

Time-optimal differential game

N.N.Krasovskii, A.I.Subbotin Game Theoretical Control Problems (1988)

- Dynamical system: $\dot{x}(t) = f(x(t), u(t), v(t)),$
 $t \geq 0, x(t) \in R^n, x(0) = x_0 \in R^n, u(t) \in P, v(t) \in Q$

- Payoff functional: $M \subset R^n$ is a given terminal set,

$$J(x(\cdot); M) = \begin{cases} \infty, & \text{if } x(t) \notin M \text{ for any } t \geq 0, \\ \min\{t \geq 0: x(t) \in M\} & \text{in other case} \end{cases}$$

- Positional strategies of the players:

$$U: [0, \infty) \times R^n \rightarrow P, \quad V: [0, \infty) \times R^n \rightarrow Q$$

- Value function of the game: $x_0 \mapsto T(x_0; M) \in [0, \infty]$

$$T(x_0; M) = \inf_U \sup_{x(\cdot) \in X_1(x_0, U)} J(x(\cdot); M) = \sup_V \inf_{x(\cdot) \in X_2(x_0, V)} J(x(\cdot); M)$$