

## Papers by A.A.Markov and L.E.Dubins

$$\dot{x}_p = \sin \theta$$

$$\dot{y}_p = \cos \theta$$

$$\dot{\theta} = u$$

$$|u| \leq 1$$

**A.A.Markov (1889).** Some examples of the solution of a special kind of problem on greatest and least quantities Soobschenija Charkovskogo matematicheskogo obščestva, Vol. 2, 1, N° 5, 6, 250–276.

**L. E. Dubins (1957).** On curves of minimal length with a constraint on average curvature and with prescribed initial and terminal positions and tangents. Amer. J. Math., Vol. 79, 497–516.

Нѣсколько примѣровъ рѣшенія особаго рода задачъ о наибольшихъ и наименьшихъ величинахъ.

А. А. Маркова.

ЗАДАЧА 1.

Между данными точками  $A$  и  $B$  (см. фиг. 1-ю) провести кратчайшую кривую линию при слѣдующихъ двухъ условіяхъ: 1) радиусъ кривизны нашей кривой повсюду долженъ быть не меньше данной величины  $\rho$ , 2) въ точкѣ  $A$  касательная къ нашей кривой должна имѣть данное направленіе  $AC$ .

РѢШЕНИЕ.

Пусть  $M$  одна изъ точекъ нашей кривой, а прямая  $NMT$  соответствующая касательная.

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ON CURVES OF MINIMAL LENGTH WITH A CONSTRAINT ON  
AVERAGE CURVATURE, AND WITH PRESCRIBED INITIAL  
AND TERMINAL POSITIONS AND TANGENTS.\*<sup>1</sup>

By L. E. DUBINS.

We have now established our main result:

**THEOREM I.** *Every planar  $R$ -geodesic is necessarily a continuously differentiable curve which is either (1) an arc of a circle of radius  $R$ , followed by a line segment, followed by an arc of a circle of radius  $R$ ; or (2) a sequence of three arcs of circles of radius  $R$ ; or (3) a subpath of a path of type (1) or (2).*