The words of the human locomotion*

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Abstract. This paper overviews a multidisciplinary research effort on the understanding of human locomotion. It addresses the computational principles of locomotion neuroscience via the geometric control of nonholonomic systems. We argue that a human locomotion model can be derived from a top-down approach, by exclusively looking at the shape of locomotor trajectories and by ignoring all the body biomechanical motor controls generating the motions.

1 Introduction

The walking of people in the snow or in the sand leave traces. Such traces are the static images of the time-parameterized walking trajectories. What are the underlying locomotion principles explaining their shapes? Let us state the question in a more accurate manner. Ask somebody to enter a large empty room by a door and to leave it by another one. The resulting motion is an intentional motion motivated by a well defined goal. A lot of trajectories solve possibly the task. The subject will choose one of them. Why that one instead of all the other possible ones (see Figure (1).a)? Would anybody else choose the same trajectory? If we change a little bit the position and the direction of the goal, how is the initial trajectory reshaped? How is this reshaping smooth? For instance, let us consider the real study cases illustrated in Figures (1).b and (1).c. In both figures both goals are at the same position. Their respective directions differ by a small margin. In the case of Figure (1).b a subject has chosen two very close trajectories. In the case of Figure (1).c the same subject has chosen two completely different strategies. How to explain this strategy change?

* Note: The current version of the paper is 18 pages long. It can be reduced to 12 pages by changing Section 1 to a single paragraph. It can be extended in two 10 pages papers with more details on Section 4. In case of acceptation we will follow reviewer advices.

In this paper we address all these questions via the study of a single optimal control problem:

1. We show that the human locomotion trajectories are well approximated by the trajectories of a nonholonomic system optimizing the derivative of the curvature. Such trajectories are piecewise made of elementary clothoid arcs.
2. We provide an optimal control synthesis (i.e., a complete description of all the finite possible sequences of such elementary arcs). Such sequences may be phrased as, for instance, "Start turning left while increasing the curvature during τ_1 second, then decrease the curvature during time τ_2 and finally increase the curvature during time τ_3". These combinations generate "words" that account for the locomotion strategy used by the subjects according to the placement of the 3-dimensional goal.

The results of the first item have been already published in [8, 9, 11]. Those of the second item are new. The objective of the paper is to present a global view of the methodology followed by the authors.

1.1 Related work: optimal control in neuroscience

A redundant system can be viewed as a mapping from a control space with dimension q onto a task space with dimension n, with $n < q$. Such a mapping is not one-to-one. As a consequence performing a given task can be done by an infinite number of trajectories. In Robotics, it is a well known practice to benefit from the system redundancy to optimize some criteria (see for instance [2, 23, 29] and for an overview see [33]).
The human body is a highly redundant system. For most tasks (e.g., walking, grasping) we get $n << q$. Since pioneering works like those of Bernstein [22] we know that the central nervous system do not explore the entire q-dimensional manifold each time a task has to be performed. For instance, when we are walking the rhythmic motions of the arm follow the same rhythmic motions of the legs. Such synchronization of motions reduces the dimension of the motor space to be explored. The human motor learn by discovering motion patterns that reduce the dimension of the motor space. It tends to build a control space with lower dimensions than the motor space. This is the challenge of modern computational neuroscience: to propose control space models that can be generic enough to account for large classes of tasks [6]. Today, optimal control plays a central role [7, 21] amongst developed accepted models.
A community of problems between robotics and neuroscience gives rise today to a multidisciplinary research synergy. The synergy benefits from the sharing of common investigation tools such as optimal control theory. The research we present here provides a different spin to traditional optimal control by directing focus away from redundant systems.

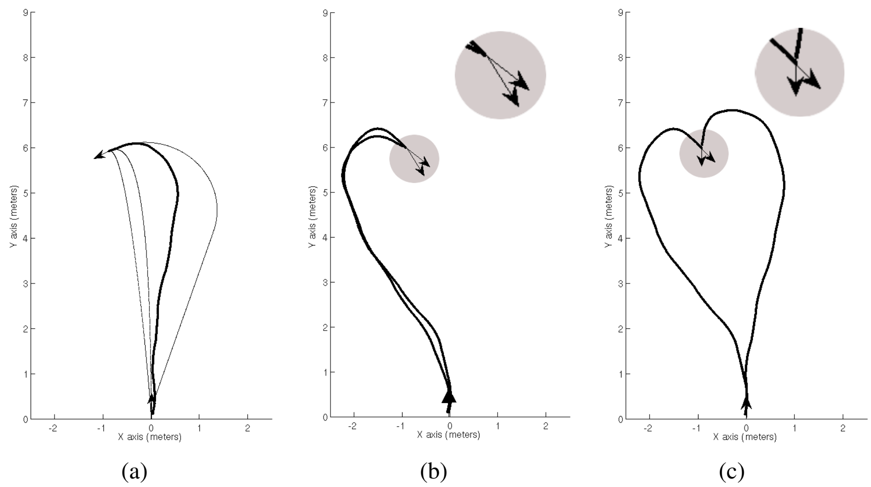


Figure 1: *(a) amongst four "possible" trajectories reaching the same goal, the subject has chosen the bold one. Why? (b) and (c) show some examples of real trajectories with the same final position. The final directions are almost the same. (b) a subject performed two very close trajectories. (c) the same subject performed two completely different trajectories. How to explain this strategy change?*

1.2 A simple statement: the nonholonomic nature of the human locomotion

Let us start with a simple statement. The placement of a body on a 2-dimensional space requires three parameters: two for the position of the body and one for its direction (with respect to a given frame of the world). The trace left by a man walking on the snow is made of the history of the various *positions* he traversed (see Figure (2)). It is printed in a 2-dimensional space. The information about the *direction* of the body seems a priori missing. However it can be logically deduced by considering that the direction of the body at a given instant of the trajectory is the direction tangent to the trace at the corresponding *position*. Looking at the trace derivative of the body position gives the body direction. This is a consequence of the fact the man is walking forward. Such a coupling between position and direction is a differential coupling.
In the presence of a differential coupling, a critical question must be asked: is the coupling *integrable* or not? And, what does this question mean?
The integrability of a differential equation is related to the dimension of the *reachable space* of the associated system.
Let us consider the watch depicted in Figure (3). It is a mechanical system made of two rotating arrows. The "motor space" is 2-dimensional. It can be represented as a

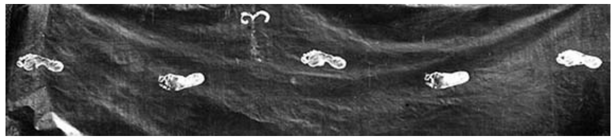


Figure 2: *Step trace. Photograph by Marey (1830 - 1940). Copyright: Cinémathèque française*

torus. The velocity of the long arrow is 12 times faster than the velocity of the small one. There is a differential coupling between both arrows. Such a coupling is clearly integrable. This means that the *position* of the small arrow depends on the *position* of the long arrow. The *reachable space* of the arrow positions is not 2-dimensional. It is only 1-dimensional. The system is said to be *holonomic*.
Let us now consider the walking man. The differential coupling between the position (x, y) and the direction θ is:

$$\dot{y}\cos\theta - \dot{x}\sin\theta = 0 \tag{1}$$

This means that the tangent vectors at any point of an admissible trajectory necessary belongs to a 2-dimensional vector space spanned by the vector fields:

$$\begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} \text{and} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{2}$$

In that vector space, sideway steps are not allowed. The "motor space" of the human walking is 2-dimensional as for the watch above. However it is a well-known fact that the man can walk everywhere he wants! Its *reachable space* is 3-dimensional. The differential equation (1) is not integrable. The corresponding system is not holonomic: a *nonholonomic* system is a system whose reachable space dimension is strictly greater than its "motor space" dimension.
Checking whether a differential coupling is integrable or not is done by the Frobenius theorem, a classical tool from differential geometry [31]. The study of nonholonomic systems generates works in the community of pure mathematics (e.g., [1]), control theory (e.g., [34]) and robotics (e.g., [17]).

1.3 Related work: optimal control and nonholonomic robots

Equation (1) is the classical differential equation governing the motions of a rolling wheel, and then the motions of mobile robots with wheels. How to steer a mobile robot from a given 3-dimensional starting configuration to a given 3-dimensional goal while the robot control space is only 2-dimensional? The question gave rise to an active

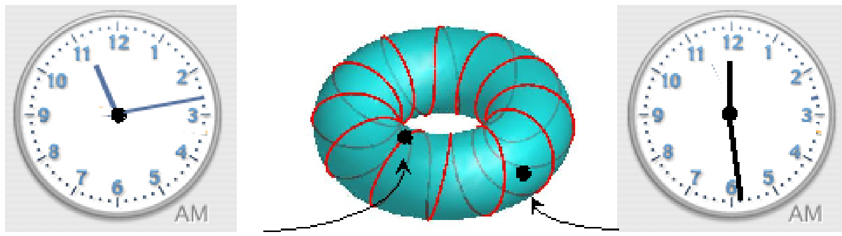
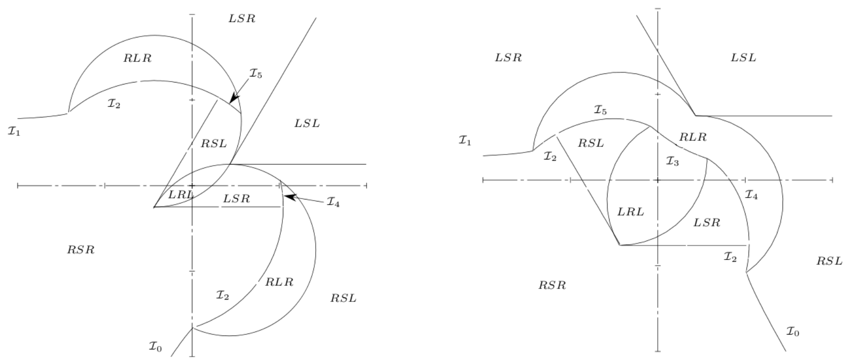


Figure 3: *The velocity of the long arrow is 12 times faster than the velocity of the small one. Such a coupling is clearly integrable. This means that the reachable space of both arrows is only 1-dimensional (the line on the torus). The time displayed by the watch of the left side corresponds to a point on the torus line. The "time" displayed by the watch on the right picture will never happen!*

research topic during the past twenty years. Among the numerous steering methods (see overviews in [17], [10] and [26]) optimal control based methods are certainly the most efficient ones. Unfortunately planning nonholonomic optimal motions is a difficult problem. It has been solved only for some classes of simple systems. Some of the popular include the Dubins' car [19] and the Reeds and Shepp's car [12, 14] (see also [5, 16] and the overview [24]).
Let us emphasize shortly on Dubins' car since it is the closest system related to our problem. Dubins' car is a car moving only forward at a constant linear velocity. It corresponds to the following control system:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u \tag{3}$$

where u is the steering wheel control. u is a map onto $[-1, 1]$. Dubins [19] (and then the proof by Sussmann et al [12] using modern optimal control theory as the maximum principle of Pontryagin [20]) shows that the shortest length paths of the system are made of finite sequences of straight line segments S ($u = 0$) and arcs of circle with constant minimal radius C ($u = \pm 1$). By considering that an arc of a circle C can turn either right (R arcs when $u = 1$) or left (L arcs when $u = -1$), Dubins shown that a sufficient family of shortest paths is the following one: $RSR, RSL, LSR, LSL, LRL, RLR$. These sequences are what we call the "words" of the Dubins' car. From this, we can say that six words are necessary to describe all the possible optimal paths. Now another question arises dealing with the uniqueness of the shortest paths: what is the partition of the (3-dimensional) space according to the



(a) (b)

Figure 4: *Partition of the slices $\theta = \frac{\pi}{3}$ and $\theta = \frac{2\pi}{3}$ induced by the Dubins' words. The pictures are borrowed from [32].*

various words? This problem has been solved in [28][1]. Figure (4) shows the partition of the slices at $\theta = \frac{\pi}{3}$ and $\theta = \frac{2\pi}{3}$ according to the Dubins' words.

Computing such a partition has been done for a few other systems [4, 25]. These contributions are based on the application of the Pontryagin principle that gives (only) necessary conditions for optimality. Necessary and sufficient conditions can be found in the works by Boltyanskii [30]. They are related to the regular synthesis of optimal control (i.e., the computation of partitions such as the Dubins' partitions we have just sketched here). Solving the regular synthesis problem in a generic way (i.e. for any kind of non linear systems) remains a highly challenging mathematical problem.

More than that, the application of the Pontryagin principle generally does not provide enough information to describe optimal trajectories with finite words. This is why, most of the time, the optimal trajectory computation is done via numerical analysis algorithms [13, 15, 18].

The objective of our current research is to provide the synthesis of the human locomotion. What optimal cost could explain the shape of the locomotor trajectories? What words describe them? What spatial partition is induced by such words?

[1] Bibliographical note: it is worth noting that the work published by T. Pecsvaradi in 1972 has remained ignored by the robotics community until now. The reference to Pecsvaradi's work has been pointed out recently to the first author by Prof. V.-S. Patsko from the Russian Academy of Sciences. The results published in [32] are nothing else than the results previously published in [28]

2 Methodology

2.1 Objective: an inverse optimal control problem

Our approach aims at explaining the shape of the locomotor trajectories by optimal control. By nature the validation of the control model we are looking for should be done by comparing the trajectories simulated from the proposed model with a set of observed trajectories. We first have to find a control system that "reasonably" accounts for the human locomotion. Then we have to find an optimal cost that "reasonably" accounts for the shape of the trajectories. "Reasonably" means that we want a human locomotion model that applies as closely as possible to a set of observed data: the "proofs" will come from statistical analysis. Our approach underlies several questions:

1. Does everybody obey the same locomotor strategy? To answer the question, we should build a data basis of trajectories performed by several subjects having to reach a same goal. Then we should prove the existence of stereotyped behaviors.
2. A data basis of trajectories being given, we should find a control model with an associated optimal cost. The inputs of a standard optimal control problem are a model and an associated cost function. The outputs are the optimal trajectories. Here we assume that the observed trajectories are optimal and we should find the corresponding system (model and cost). This problem is then viewed as an *inverse* optimal control problem.

Of course the data basis should not be made of a single trajectory. All the possible goals have a priori to be considered. The task is obviously impossible from the experimental point of view. This is why the experimental protocol considers a sampling of the reachable space.

2.2 Trajectory data basis: protocol and apparatus

The underlying hypothesis of our work is that the simple differential equation (1) contains enough information to build a model. It involves only three parameters: two for the body position and one for the body direction (with respect to an external frame). Then the idea is to sample the 3-dimensional reachable space into a set of goals to be reached.

We restrict the study to the natural forward locomotion with nominal speed. The model we study should be valid for all possible intentional goals reachable by a forward walk. We exclude from the study the goals located behind the starting position and the goals requiring side walk steps. Because the objective was to cover at best the 3-dimensional reachable region, we sampled the domain with 480 points defined by 40 positions on a 2-dimensional grid (within a 5m by 9m rectangle) and 12 directions each (see Figure (5)). The starting position was always the same. One subject performed all the 480 trajectories while other six performed only a subset of them chosen at random. Subjects walked from the same initial configuration to a randomly selected final configuration.

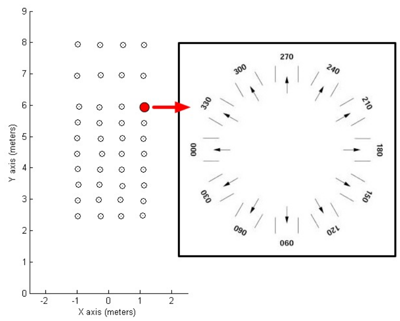


Figure 5: *The space is sampled with 40 positions. At each position the porch direction is sampled with 12 angles.*

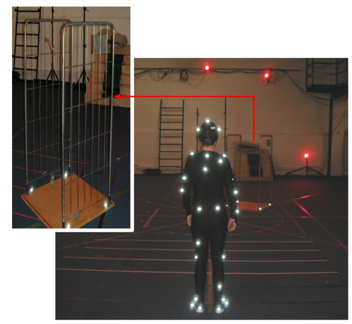
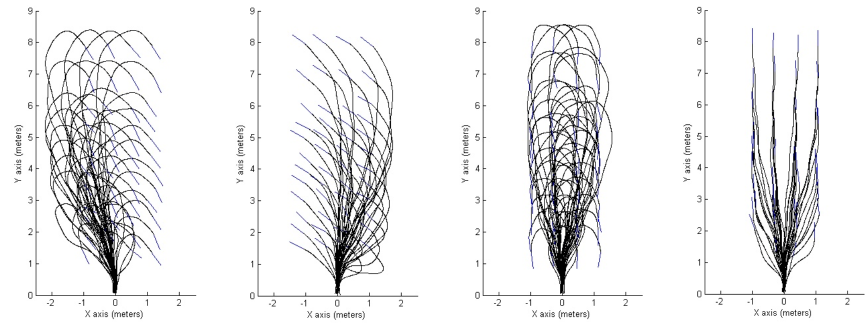


Figure 6: *The porch and the room used in the experiments.*

The target consisted in a porch which could be rotated around a fixed point to indicate the desired final direction (see Figure (6)). The subjects were instructed to freely cross over this porch without any spatial constraints relative to the path they might take. They were allowed to choose their natural walking speed to perform the task. The distances travelled by subjects ranged between 4.50 ± 0.25 meters (across subjects and trials) for the nearest target and 9.38 ± 2.54 meters for the farthest target. We used motion capture technology to record more than 1400 real trajectories. Subjects were equipped with 34 light reflective markers located on their bodies. This is the data basis used for statistical analysis and validation of the proposed models. Figure (7) displays some of the recorded trajectories. In that picture, the trajectories are those of the torso (see below the comment on the choice of the body frame).
Another important issue is the following one: dynamical effects. We wanted to prevent the dynamical effects as much as possible. The subjects were asked to walk quietly. The statistical analysis has shown that they performed the trajectories at a quasi-constant linear velocity. Moreover, in order to neglect the dynamical impulsions of motion starting and ending, the subjects were asked not to begin their motion at the starting configuration as well as not stop at the goal. They entered the room via the starting configuration while walking and they continued walking after reaching the goal. After all motions were recorded, we removed the segments before the start configuration and after the goal. Figures (7) display various samples of trajectories performed by a single subject for different goals.

2.3 Statistical analysis: stereotyped behaviors

Thanks to a statistical study reported with details in [11] it has been possible to show that general principles governed the motion strategy of all the six observed subjects.



(a) (b) (c) (d)

Figure 7: *Some examples of real trajectories with the same final direction. (a), (b), (c) and (d) show all real trajectories where the final direction is 330 deg., 120 deg., 90 deg. and 270 deg. respectively.*

While no specific constraint was provided to subjects neither at the spatial level (the path they should have followed for crossing throughout the doorway) nor at the temporal level (the velocity profile they should have produced), we observed that all subjects generated very similar trajectories both in terms of path geometry and in terms of velocity profiles. The subjects also exhibit very similar turning behaviors as quantified by their body rotation in space. In contrast, a much greater variability was observed at the level of the successive positions of the feet in space. These results show that the locomotor trajectories are planned according to goals which are expressed in terms of the body spatial displacement in space, and are generated using motor coordination *patterns*. These observations confirm the validity of the adopted top-down approach announced at the very beginning of the paper.

2.4 The choice of the body frame

In this study the choice of the body frame we should consider has been a critical issue. We are looking at a frame that best accounts for the equation (1). Three of them have considered (see Figure (8)): the head, the torso and the pelvis. The position and the direction of the frames are deduced from the motion capture markers located respectively above the ears, on the shoulders and on both sides of the hip.The picture shows a single trajectory represented in the three frames. The curves compare the direction of the frame and the tangent to the position of the frame. The nonholonomic hypothesis is verified if both curves coincide. The study in [8] shows that the torso frame accounts for that hypothesis, much better than the head or the pelvis. The statistical analysis shows that a good model of the torso frame behavior is

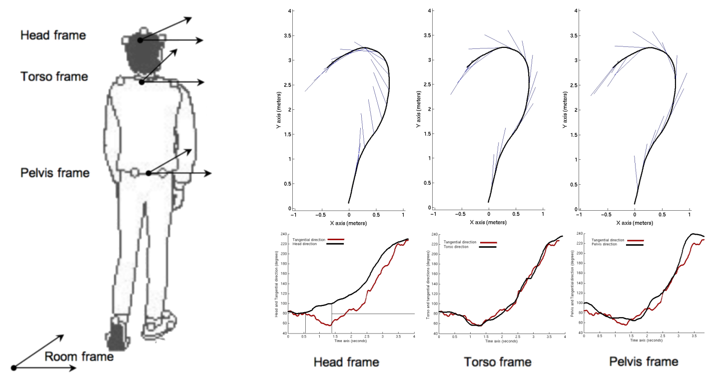


Figure 8: *Left: Markers and body frames. Right: For the same walking task, the three pictures on the top show the trajectories followed by the head, pelvis and torso positions in the room frame. The small segments indicate the respective directions of the body segments (head, pelvis, torso). The bottom pictures show two curves each: one curve is the tangent direction to the trajectories above; the second one is the corresponding body segment direction. For the torso both directions are very similar: this means that the torso frame follows "rather well" equation (1).*

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} u_1 + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u_2 \tag{4}$$

where u_1 is the linear velocity and u_2 is the angular one. The detailed study [8] shows that the shoulders behave as the front wheels of a car by anticipating up a couple tenths of second the direction change of the body.

3 Optimizing the derivative of the curvature

Considering equation (1) we made several hypothesis to find a cost function (time, length, jerk) that would explain the geometric shape of the walking trajectories. None of them can be validated by statistical analysis on the data basis. The idea presented in [9] has been to consider the following dynamic extension of (4):

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\kappa} \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \\ \kappa \\ 0 \end{pmatrix} u_1 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} u_2 \tag{5}$$

where u_1 is the linear velocity and u_2 is the control of the time-derivative of the curvature. Assuming $u_1 \in [a, b]$ with $a > 0$ (forward motion) and $u_2 \in [-c, c]$, we consider the cost function:

$$J = \frac{1}{2} \int_0^T <(u(\tau), u(\tau))> d\tau \tag{6}$$

Applying the maximum principle we found that the optimal trajectories verify locally $u_1^2 + u_2^2$ should be constant. The result is not surprising (see [27]). It has been not possible to deduce more information from the maximum principle. Now we have considered a statistical analysis performed on the trajectory data basis. It appeared that the u_1 control remains "reasonably" constant along the trajectories (we should keep in mind that the subjects were asked to enter the room by the starting configurations while not stopping at the goals). Then we deduced that u_2 is a piecewise constant function. A curve followed at constant velocity while linearly increasing or decreasing the derivative of the curvature is known as being a clothoid[2]. Finally we concluded on the conjecture that locomotor trajectories are made of clothoid arcs.

The proof of that conjecture required the effective computation of the optimal trajectories for the considered system (5) with the considered cost (6). Analytical solutions are out of the scope of the current state of the art. Then we fell back on numerical optimization algorithms. Because the system (5) is non linear, we made use of the numerical algorithm proposed in [3] (see [9] for detailed development). Figure (9) from [9] shows a representative example of statistics computed over the entire movement for the same initial and final configurations. Each subject has done 3 trials. Figure (9).a shows the real (thin) trajectories performed by five subjects with respect to predicted (bold) optimal trajectory computed by the numerical algorithm from our optimal control model. All these trajectories correspond to the same initial and final configuration. It illustrates the variability of the pattern and the predicted trajectory. Figure (9).b shows an averaged trajectory from 15 real ones performed by the subjects (thin) and the predicted optimal trajectory linking the same initial and final configurations (bold). Such a statistical analysis were performed for all the study cases: the optimal control model we propose approximates 90 percent of the real trajectories with a precision error of less than 10 cm in position, along all the considered trajectories. The geometric shape of the locomotor trajectories is then similar to the one of the optimal control model (see Figure 10).

This study shows that, when walking in an empty room towards an intentional goal defined both in position and in direction, the subjects tend to optimize the time-derivative of the curvature.

In the following section, we present the (numerical) synthesis of the proposed optimal control problem. As per the Dubins' model sketched above, the related questions

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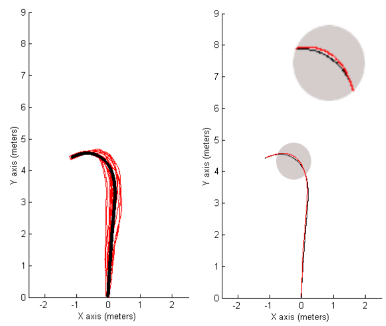
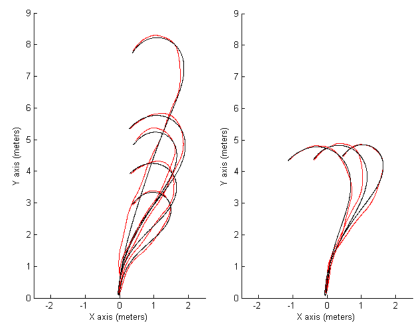
[2] A clothoid is a curve of equation $\rho(s) = \frac{1}{s}$ where ρ is the radius of curvature and s is the curvilinear abscissa.



(a) (b)

Figure 9: *Representative example of statistics computed over the entire movement for the same initial and final configurations. (a) shows the real (thin) trajectories performed by five subjects with respect to predicted (bold) optimal trajectory. (b) shows the mean trajectory (thin) and the predicted optimal trajectory (bold).*



(a) (b)

Figure 10: *Representative examples of comparisons between real (thin) and predicted (bold) locomotor trajectories. (a) shows the behavior of the real and predicted trajectories by translating the final position in the vertical axis with a fixed final direction. (b) shows the behavior of the real and predicted trajectories by translating the final position in the horizontal axis with a fixed final direction.*

are: How many clothoid arcs the trajectories contain? How are the sequences of arcs organized? What is the number of the different sequences?

4 Synthesis: the words of the human locomotion

4.1 Synthesis

In this section, we numerically characterize the optimal paths by the number of concatenated clothoid arcs. We can define a clothoid as the curve satisfying the following equation:

$$\kappa(\tau) = \pm c\tau, \quad \tau \in (-\infty, \infty) \tag{7}$$

where the sign $\pm$ defines the orientation of each piece of clothoid: $u_2(\tau) \equiv c, \kappa(\tau) \to \infty$ or $u_2(\tau) \equiv -c, \kappa(\tau) \to -\infty$.

The concatenation point between two clothoid arcs is called a *switching point*. At each switching point the curvature function contains a local extremum and the derivative of the curvature has a discontinuity. From the preceding reasoning, our numerical

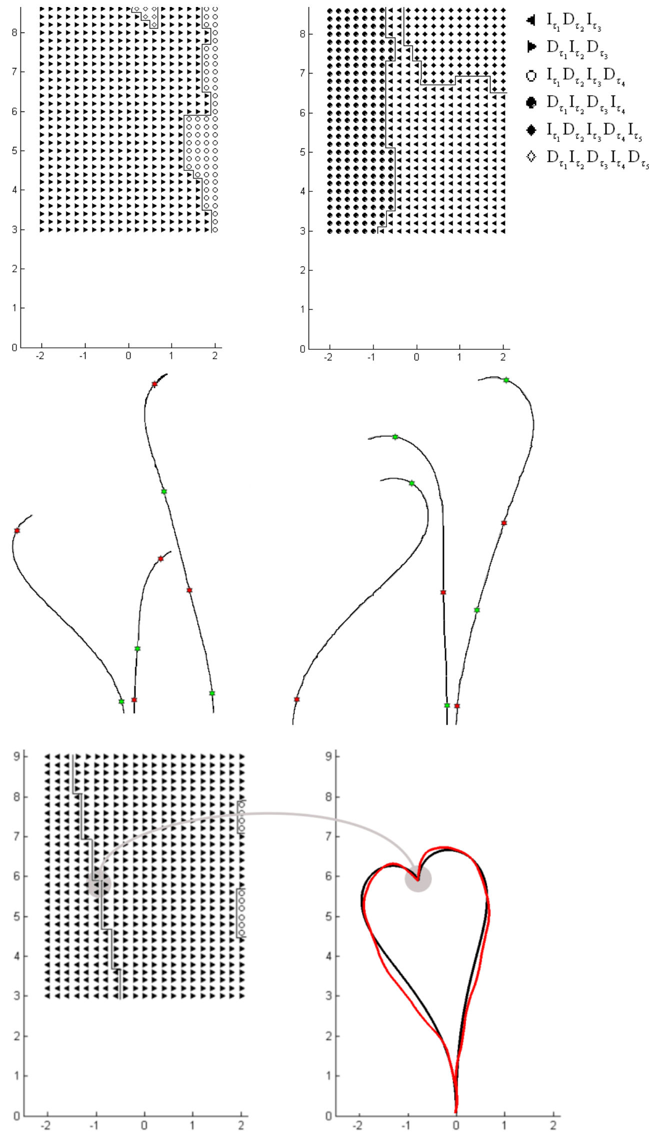


Figure 11: *Top: The partition of the slices $\theta = 150deg.$ and $\theta = 340deg.$ induced by words of human locomotion. Middle: Some examples of representative optimal trajectories for different regions. Bottom-left: The partition of the slice $\theta = 260deg.$. Bottom-right: An example of two configurations belonging to two adjacent cells governed by Case 2. The real (thin black) trajectories performed by the same subject with respect to predicted (bold) optimal trajectory. Cut-locus explains the strategy change.*

approach consists to determine the number of switches and the order of switching points of each optimal path. The method is based on the local analysis of the curvature. To be more precise, at each local extremum of the curvature there exists a switching point. A description of the regions in the configuration space is obtained by repeating the above process for all possible optimal trajectories. We have indeed identified the sets of optimal trajectories uniquely defined by the orientation of each piece of clothoid curve and the total number of clothoid arcs.

The aim of the synthesis problem is the characterization of an optimal path linking any two configurations in the space. The synthesis problem is solved when from any initial configuration we can determine the optimal path joining such a configuration to the origin (or final configuration). To compute the synthesis for our optimization problem, we first sampled the portion of the reachable space considered in the experimental protocol $(x, y, \theta) \in \mathbb{R}^2 \times \mathbb{S}^1$. Hence, we only considered the space in front of the starting configuration. We defined the point $(0, 0, \pi/2)$ as the origin of the configuration space ($\kappa = 0$ at the starting and goal configurations). We computed the approximation of the space by a grid decomposition technique. The grid resolution was $(0.2m \times 0.2m \times 10deg)$ and the grid range from $[-2, 2] \times [3, 9]$ in position. Then, the analysis has been carried out by the following steps:

1. $\mathbb{R}^2 \times \mathbb{S}^1$ is partitioned with 36 slices according to the direction θ.
2. For each θ-slice, we computed numerically all the optimal trajectories from the origin to each 2-dimensional vertex in the grid.
3. At each θ-slice, we determined the regions mapping the types of optimal paths. Each region corresponds to a set of optimal trajectories containing the same ordered combination of clothoid arcs in terms of their orientations.

By considering that the curvature κ of an arc of a clothoid can either increase (I_τ when $u_2 = c$) or decrease (D_τ when $u_2 = -c$) for a given time τ, we then found experimentally that only 6 combinations appear: $I_{\tau_1} D_{\tau_2} I_{\tau_3}$, $D_{\tau_1} I_{\tau_2} D_{\tau_3}$, $I_{\tau_1} D_{\tau_2} I_{\tau_3} D_{\tau_4}$, $D_{\tau_1} I_{\tau_2} D_{\tau_3} I_{\tau_4}$, $I_{\tau_1} D_{\tau_2} I_{\tau_3} D_{\tau_4} I_{\tau_5}$, $D_{\tau_1} I_{\tau_2} D_{\tau_3} I_{\tau_4} D_{\tau_5}$. We call these sequences the words of human locomotion.

4.2 Geometric analysis and motor control interpretation

The words above induce a partition of the configuration space $\mathbb{R}^2 \times \mathbb{S}^1$ into cells (see Figure (11)).

Let us consider a word, e.g. $I_{\tau_1} D_{\tau_2} I_{\tau_3}$. The mappings from $\mathbb{R}^3$ onto $\mathbb{R}^2 \times \mathbb{S}^1$ associating the triplet (τ_1, τ_2, τ_3) to a configuration in the cell of the corresponding word is a local diffeomorphism. This means that the motor controls used to reach two different configurations in a same cell follow the same pattern. They just differ by the position of the switching points on both trajectories. When a configuration varies continuously within a given cell, the switching times vary continuously. What happens between two adjacent cells? Two cases may arise:

- Case 1: Traversing the border can be be done by a continuous change of the motor control. For instance the cells $I_{\tau_1} D_{\tau_2} I_{\tau_3}$ and $D_{\tau_4} I_{\tau_5} D_{\tau_6}$ have common borders of respective equation $\tau_1 = 0$ and $\tau_6 = 0$. This means that there is a *continuous* reshaping of the geometric shape of the trajectories (as in Figure (10)).
- Case 2: Traversing the border induces a discontinuity on the motor control. This would be the case of the same cells $I_{\tau_1} D_{\tau_2} I_{\tau_3}$ and $D_{\tau_4} I_{\tau_5} D_{\tau_6}$ if their common border is obtained respectively for $\tau_3 = 0$ and $\tau_6 = 0$. In that case there is a *discontinuity* of the geometric shape of the trajectories when traversing the border.

For most of the cells, the cell adjacency is governed by Case 1. The interesting case is Case 2. It is a subtle case: at a configuration on the common border of both cells, there exist exactly *two* optimal trajectories with the *same* cost and a completely *different* shape. Such borders arise for symmetric nonholonomic systems (e.g. a car that moves forward and backward [25]). In that cases they are known as the cut-locus in sub-Riemannian geometry (see [1]).
What is worth noticing is that the cut-locus of the synthesis above accounts for the locomotion *strategy*. Figure (11) shows an example of two configurations belonging to two adjacent cells governed by Case 2. *The trajectories obtained by simulating our control model fit with the observed trajectories*. This means that the *two apparently completely different strategies* used by the subject to reach two close configurations obeys *de facto* a same strategy tending to optimize the derivative of the curvature. The model we propose answers the question asked at the beginning of the paper (see Figure (1).c).

5 Perspectives

The study presented in this paper deserves comments and opens questions:

- The optimal control synthesis has been done by numerical computation. A presentation of the numerical algorithms as well as an analysis of their robustness in capturing singularities as the cut-locus would deserve a much more detailed presentation which has been skipped here due to the lake of place. Details can be found in [9].
- The same comment holds for the statistical analysis. Details can be found in [8, 11].
- The reachable space of the control system (5) covers the entire $\mathbb{R}^2 \times \mathbb{S}^1$ configuration space of the position and the direction of the human body. However our model accounts only for a part of a human locomotion strategies. Indeed when the configuration to be reached is just behind you, you will certainly perform a *backward* step motion. If the configuration is just on your left, then you will perform a *sideway* step. Both backward and sideway motions are not accounted

by our model. We just focus on forward natural locomotion when the goal is defined *in front* of the starting configuration. The scope of our model remains to be defined in terms of the shape of the reachable space it accounts for.

- The current study opens intriguing mathematical questions. Usually, in optimal control, the considered costs induce metrics in the state space. Here the cost of a trajectory does *not* induce any metric: for instance, as for Dubins' model, the considered cost is not symmetric at all. We can say that the locomotion space is not a metric space. What special geometry accounts for the locomotion space? We have seen that the presence of special structures as the observed cut-locus is related to sub-Riemannian geometry. However the space is not equipped with a sub-Riemannian metric. The problem to understand the topology of such a space is a challenging mathematical problem as such.
- Finally, from a pure neuroscience point of view, our study validates the top-down methodology approach. It appears that decisional problems, such as the problem to decide *before moving* what strategy the subject selects (Figure (1).c), is accounted by the *same* model explaining how the trajectories are reshaped locally with respect to the goal to be reached (Figure (1).b). As a consequence the brain *plans* its actions. It has a *global* point of view of the task to be performed. A reasonable conjecture to explain the shape of the locomotor trajectories could have be a simple *local* sensory feedback control assumption: the gaze seems to be the only sensor used by the subjects in our experiments; we have checked that the gaze is always directed towards the goal; then it would have been possible to conclude that the body *follows* the gaze (as the rear wheels of a car follow the front wheels). This conjecture is not true: as depicted in the study case of Figure (8) (see [8] for details) it may appear that the body is turning right while the head is turning left. As a consequence locomotion does not obey such a simple sensory feedback control model tending to reduce locally some sensory distance to the goal. Our study shows that *planning* (i.e. open-loop control) gives an explanation that pure *feedback control* (i.e. close-loop control) cannot give. The open question now is: how planning and feedback control are related? This is the next step of our current research.

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