Theorem on sufficient conditions

Suppose that $\Omega \subseteq R^n$ and $M \subseteq \Omega$ are closed sets, a function

$$\varphi(\cdot) : \Omega \to [0, \infty]$$

is given, and

$$\Theta = \sup_{z \in \Omega} \varphi(z), \quad D(t) = \{x \in \Omega : \varphi(x) \leq t\}, \quad t \in [0, \Theta),$$

$$F(t) = \{x \in \partial D(t) : \varphi(x) = t\}, \quad B(t) = \{x \in \partial D(t) : \varphi(x) < t\},$$

$$S(t) = \overline{F(t)} \cap \overline{B(t)}, \quad G(t, \varepsilon) = \begin{cases} \varnothing, & S(t) = \varnothing, \\ S(t) + \operatorname{int} \mathbf{B}(0, \varepsilon), & S(t) \neq \varnothing, \end{cases} \quad \varepsilon > 0.$$

Suppose also that the functuion $\varphi(\cdot)$ is lower semicontinuous, $D(0) = M$, $\mathcal{T} \subset (0, \Theta)$ is a finite (possibly, empty) set, and the following conditions hold.