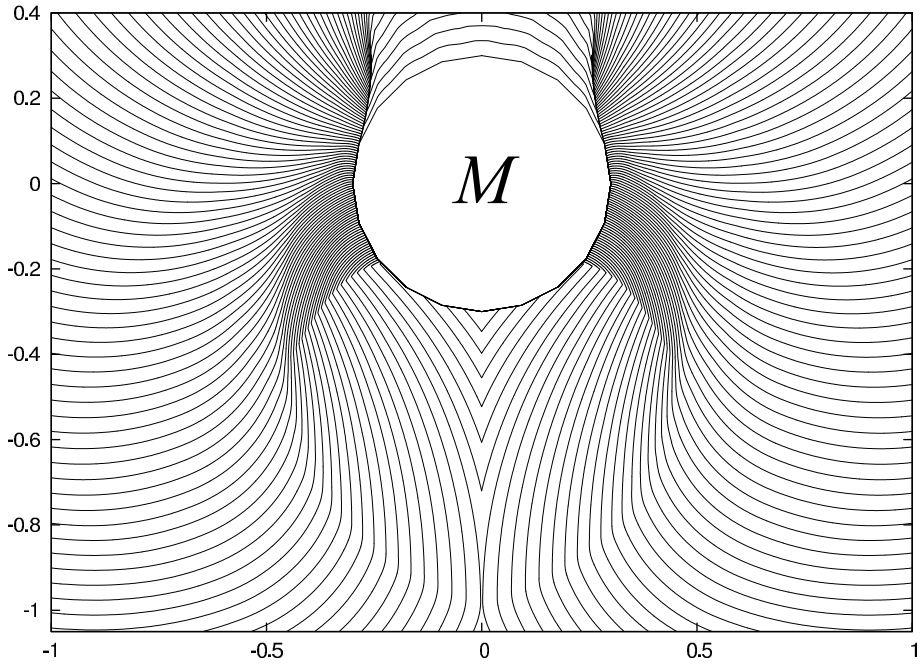
In variant 4, player *E* is immovable. The computation shows that the value function is continuous everywhere outside set *M*.

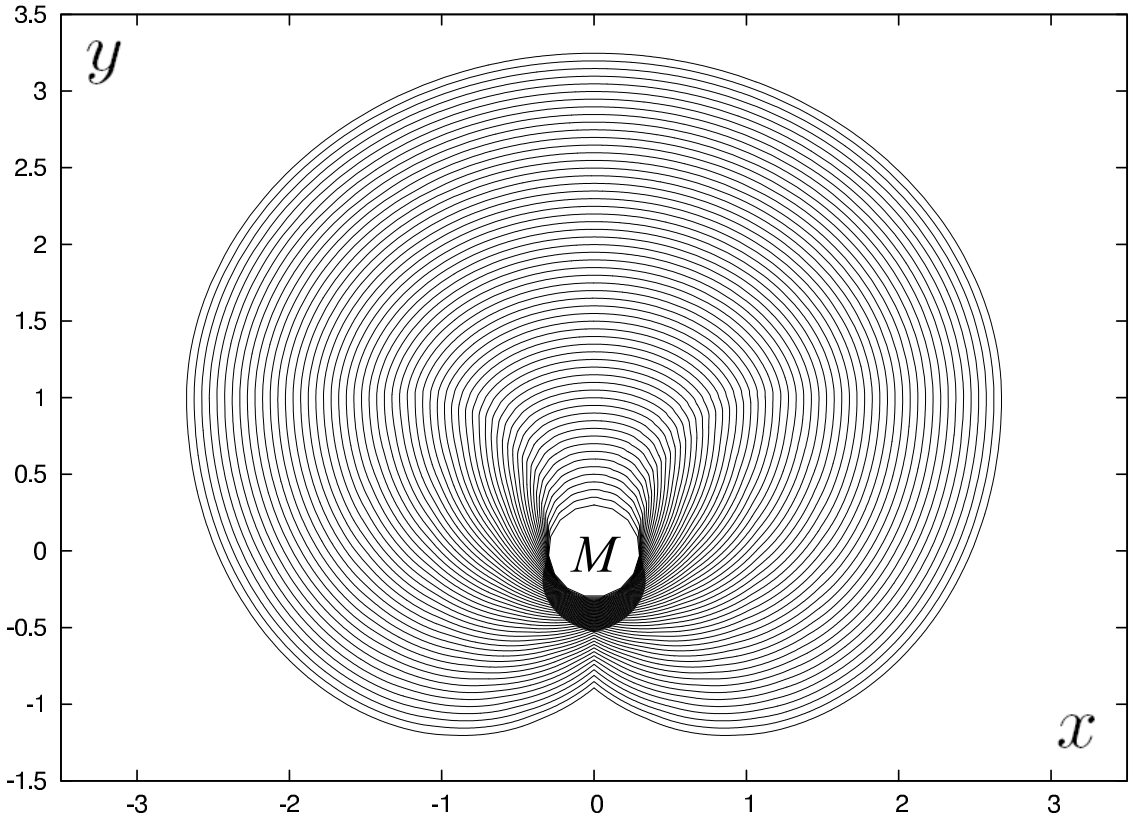


$$a = -0.1,\ \nu = 0.3$$

$$\tau_f = 4.88$$

In variant 5, the value of parameter a is negative. Note that barrier lines become shorter than those ones in variant 3. The knowledge of families of semipermeable curves allows us to predict this fact before computing level sets. After bending round the barrier lines, the ends of the front go down along the negative sides of the barriers and then move along the boundary of the terminal set. Parts of the front near the right and left sides of the terminal set and far from it move with different velocities.

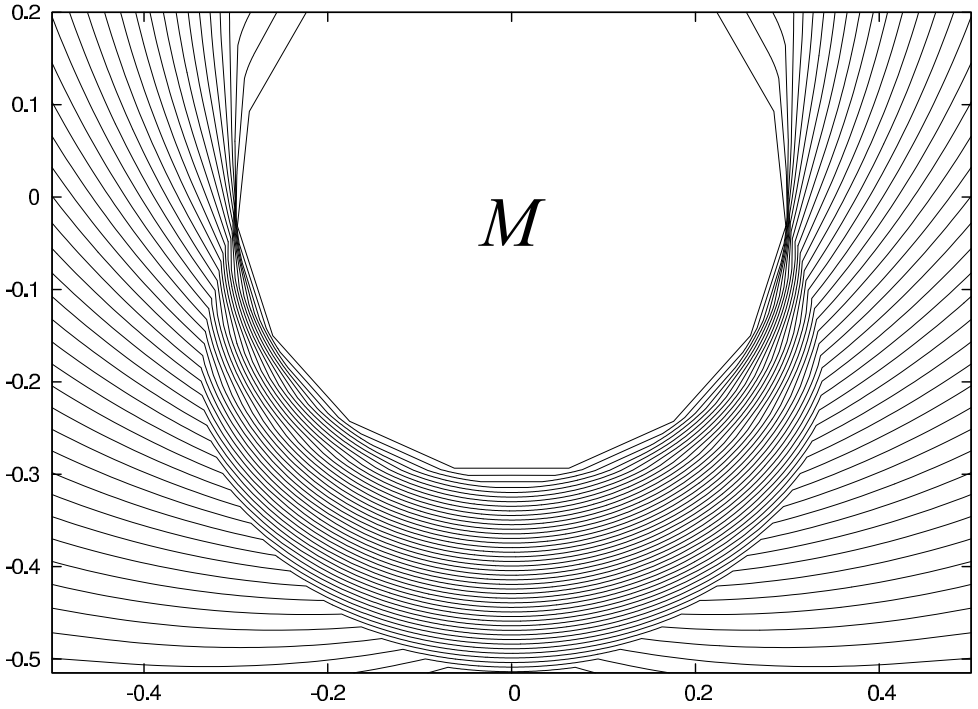


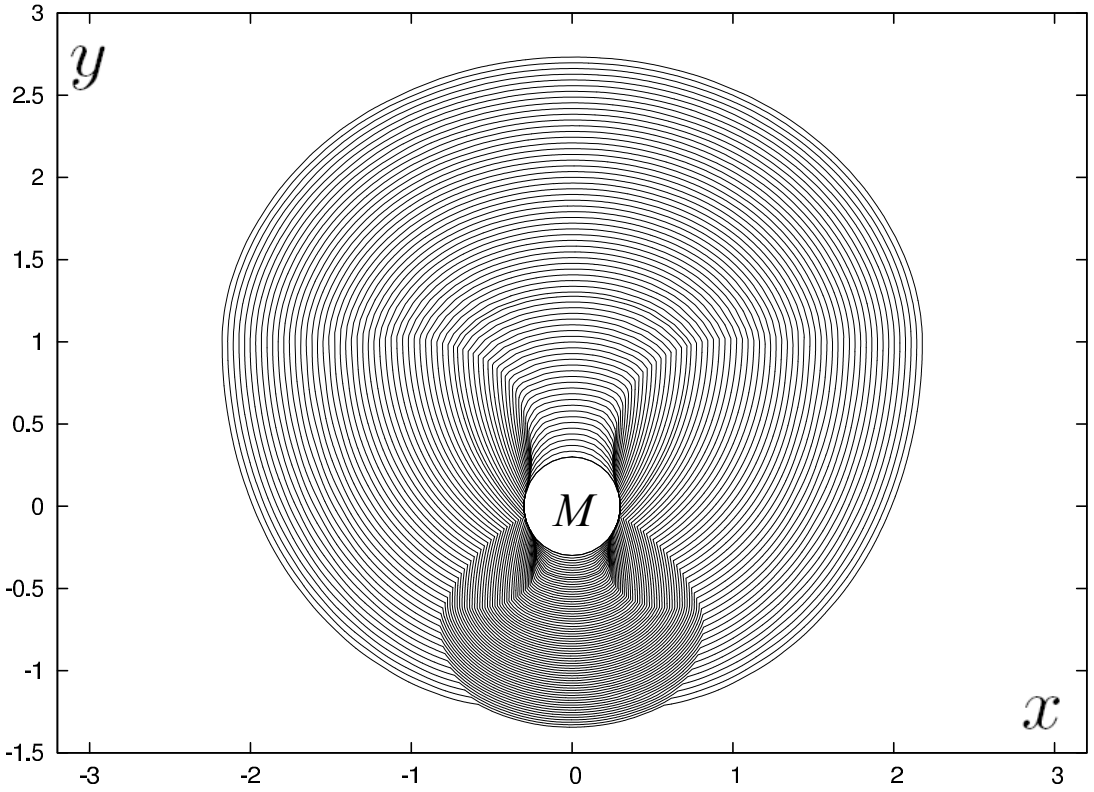


$$a = -0.1, \; v \equiv 0$$

$$\tau_f = 3$$

If we consider immovable player *E* for the same value a = -0.1, an additional usable part in the form of lower semicircle occurs on ∂M. The upper usable part is the upper semicircle on ∂M. Parts of the fronts above set *M* and below it propagate at different velocities.



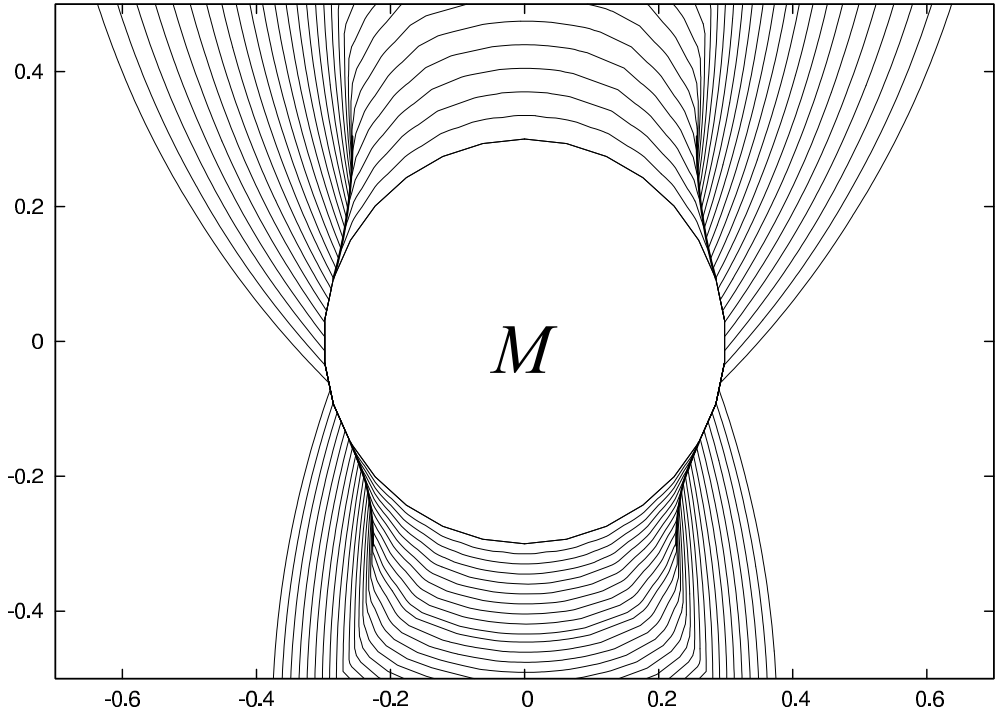


$$a = -0.6,\ \nu = 0.3$$

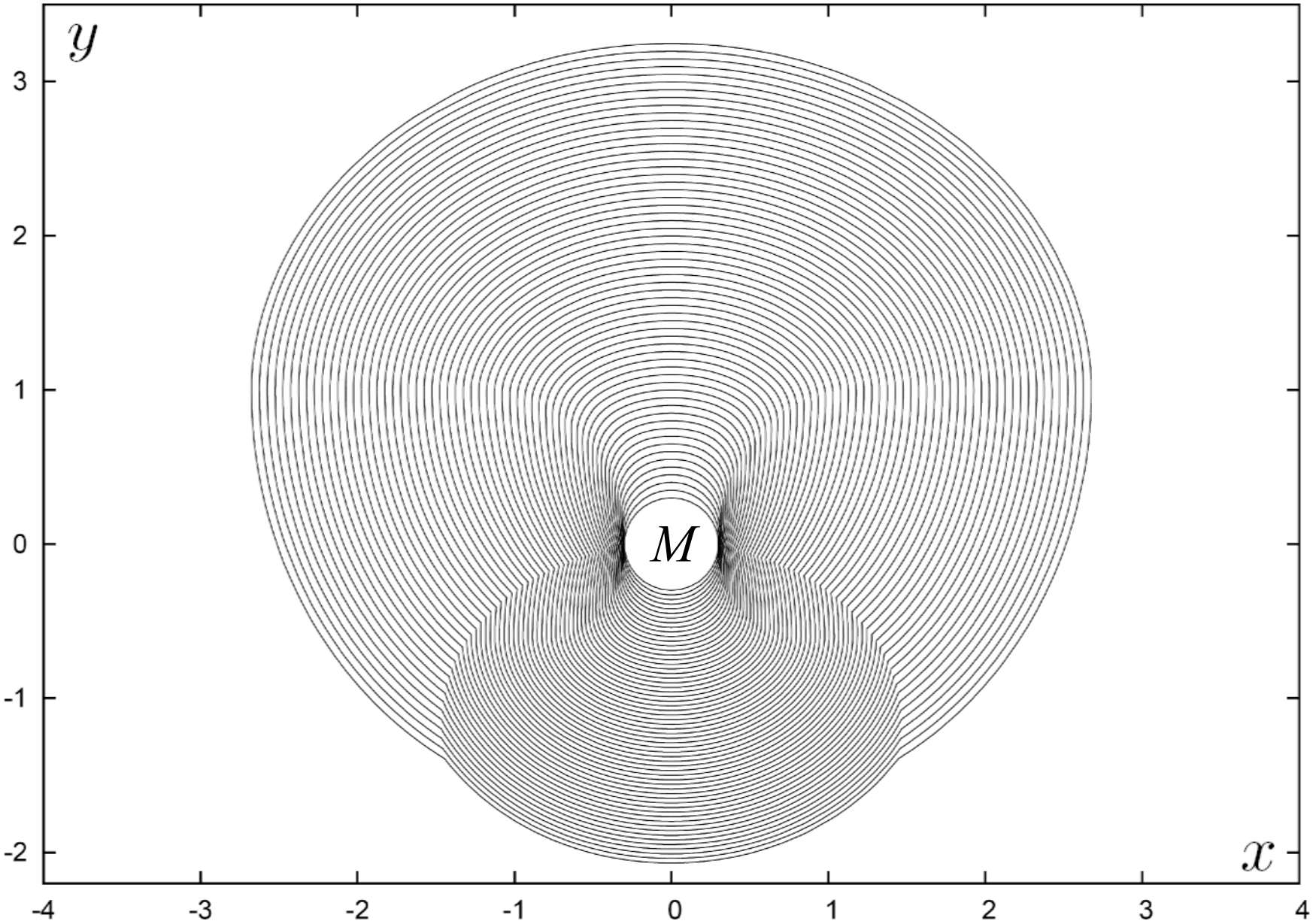
$$\tau_f = 3.5$$

For variant 7, there is also an additional lower usable part. However, both lower and upper usable parts are smaller than a semicircle. The right and left barriers emanating from the endpoints of the upper usable part are semipermeable curves of the families $\Lambda^{(1),1,1+}$ and $\Lambda^{(2),2,1+}$, respectively. They terminate, as expected, on horizontal line $y = 0.3$. The left and right barriers emanating from the endpoints of the lower usable part are semipermeable curves of the families $\Lambda^{(1),1,2-}$ and $\Lambda^{(2),2,2-}$. They terminate on horizontal line $y = -0.3$.

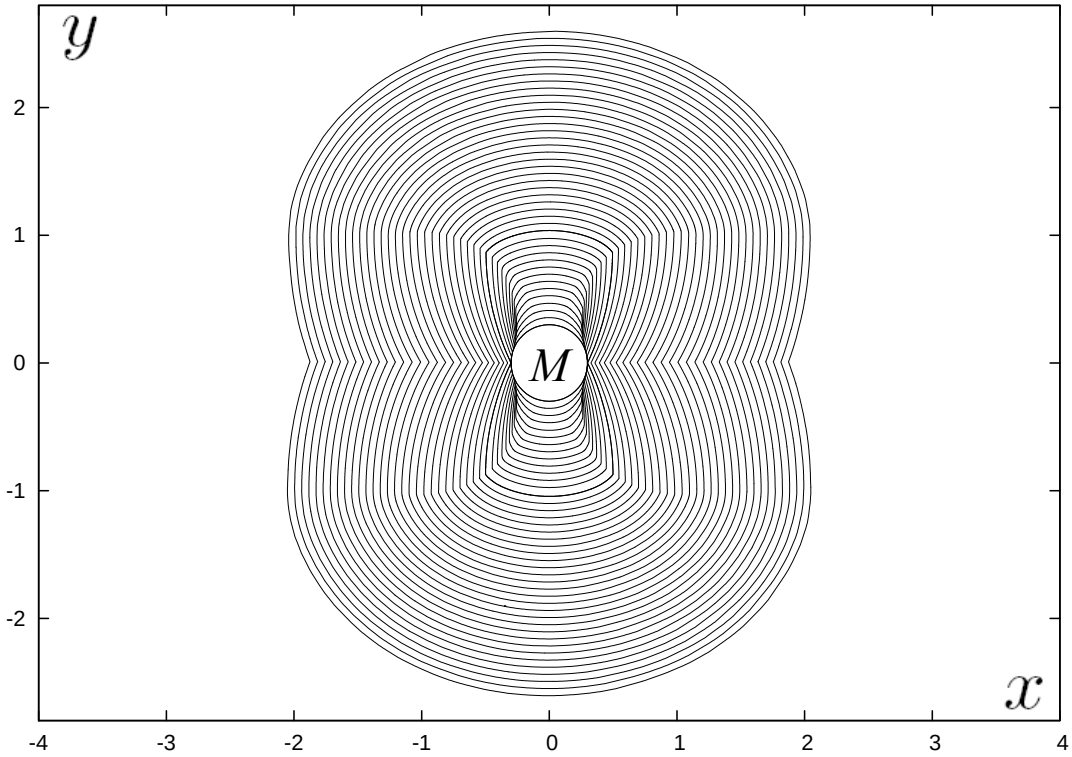
Fronts generated by the upper and lower usable parts encounter at time $\tau = 1.41$.



$$a = -0.6,\ v \equiv 0,\ \tau_f = 3$$



Here, we see that lower parts of the fronts propagate more rapidly comparing to variant 6 where $v \equiv 0$ as well but the value of a is greater.



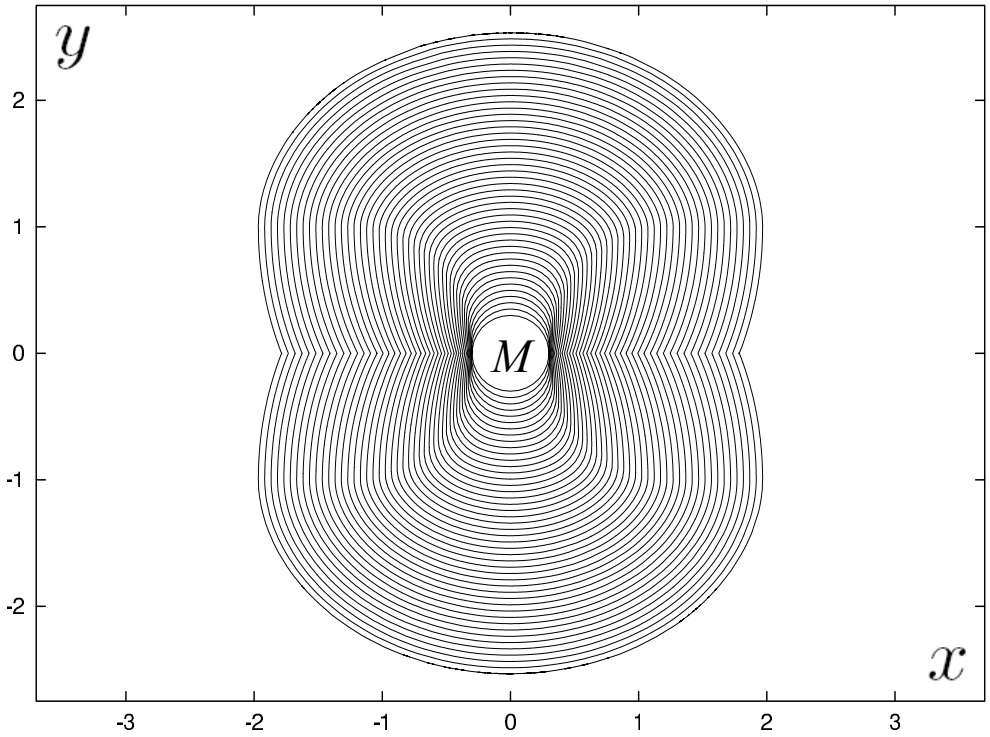
$$a = -1,\ \nu = 0.3$$

$$\tau_f = 3.28$$

Reeds and Shepp´s car

$$a = -1,\ v \equiv 0$$

$$\tau_f = 2.25$$



Here, computation results for variants 9 and 10 are presented. The level sets are symmetrical with respect to both horizontal axis *x* and vertical axis *y*. For variant 9, there still exist discontinuity lines of the value function near the set *M*.

Reachable sets for a point initial state

Let us now present results of construction of level sets for a point terminal set. It will be assumed that player *E* is immovable. As it was already mentioned, in this case, level sets of the value function coincide with reachable sets of player *P* written in geometric coordinates.
Due to specifics of our numerical procedure, the terminal set is considered to be a small circle of radius 0.01 instead of a point.

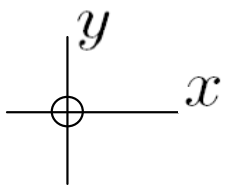
Car with the reinforced dynamics

$$\dot{x}_p = w \sin\theta$$
$$\dot{y}_p = w \cos\theta$$
$$\dot{\theta} = u \qquad\qquad |u| \le 1, \quad a \le w \le 1$$

$$x_{p0} = 0, \; y_{p0} = 0, \; \theta_0 = 0$$

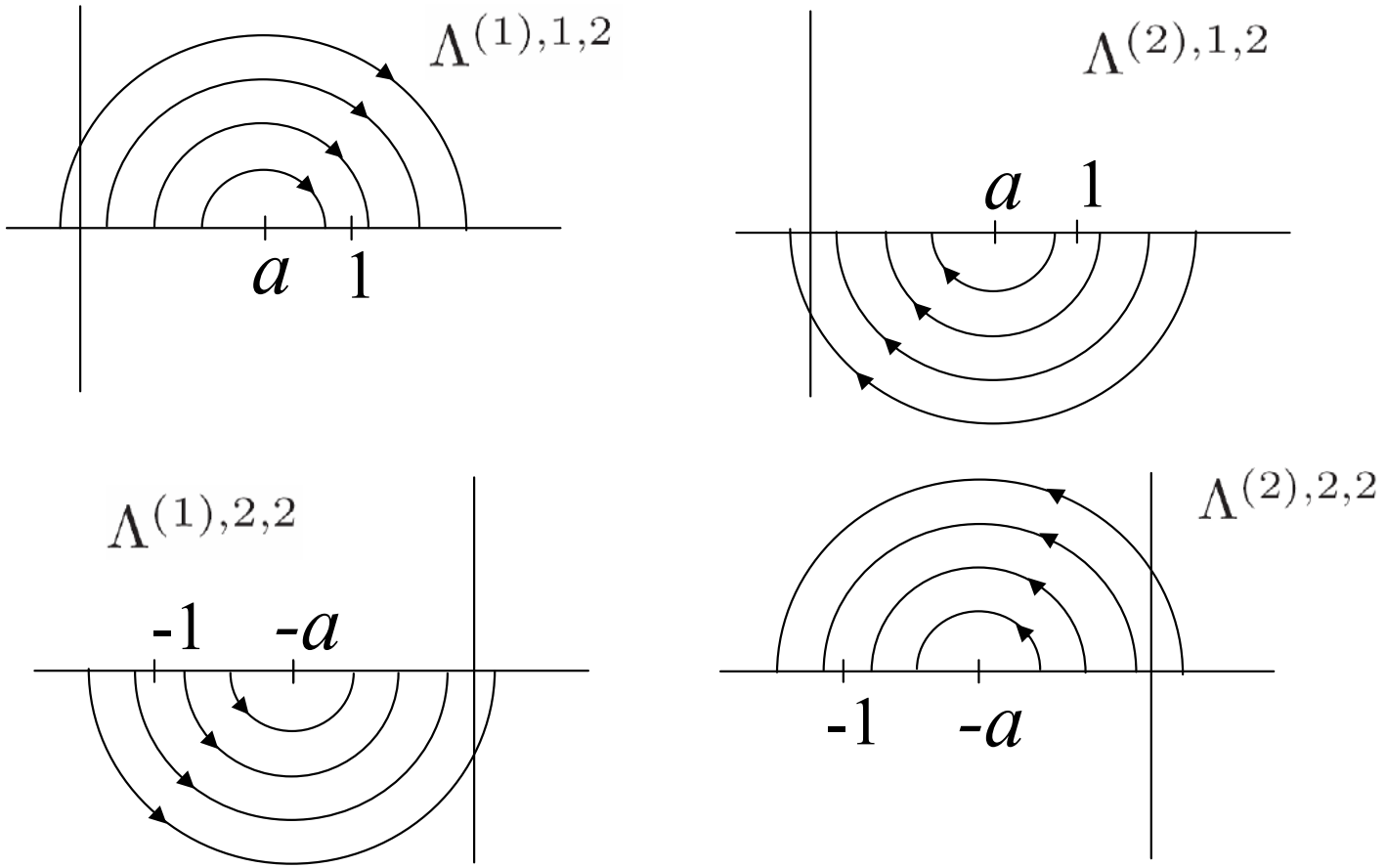
$$\dot{x} = -y \; u$$
$$\dot{y} = \quad x \; u - w \qquad |u| \le 1, \quad a \le w \le 1$$

$$M = \{0\}$$



The dynamics of player *P* in original and reduced coordinates are presented.

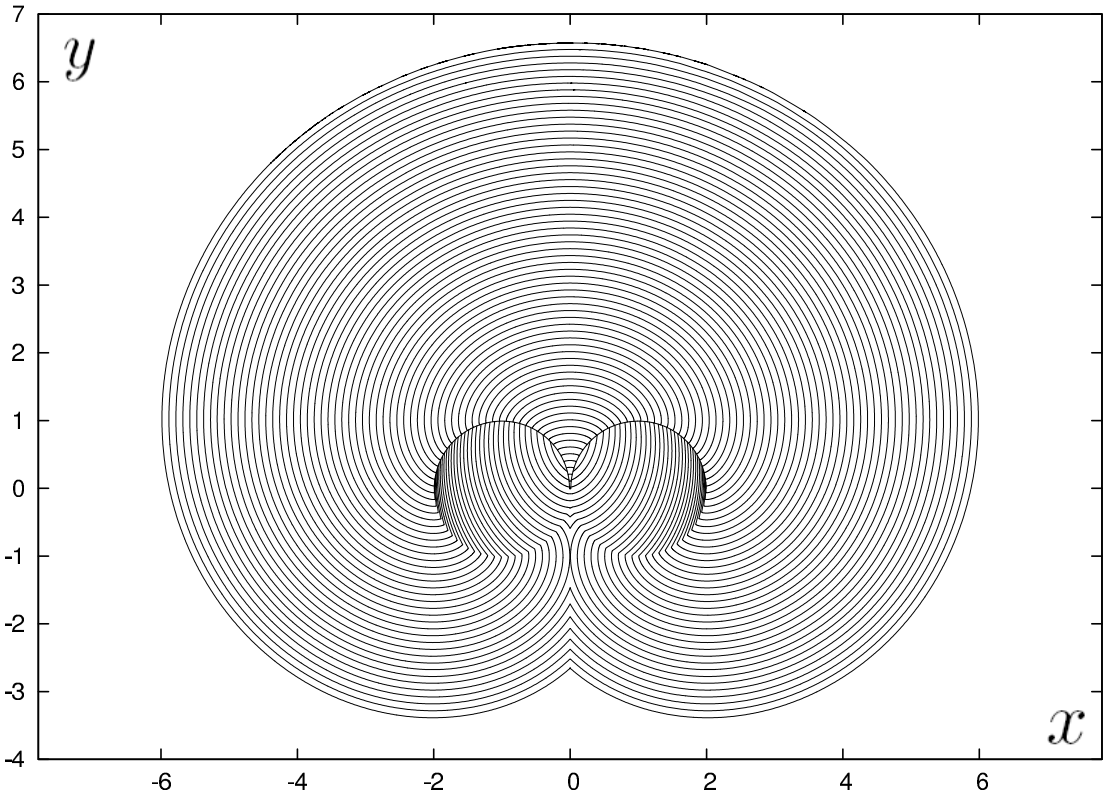
Families of semipermeable curves for the reinforced car



Here, families of semipermeable curves of the first and second type are shown for $a > 0$. Every semipermeable curve is a semicircle. Those curves of the family $\Lambda^{(1),2,2}$ which come to the interval $(-a,a)$ on the horizontal axis are smoothly glued to curves of the family $\Lambda^{(1),1,2}$. Similarly, curves of the family $\Lambda^{(2),1,2}$ are smoothly glued to curves of the family $\Lambda^{(2),2,2}$.

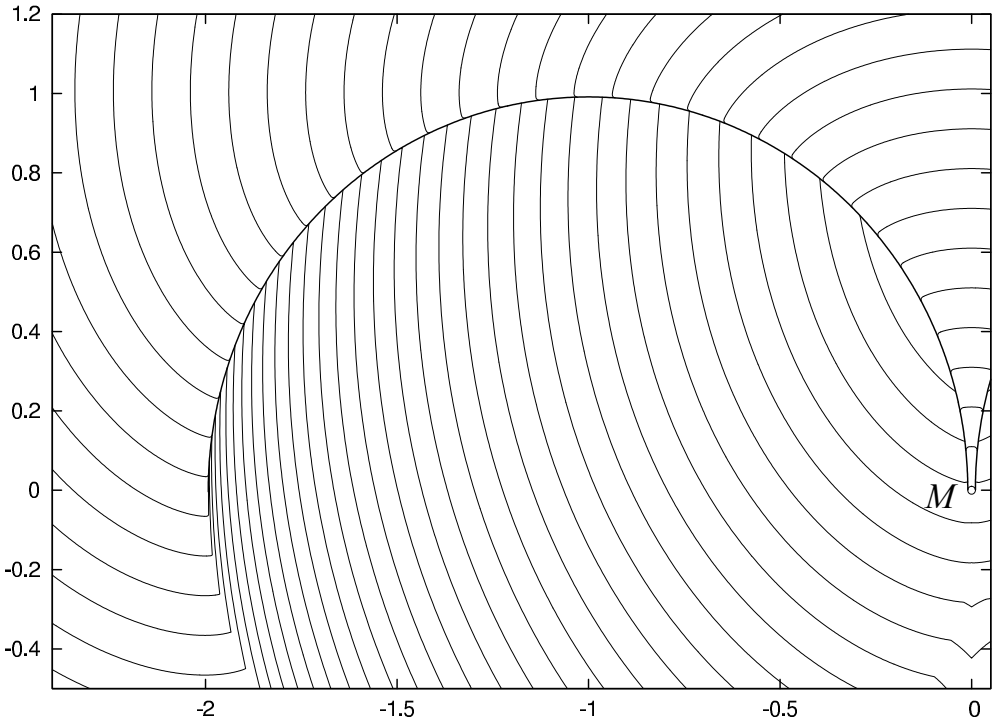
If $a = 0$, the shape of the families remains the same but the above mentioned smooth junction is not possible.

If $a < 0$, semipermeable curves do not exist.

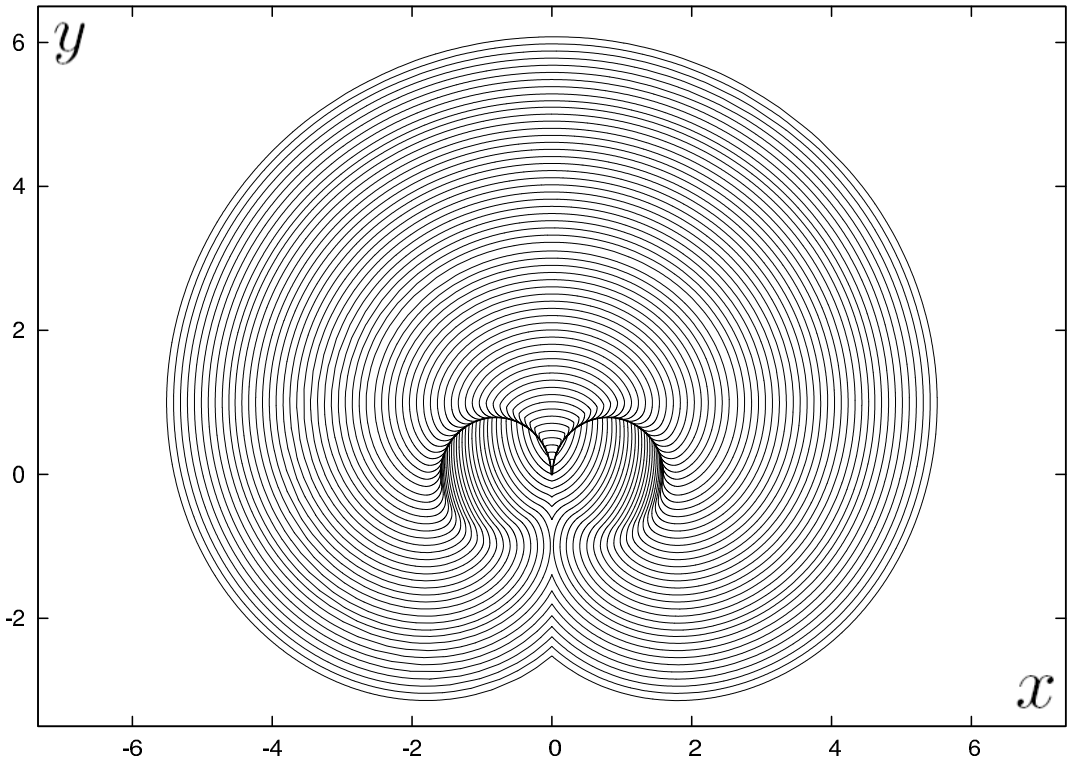


$$a = 1, \quad t_f = 6.5$$

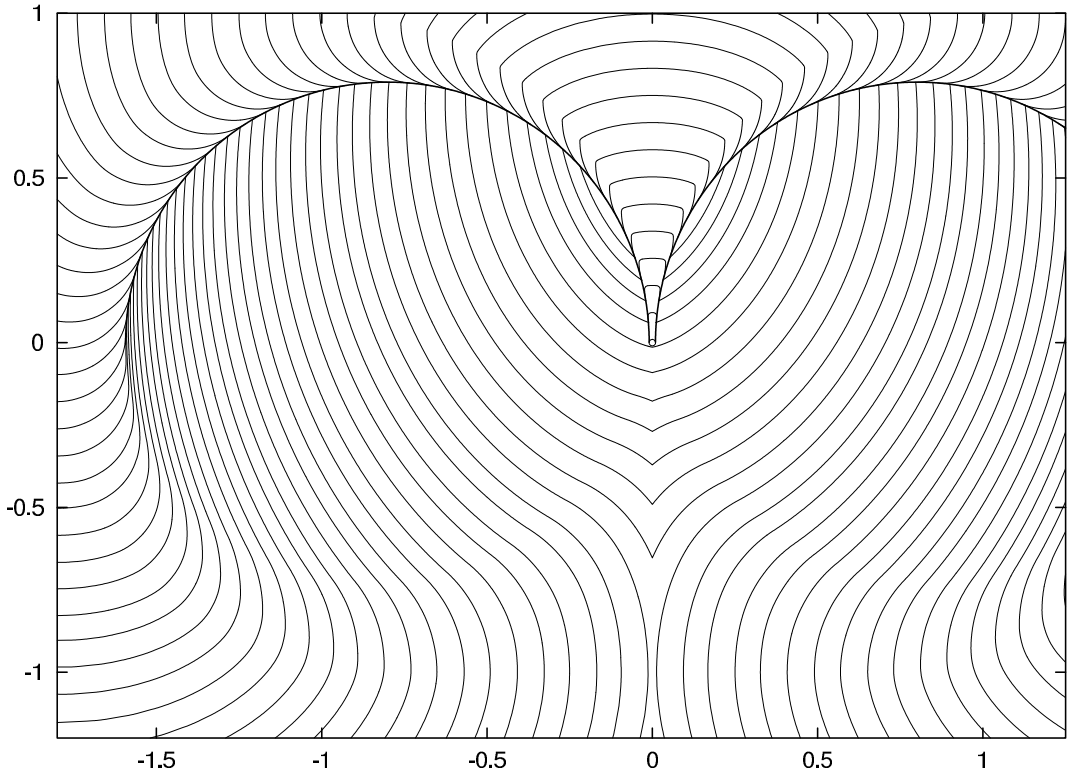
Isaacs-Dubins´ car

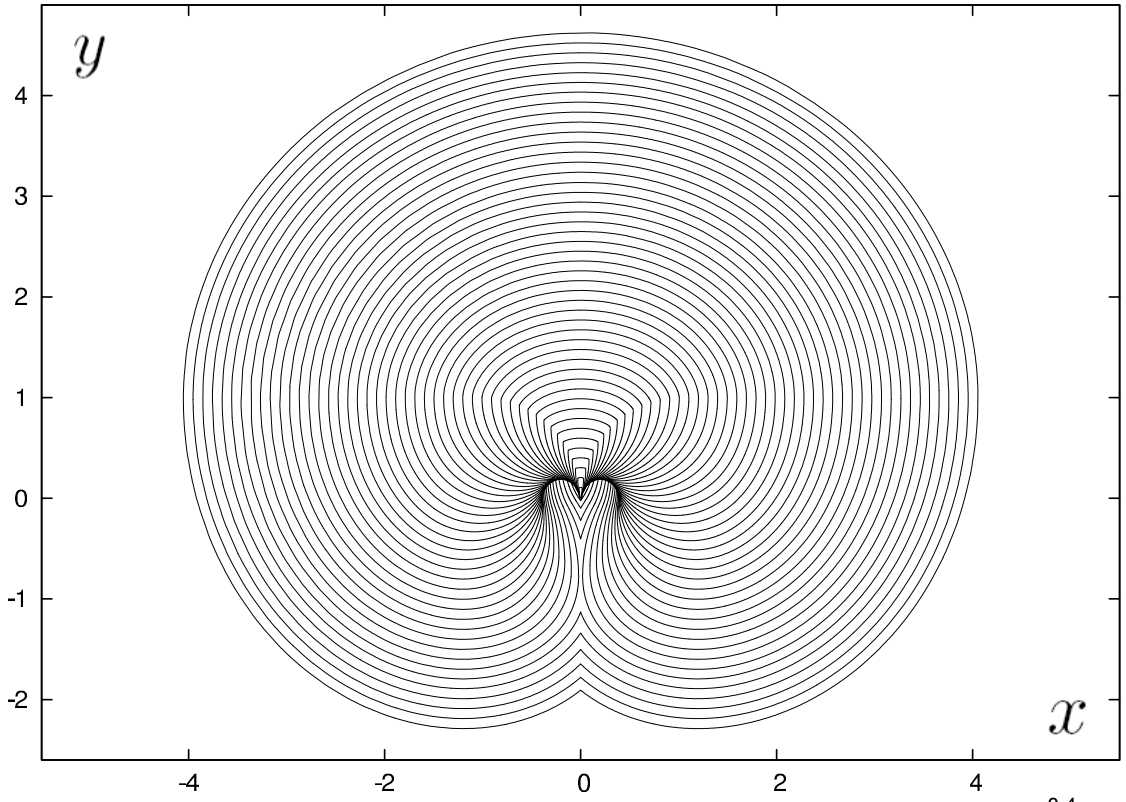


On this and two next slides, one should pay attention to the size of the area which is centered at the origin and where the value function is discontinuous.

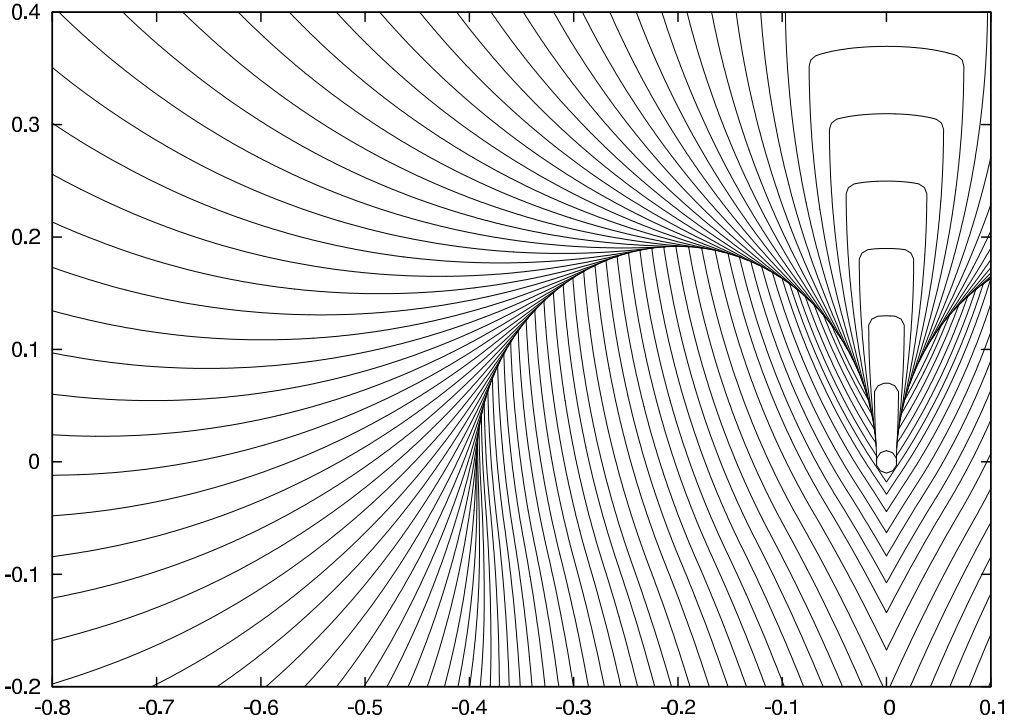


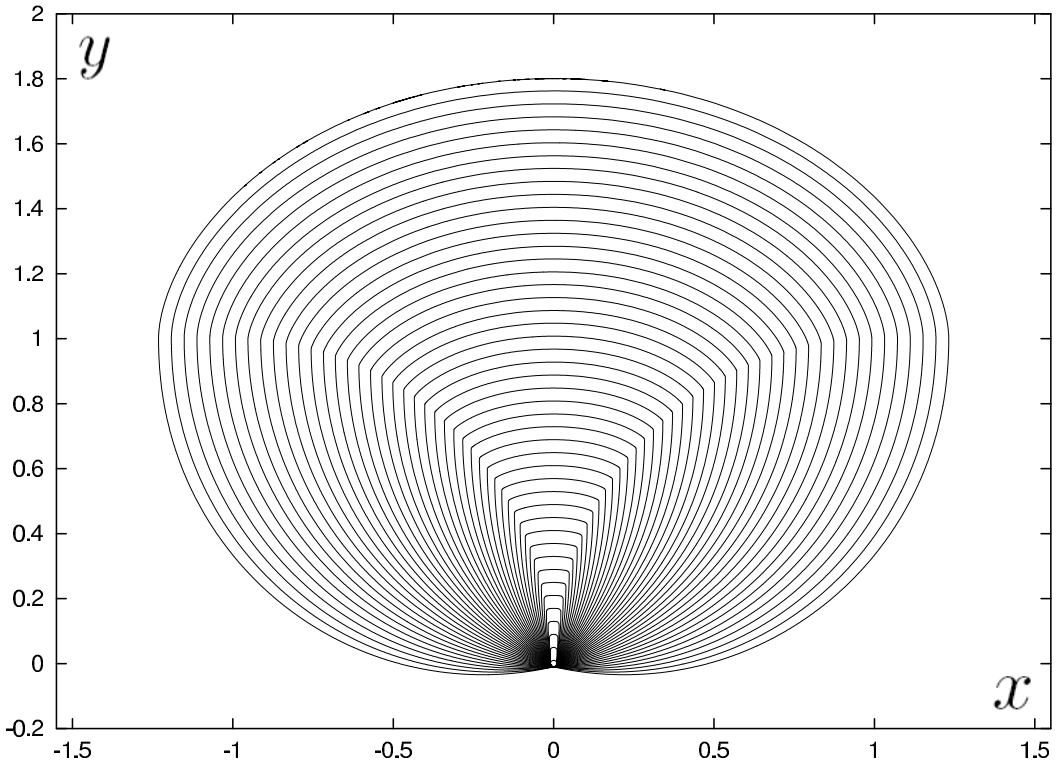
$$a = 0.8, \quad t_f = 6.1$$



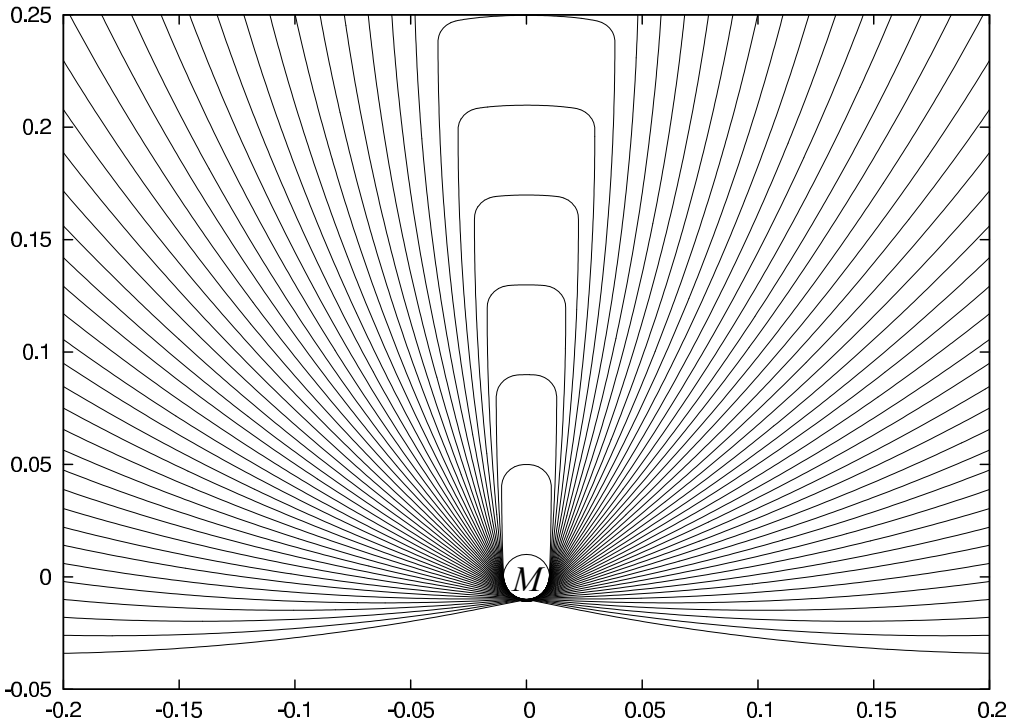


$$a = 0.2, \quad t_f = 4.7$$

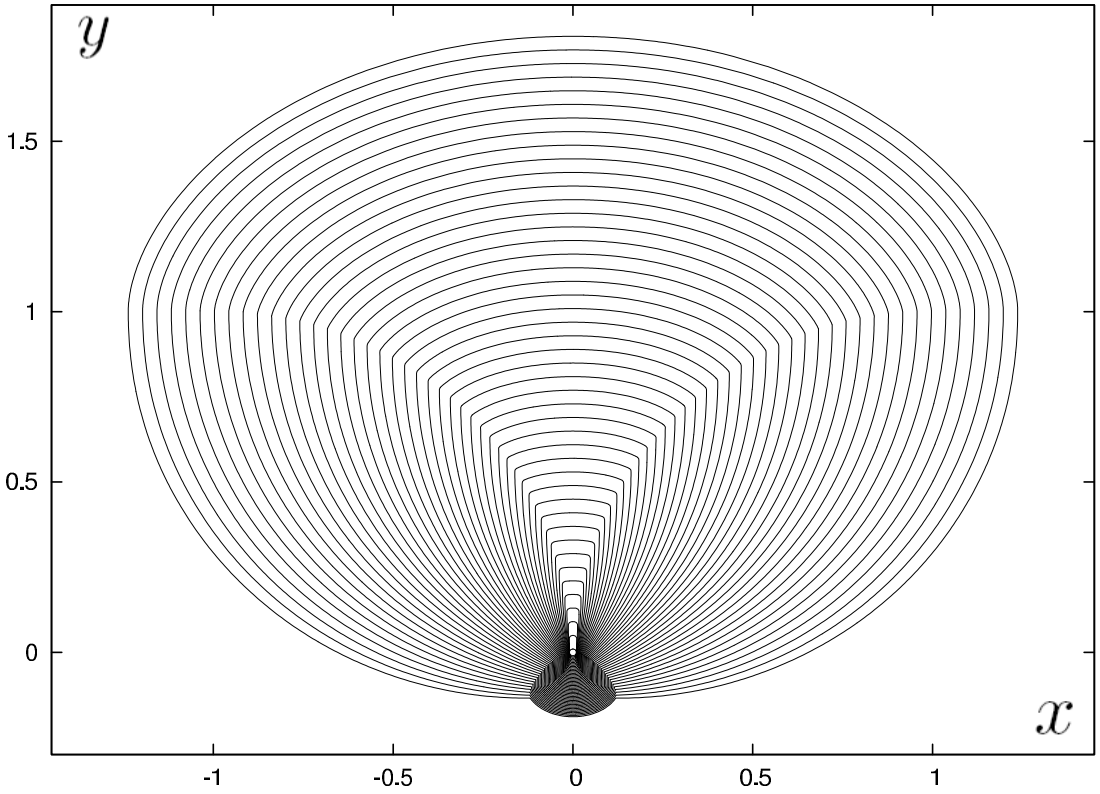




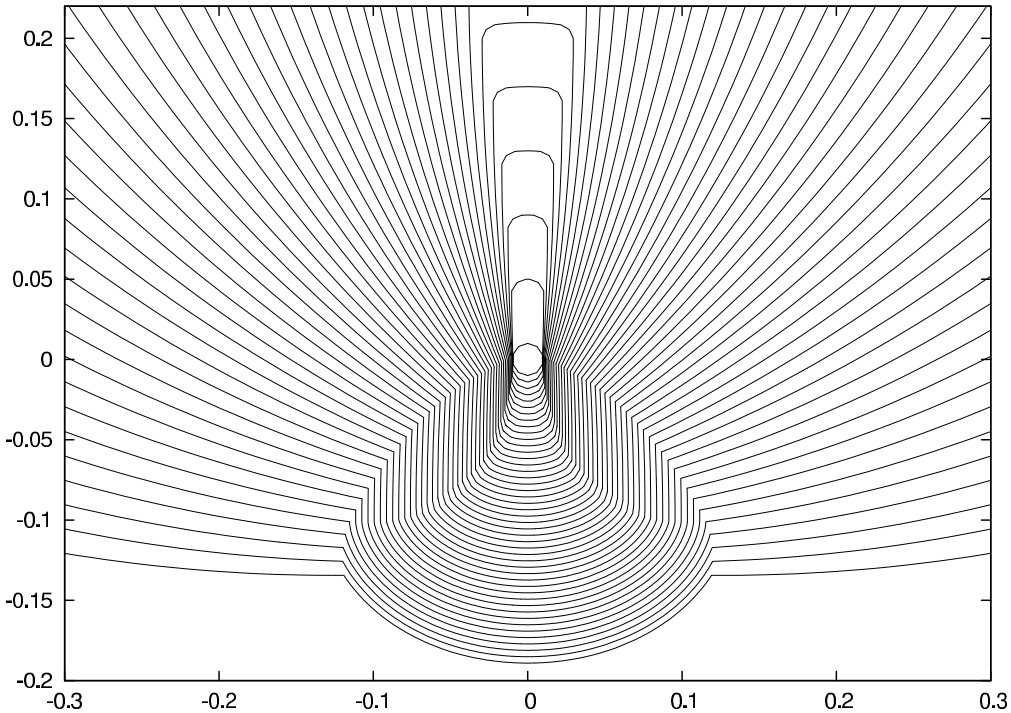
$$a = -0.001,\ t_f = 1.8$$



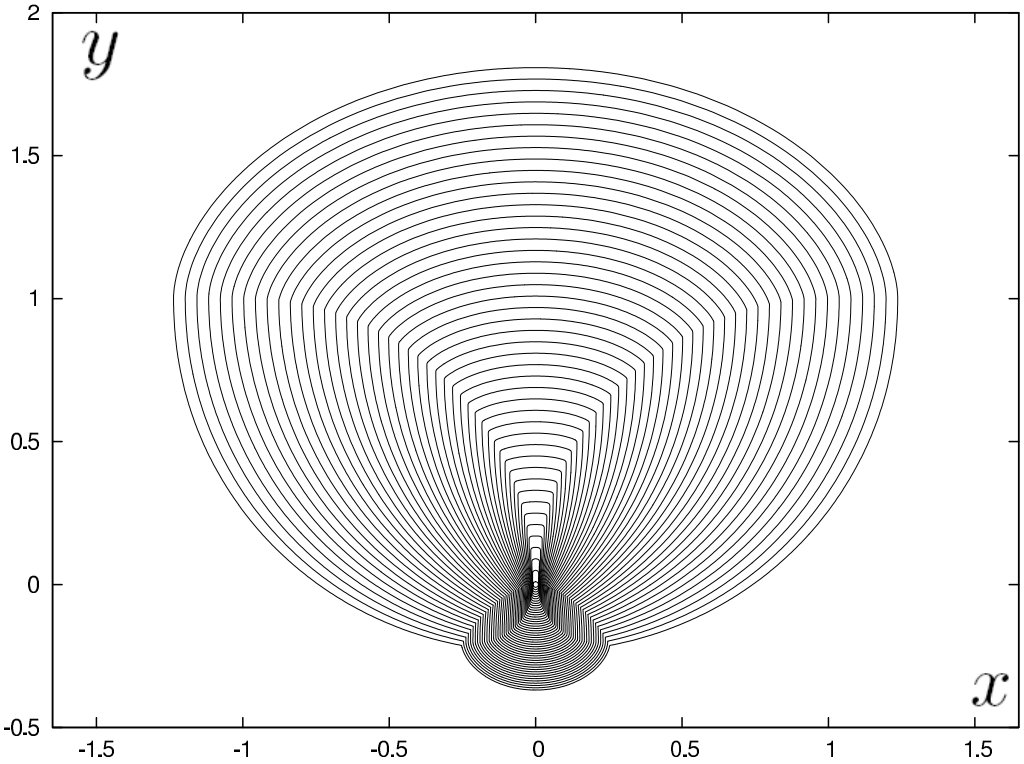
Here, the value of a is close to zero. As a consequence, barrier lines are absent.



$$a = -0.1, \quad t_f = 1.8$$

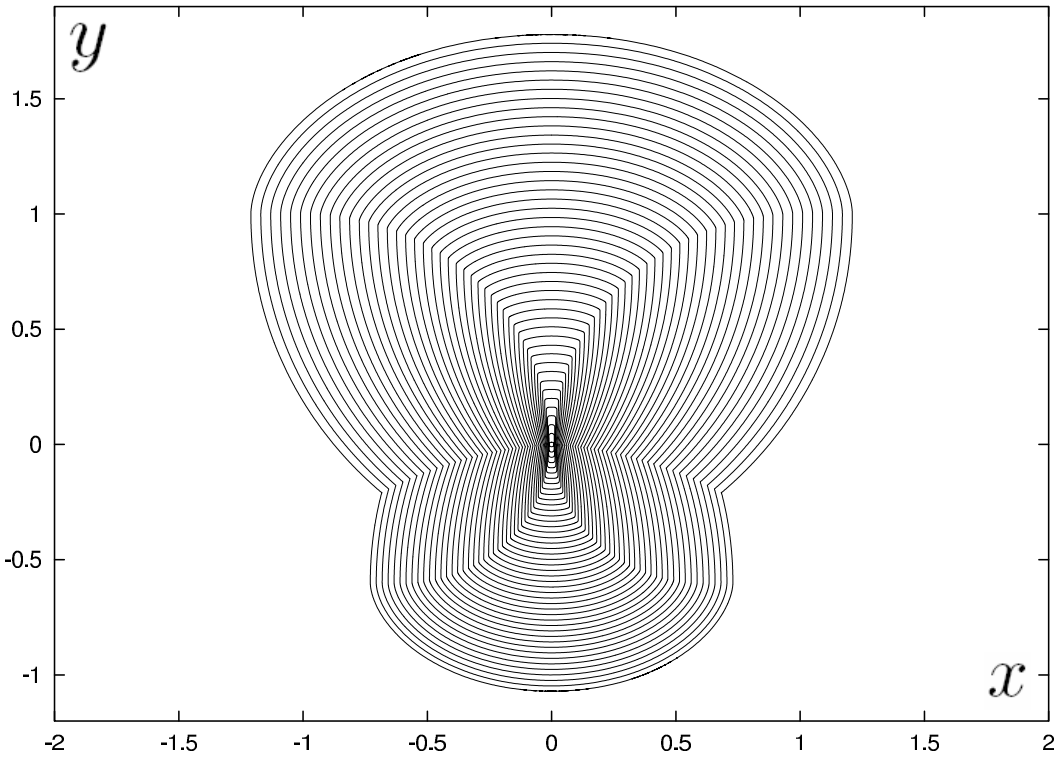


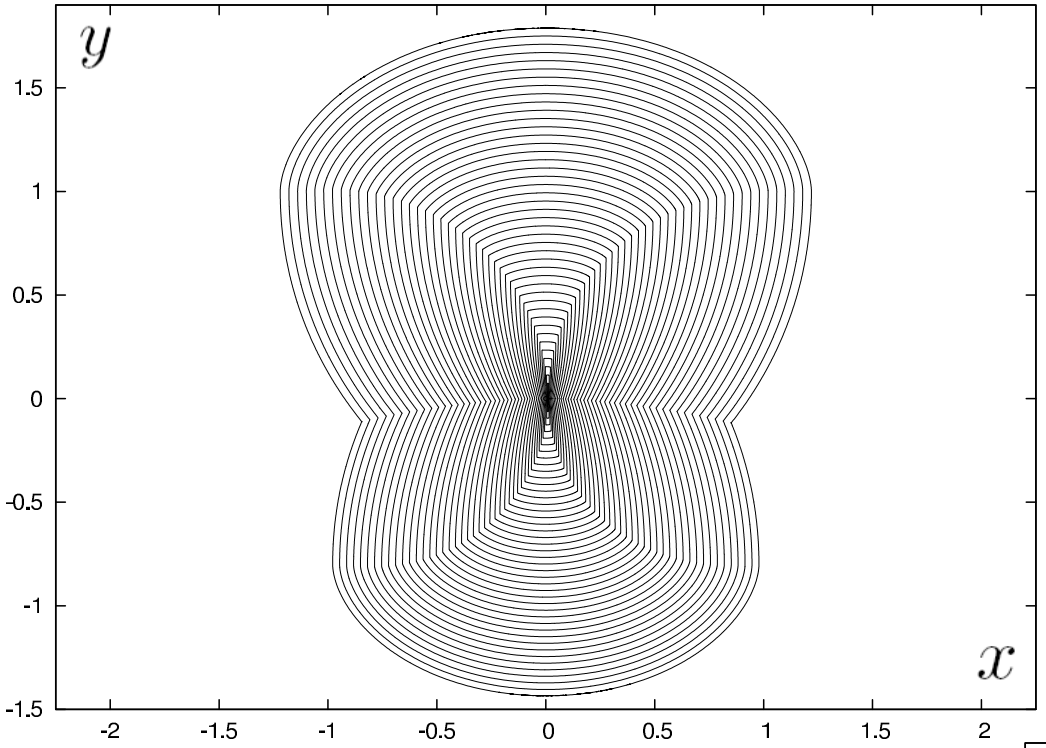
For negative values of a, the level set of the value function grows in all directions (although with different velocities) as τ increases.



$$a = -0.2, \quad t_f = 1.8$$

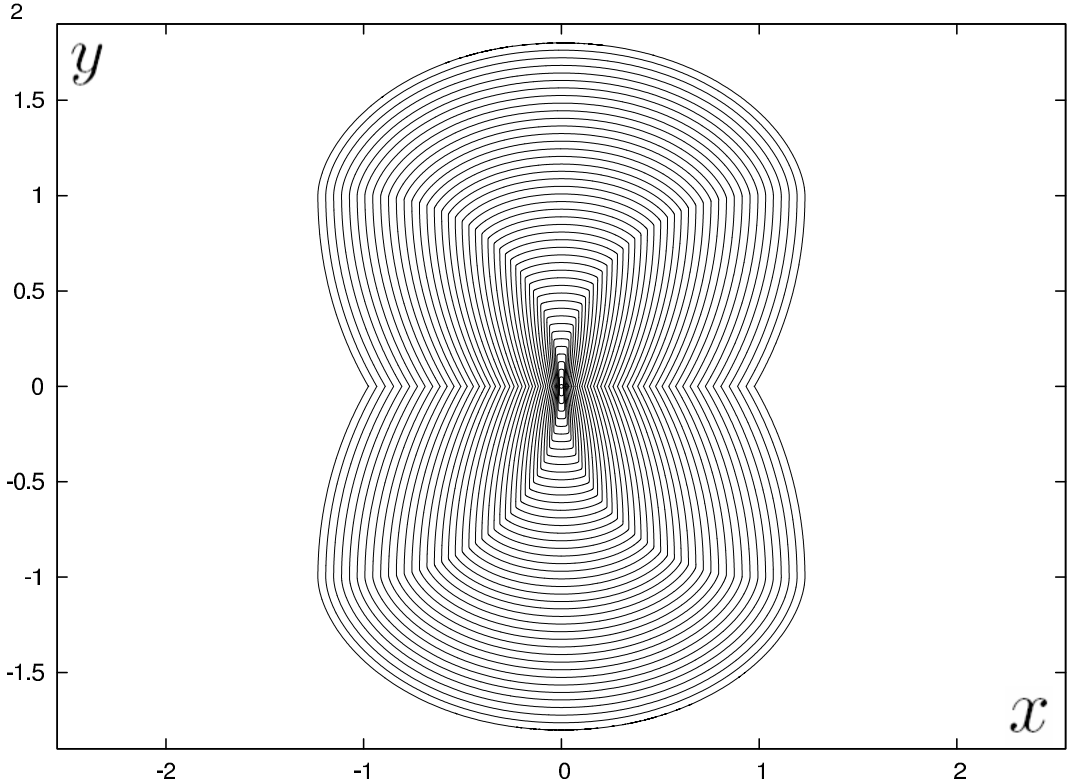
$$a = -0.6, \quad t_f = 1.8$$





$$a = -0.8,\ t_f = 1.8$$

$$a = -1,\ t_f = 1.8$$



For a = -1, we obtain Reeds and Shepp´s car.

In some papers, the reachable sets for Reeds and Shepp´s car are studied analytically. Here, an abstract of the paper by P.Soueres, J.-Y.Fourquet, and J.-P.Laumond is presented. Also, a figure showing reachable sets for different time instants is given (in our notation, axis *y* corresponds to the horizontal axis).

IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 39, NO. 8, AUGUST 1994

Set of Reachable Positions for a Car

Philippe Souères, Jean-Yves Fourquet, and Jean-Paul Laumond

***Abstract*—This paper shows how to compute the reachable positions for a model of a car with a lower bounded turning radius that moves forward and backward with a constant velocity. First, we compute the shortest paths when the starting configuration (i.e., position and direction) is completely specified and the goal is only defined by the position with the direction being arbitrary. Then we compute the boundary of the region reachable by such paths. Such results are useful in motion planning for nonholonomic mobile robot.**

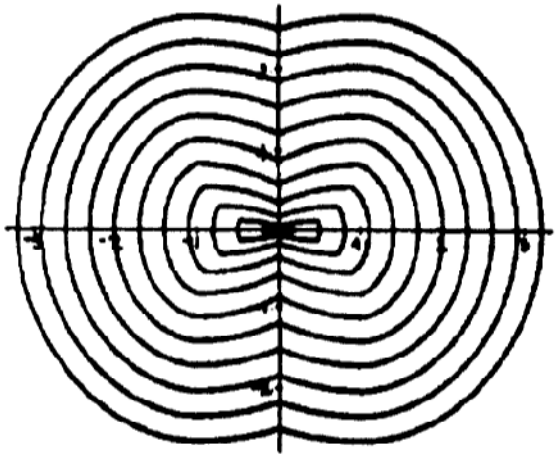


Fig. 7. Variation of the set of reachable positions.

Reachable sets at a given time for $a > 0$

For $a < 0$, reachable sets at a given time coincide with time-limited reachable sets. For $a > 0$, they are different.

Let us show how the reachable sets are looking for $a > 0$.

SIAM J. CONTROL
Vol. 13, No. 1, January 1975

PLANE MOTION OF A PARTICLE SUBJECT TO CURVATURE CONSTRAINTS*

E. J. COCKAYNE AND G. W. C. HALL†

Abstract. A particle P moves in the plane with constant speed and subject to an upper bound on the curvature of its path. This paper studies the classes of trajectories by which P can reach a given point in a given direction and obtains, for all t, the set $R(t)$ of all possible positions for P at time t, thus extending the results of several recent authors.

Appendix. Diagrams of $R(t)$ drawn by computer plotter.

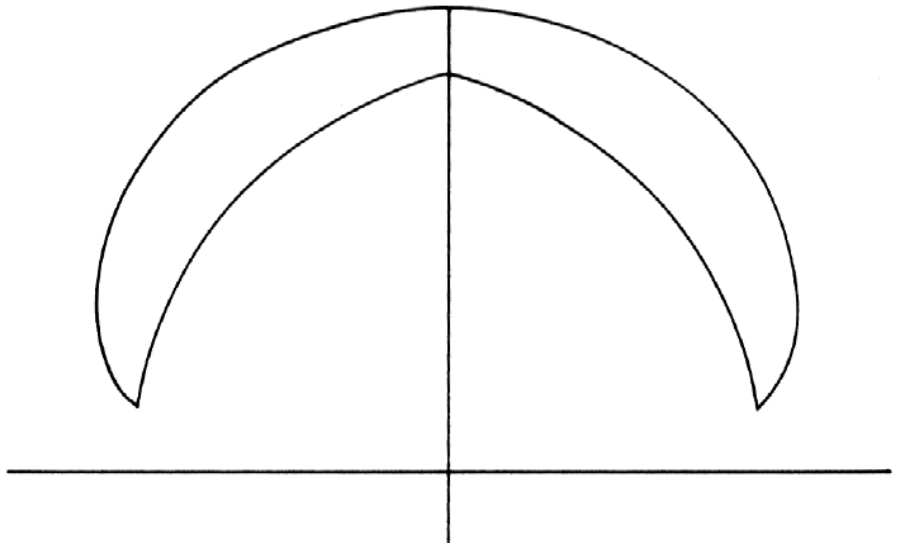
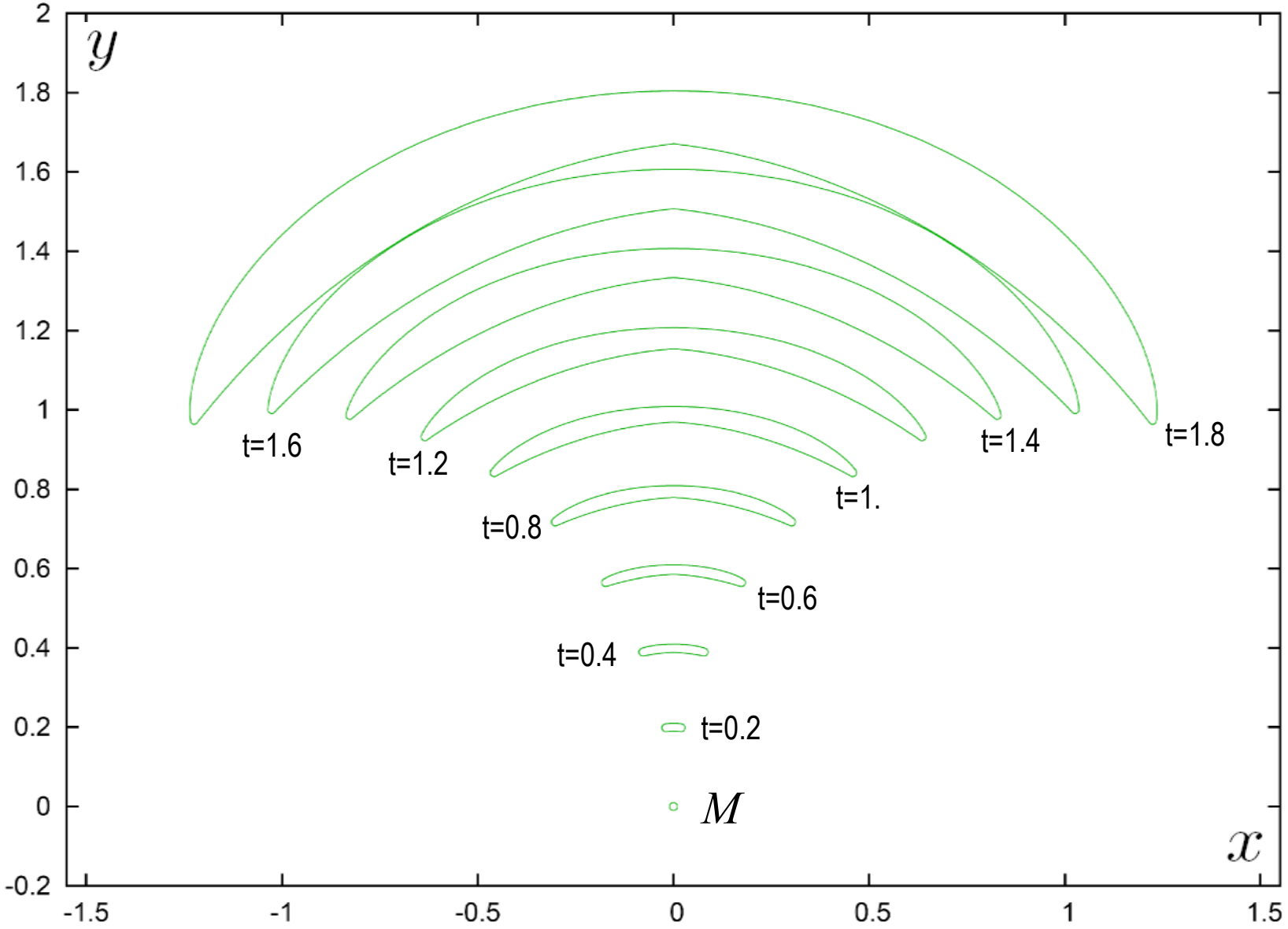


FIG. A.1. *Region $R(t)$ for $vt/\rho = 7\pi/8$*

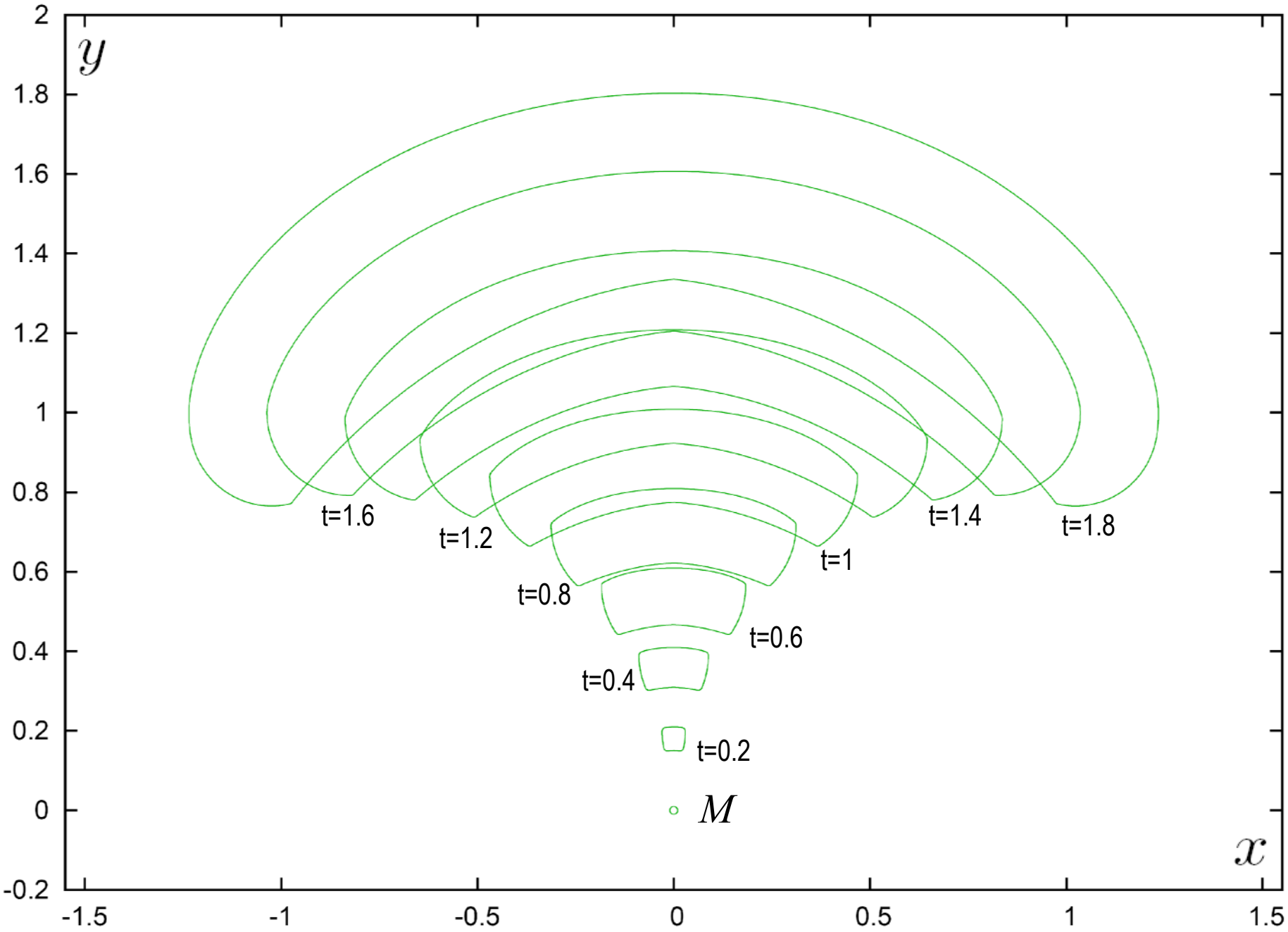
For the case $a = 1$ (Isaacs-Dubins´car), a theoretical investigation of the structure of reachable sets at a given time is done in the paper by E.J.Cockayne and W.C.Hall. Here, an abstract of this paper and a figure showing a reachable set are presented.

$$a = 0.999,\ t_f = 1.8$$

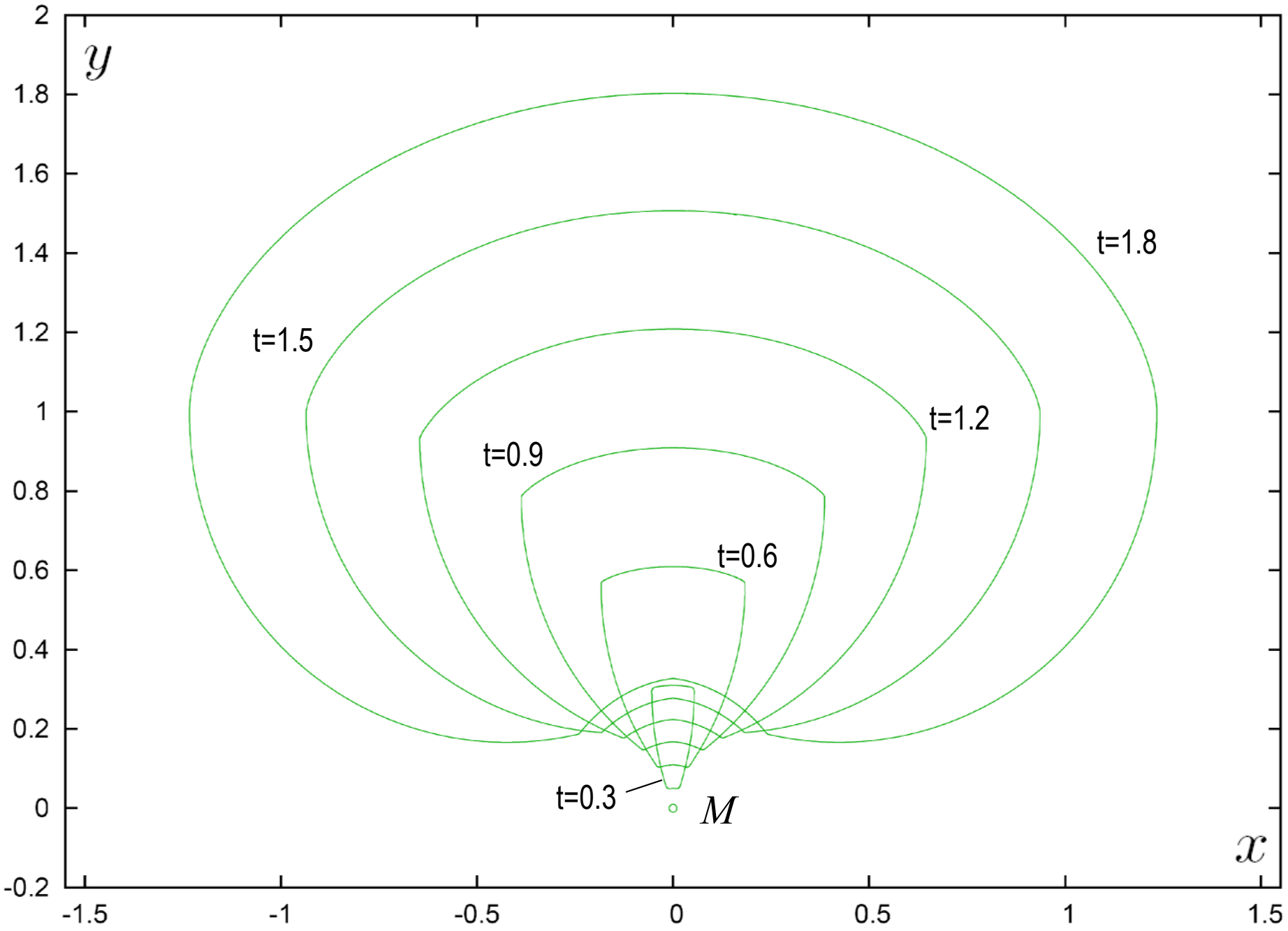


On this and two next slides, computed reachable sets at a given time are presented for three values of parameter a. The computation is done up to t_f = 1.8.

$$a = 0.8,\ t_f = 1.8$$


y
x
M
t=1.6
t=1.2
t=0.8
t=0.4
t=0.2
t=1
t=1.4
t=1.8
t=0.6
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-1.5
-1
-0.5
0
0.5
1
1.5

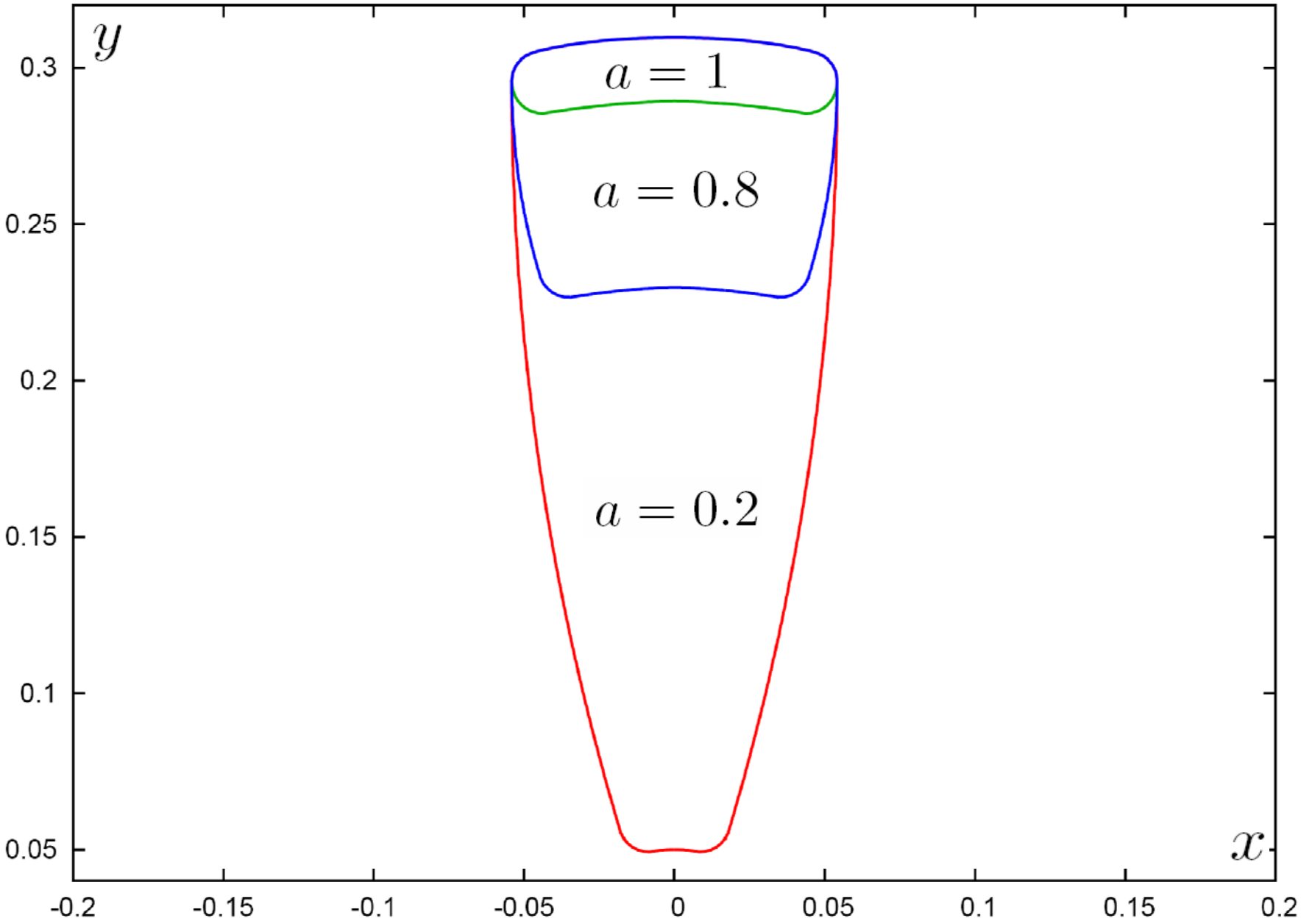
$$a = 0.2,\ t_f = 1.8$$


y
x
M
t=1.8
t=1.5
t=1.2
t=0.9
t=0.6
t=0.3
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-1.5
-1
-0.5
0
0.5
1
1.5

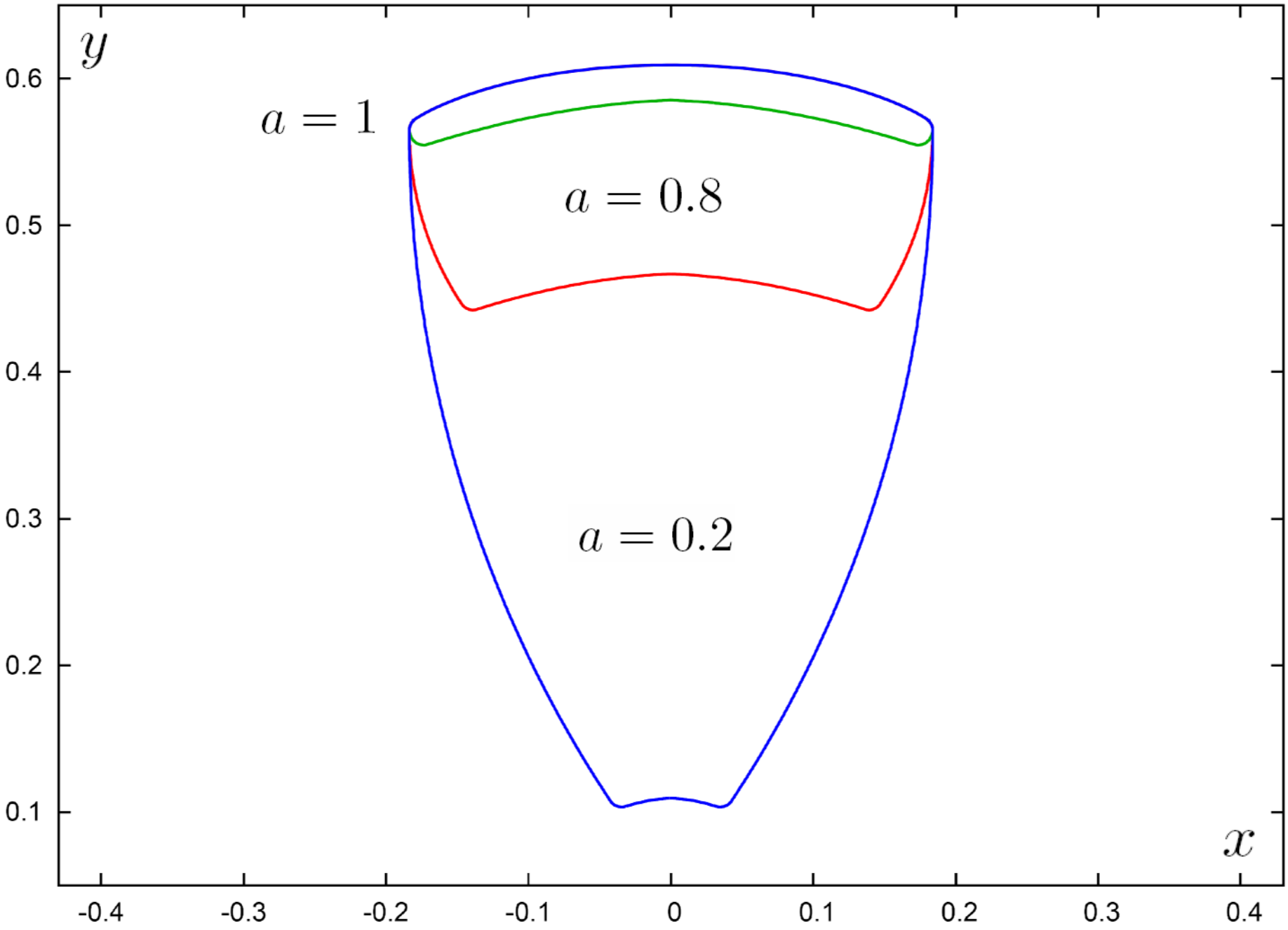
Influence of parameter a on the size of reachable sets

The final group of figures shows the influence of parameter a on the size of both reachable sets at a given time and time-limited reachable sets. Three values of a are used: a = 1, 0.8, 0.2. On each slide, the time instant for which the reachable sets are computed is given at the upper line.

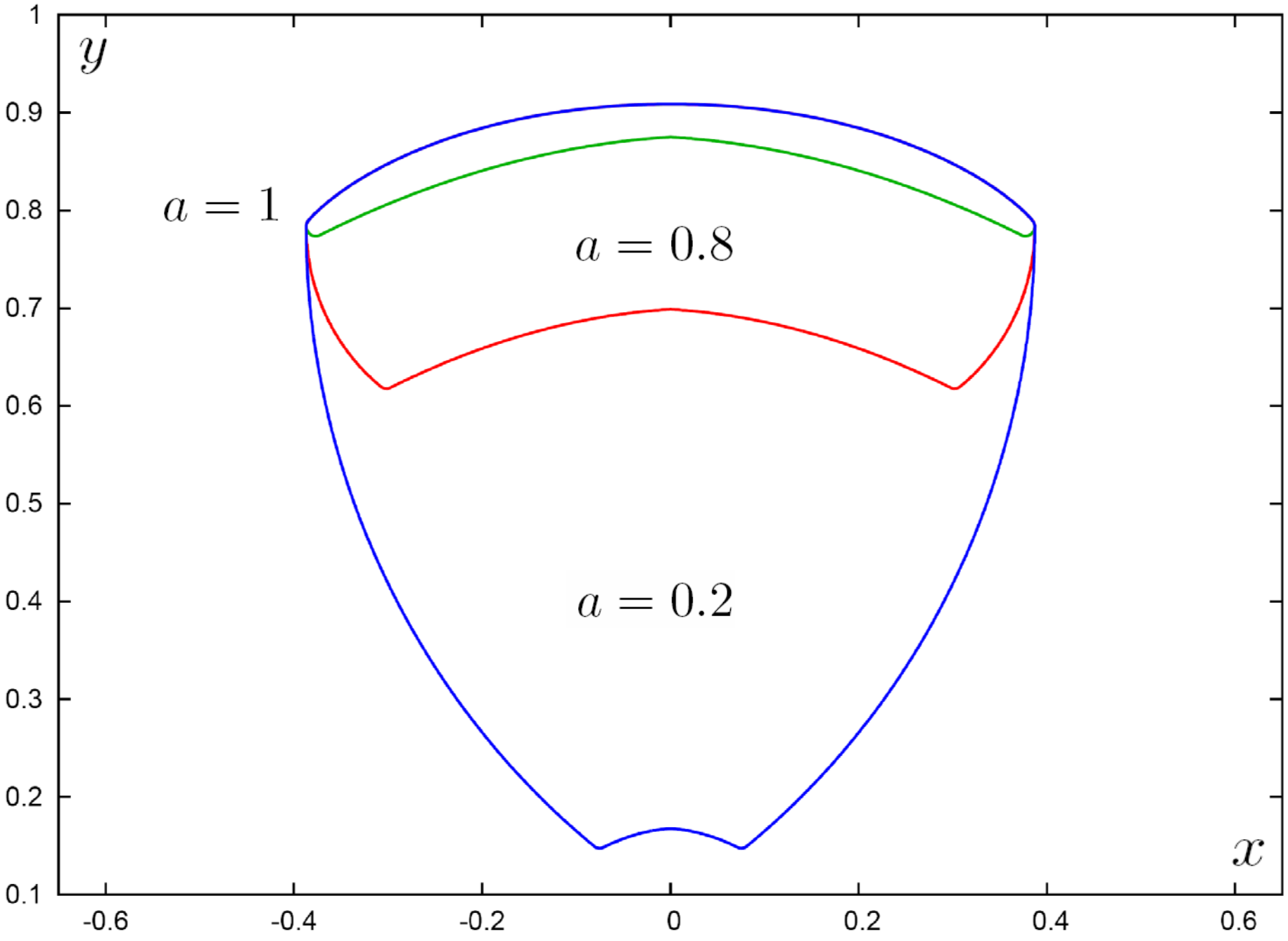
Reachable sets at given time, $t = 0.3$



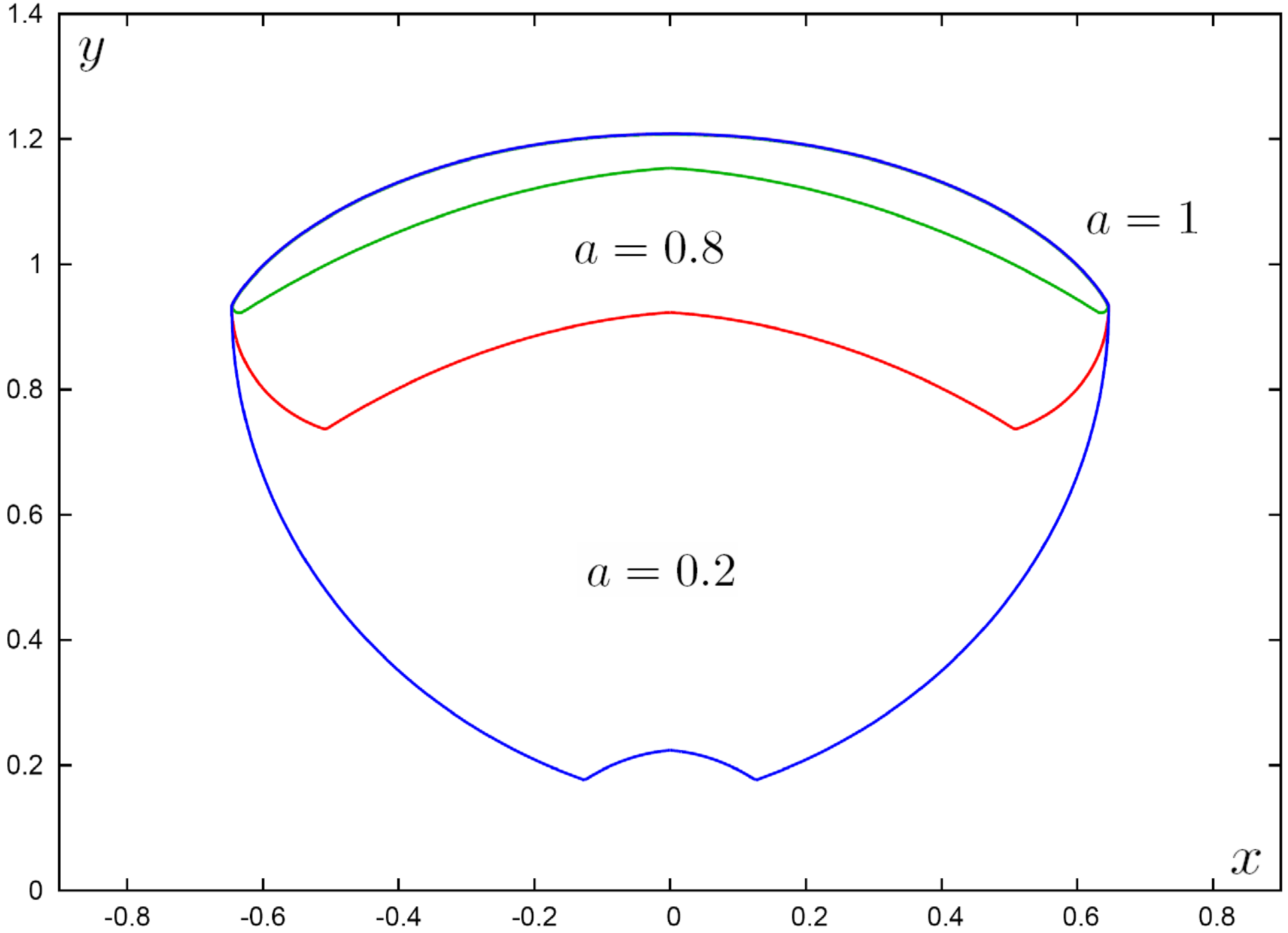
Reachable sets at given time, $t = 0.6$



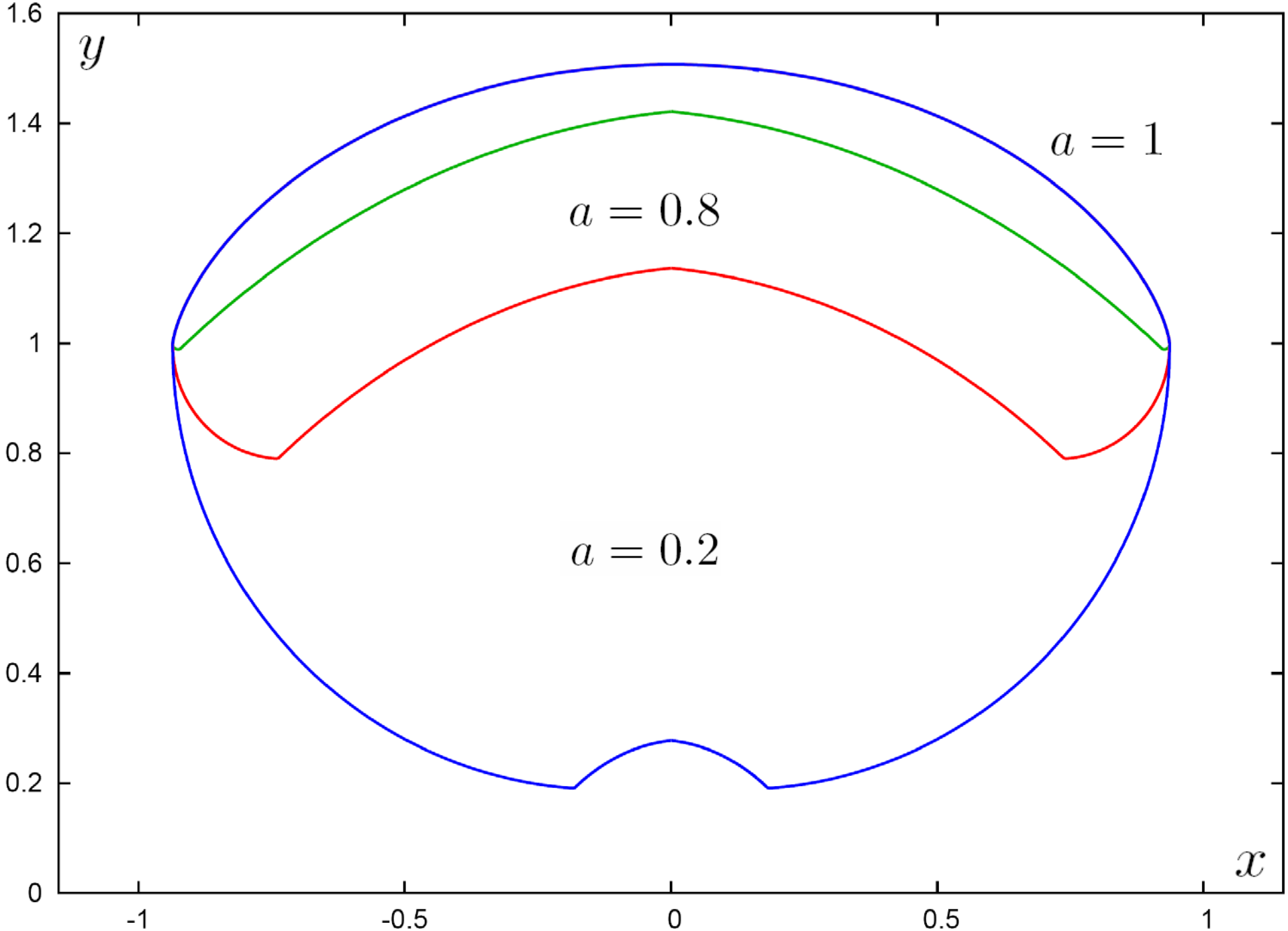
Reachable sets at given time, $t = 0.9$



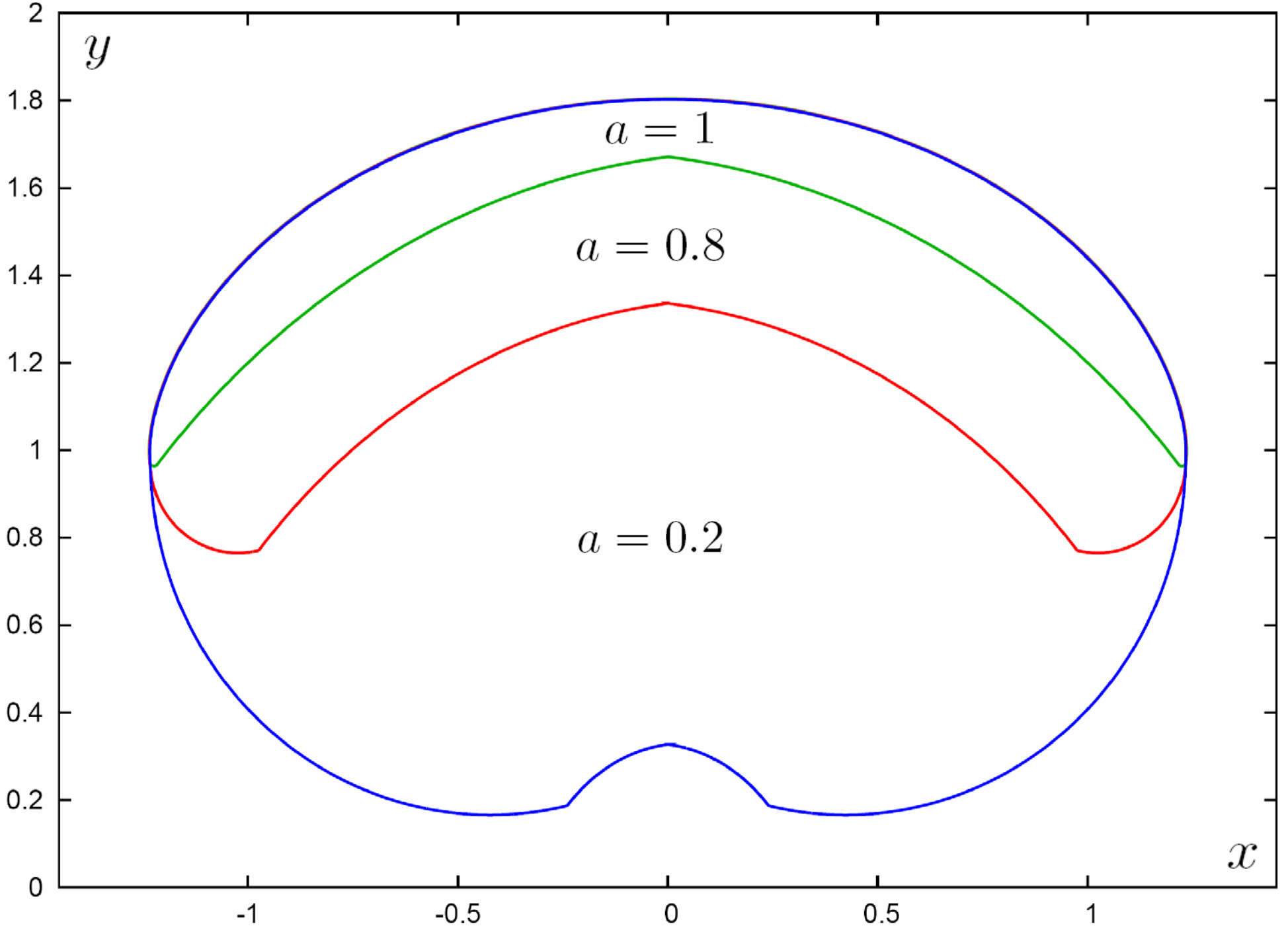
Reachable sets at given time, $t = 1.2$



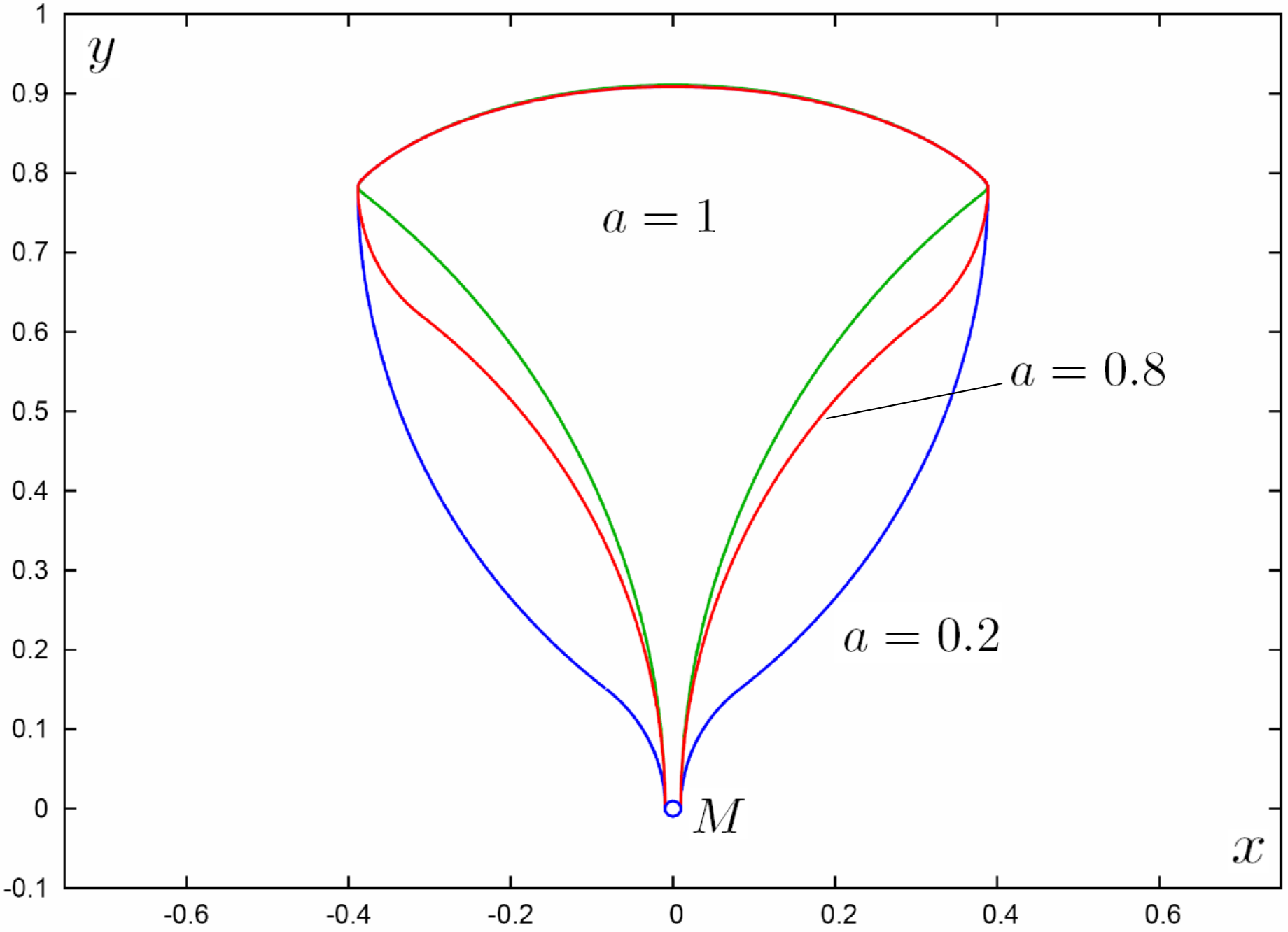
Reachable sets at given time, $t = 1.5$



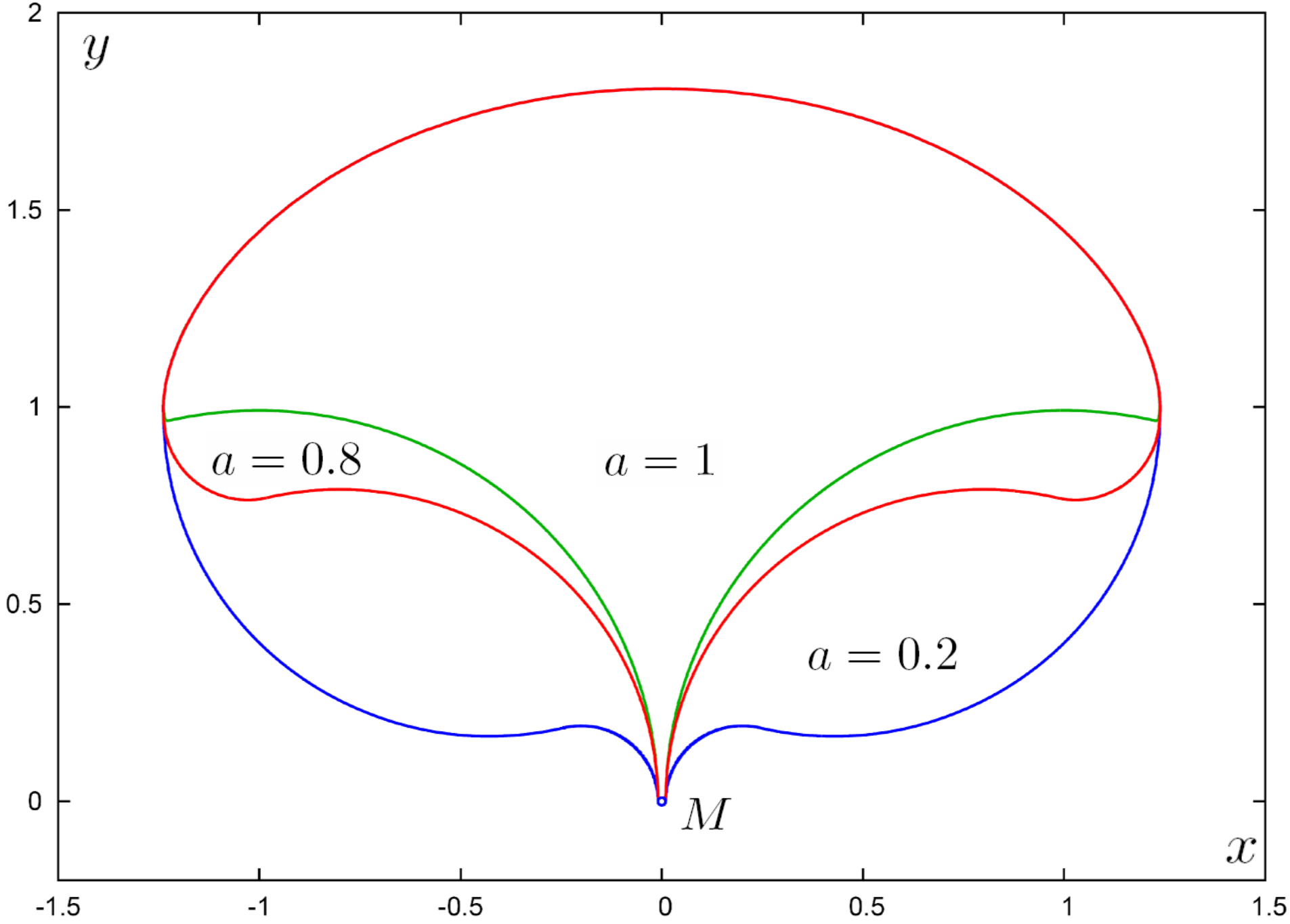
Reachable sets at given time, $t = 1.8$



Time-limited reachable sets, $t = 0.9$



Time-limited reachable sets, $t = 1.8$



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