

An example of time-optimal differential game

$$\ddot{y}_P = u, \quad \dot{y}_E = v, \quad |u| \leq \mu, \quad 0 \leq v \leq \nu, \quad \mu > 0, \quad y_P, y_E \in \mathbb{R}^1.$$

The pursuer P aims to an exact capture of the evader E as soon as possible in such a way that his velocity \dot{y}_P is equal to a given value $a > \nu$ at the capture.

Change the variables

$$x_1 = y_P - y_E, \quad x_2 = \dot{y}_P.$$

We have

$$\dot{x}_1 = x_2 - v, \quad \dot{x}_2 = u, \quad |u| \leq \mu, \quad 0 \leq v \leq \nu.$$

The first player minimizes the time of reaching the terminal set $M = (0, a)^T$, $a > \nu$, from the initial position x_0 . The interests of the second player are opposite.