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INFORMATIONAL SETS IN MODEL PROBLEMS OF AIRCRAFT TRACKING

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16-th IFAC World Congress

Prague, 2005

Here, materials of the report submitted on Section "Control and Estimation under Set-Membership Uncertainty" are presented. Organizers: Alexander B. Kurzhanski and Felix L. Chernousko.

We shall talk about simple problems of observation of an aircraft motion in the horizontal plane.

Informational set at a current instant is a totality of all phase states consistent with description of the dynamics, constraints on measurement errors, and history of the observation – control process.

Terms equivalent to the term **“informational set”** are

“feasible set”, “membership set”,

“likelihood set”, “uncertainty set”.

The approach is often called the

“set membership estimation”,

“unknown but bounded error description (UBB approach)”.

We use an approach based on set membership estimation. The main term used is the informational set. A feasible set, membership set, and likelihood set are equivalent terms.

N.N. Krasovskii,

A.B. Kurzhanski, Yu.S. Osipov, A.I. Subbotin,

F.L. Chernousko,

V.M. Kuntsevich, B.N. Pschenichnyi,

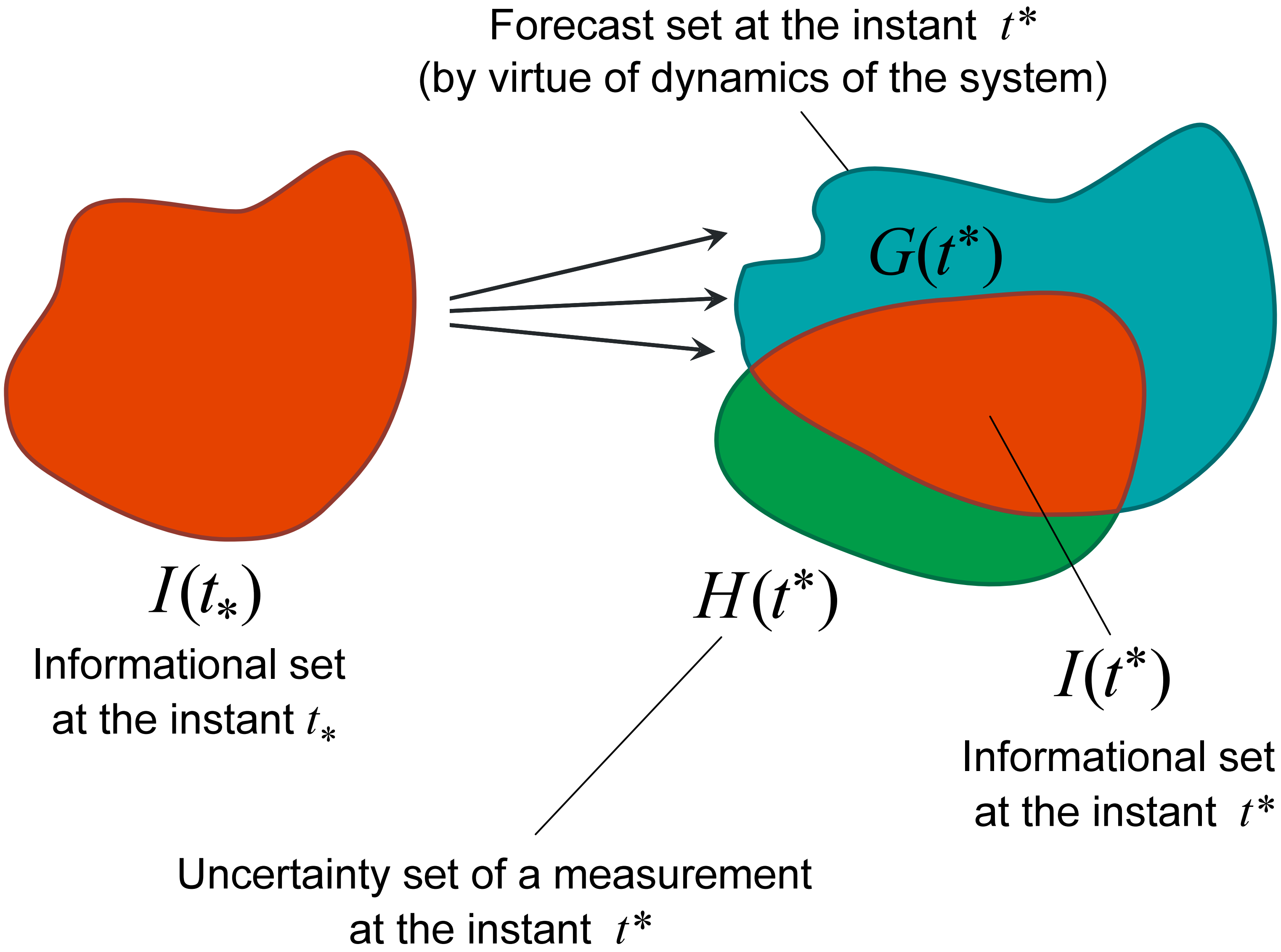
B.T. Polyak.

D.P. Bertsecas, I.B. Rhodes, F.C. Schweppe, H.S. Witsenhausen,

M. Milanese, J. Norton, H. Piet-Lahanier, E. Walter.

These are Russian and Western scientists, which have significant contribution in the theory and effective methods of construction of informational sets.

Transformation of informational sets

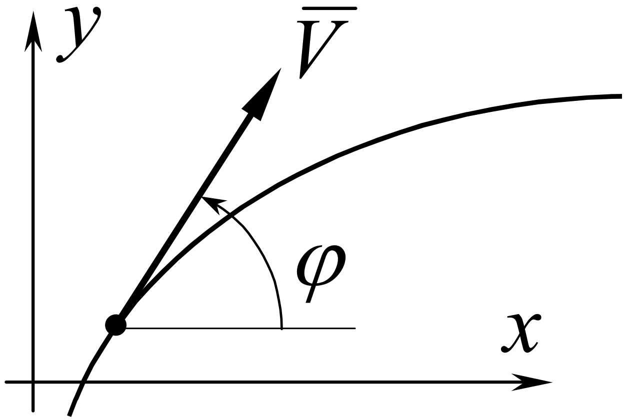


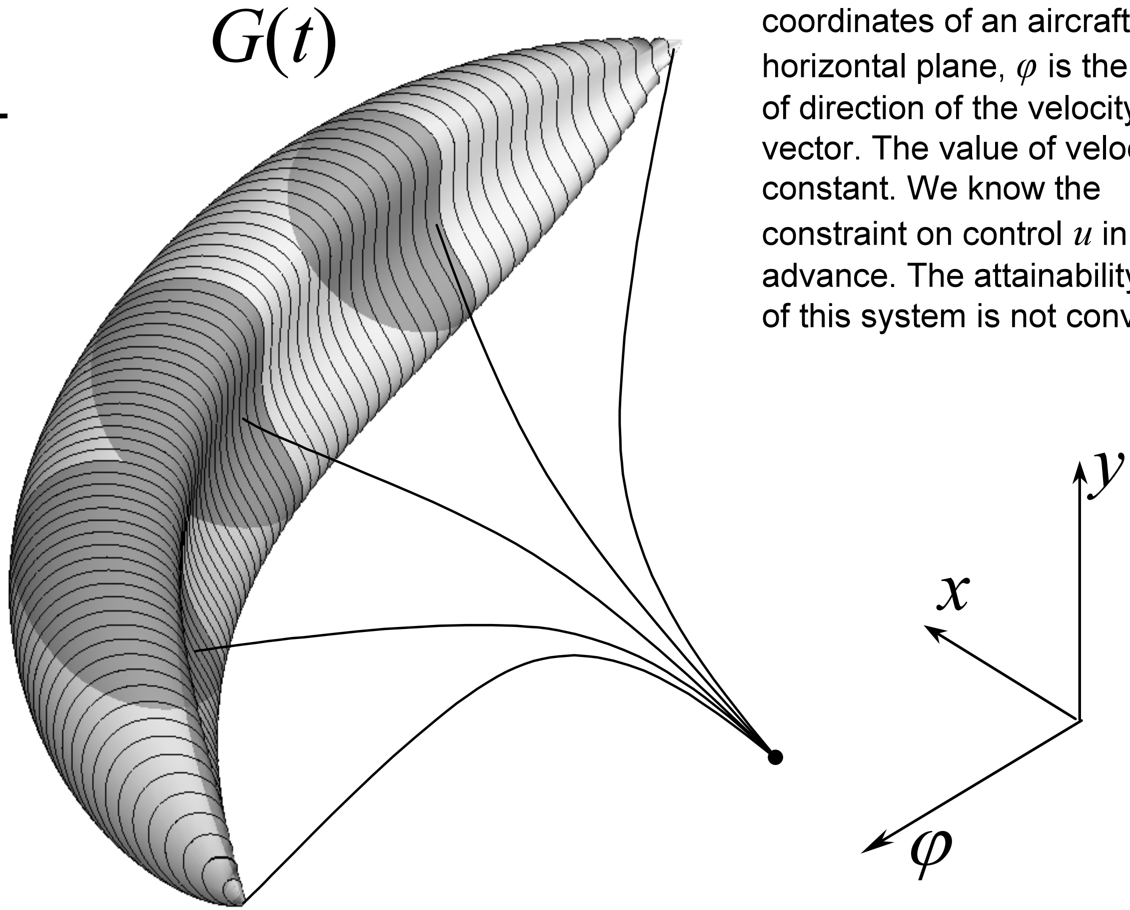
Having observations at the given instants, construction of the next informational set is implemented by building the forecast set and intersection of this set with uncertainty set of a measurement got at the instant t^*. The uncertainty set of each measurement is constructed using a priori known geometric constraint on the error of measuring.
Everywhere below we assume that only geometric coordinates of an aircraft are measured.

Informational sets in a three-dimensional problem

The first model problem.

Nonlinear control system of the third order





Here, x and y are geometric coordinates of an aircraft in the horizontal plane, φ is the angle of direction of the velocity vector. The value of velocity is constant. We know the constraint on control u in advance. The attainability set of this system is not convex.

$$\dot{x} = V\cos\varphi,$$
$$\dot{y} = V\sin\varphi,$$
$$\dot{\varphi} = \frac{k}{V}u;$$

$$|u| \le 1, \quad k = \text{const} > 0, \quad V = \text{const} > 0$$

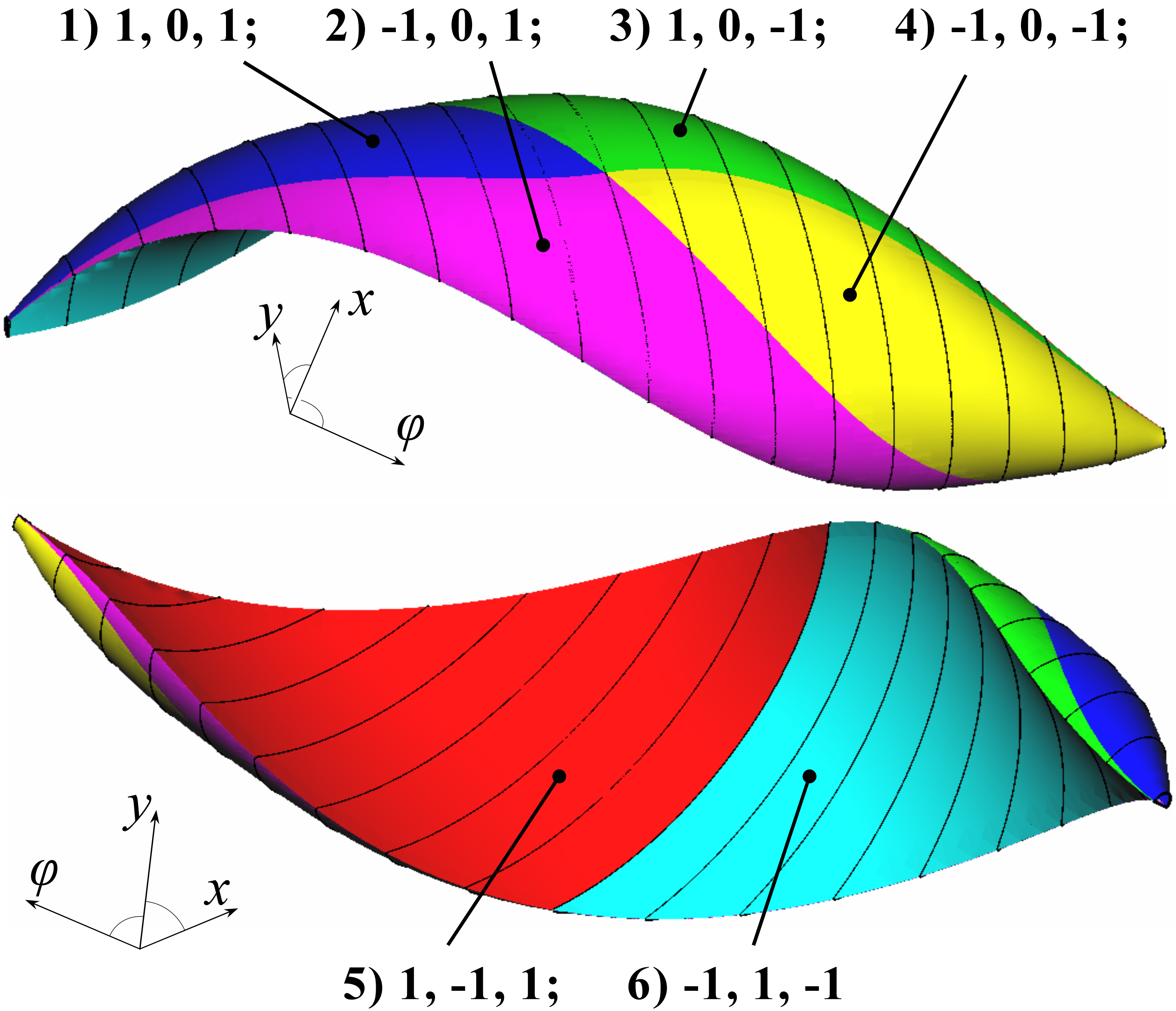
Who has investigated systems with such a dynamics:

- R. Isaacs – A car model, game statements.
- L.E. Dubins – A time optimal problem (on the position and velocity direction), the theorem on a number and character of switches of the optimal control.
- T. Pecsvaradi – The feedback optimal control in a time optimal problem (on the position and velocity direction).
- E.J. Cockayne, G.Hall; Yu.I. Berdyshev – Description of the attainability set in projection into the plane of geometric coordinates.
- J.-P. Laumond (Ed.). Robot Motion Planning and Control. 1998 – Collection of review articles on control by cars.

Many authors considered problems with such a dynamics. Here, we recall only several of them and note the problems that were researched.

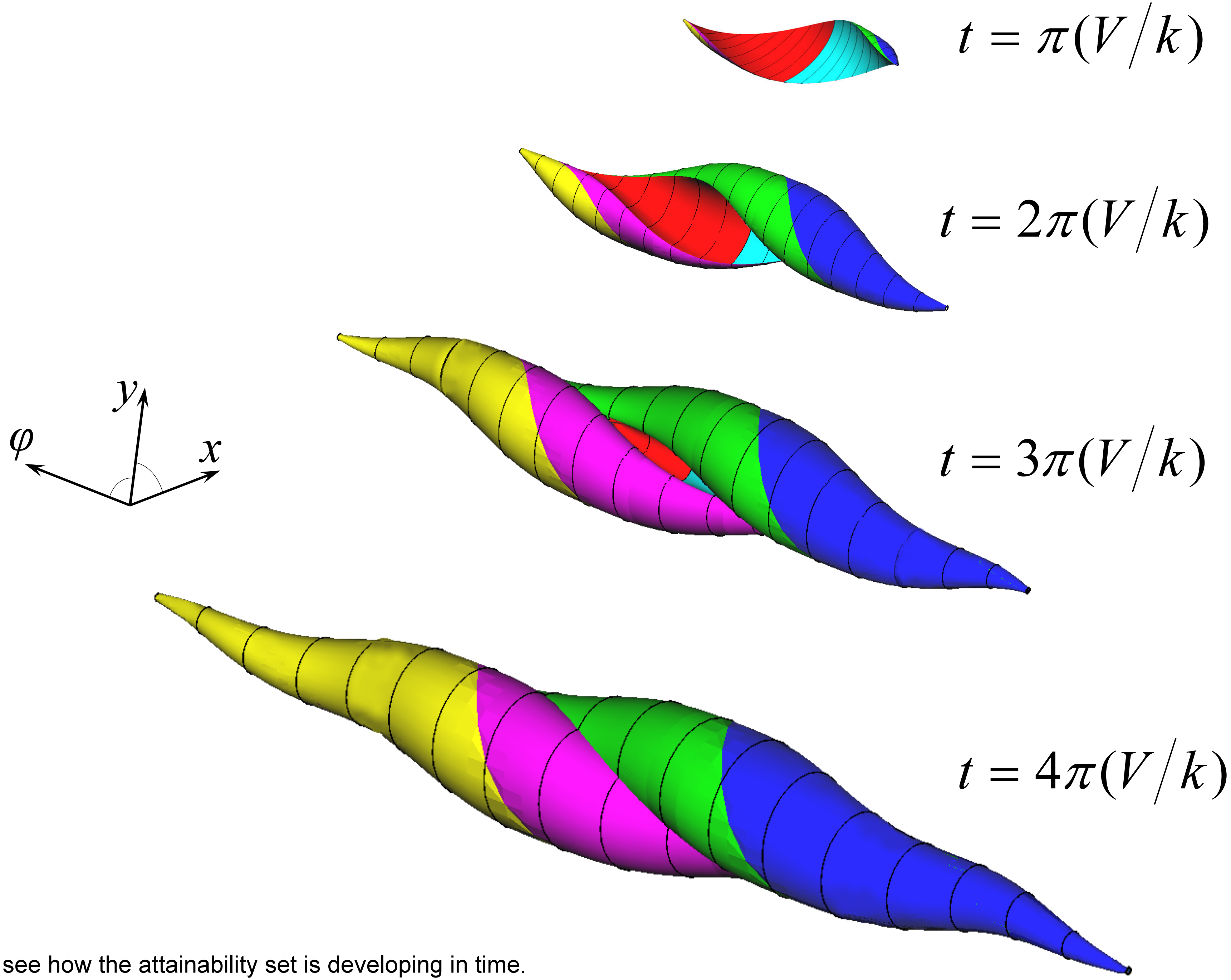
Structure of boundary of the attainability set

Variants of control leading to the boundary:



Each forecast set is the attainability set. Here is a three-dimensional attainability set for an instant not far from the initial one. Parts of the boundary with similar type of a piecewise-constant control that brings a motion onto the boundary are marked in their own colors. Each point on the boundary is reached under the control with not more than two switches. For the case shown, the initial set is a point in the three-dimensional space.

Development of the attainability set

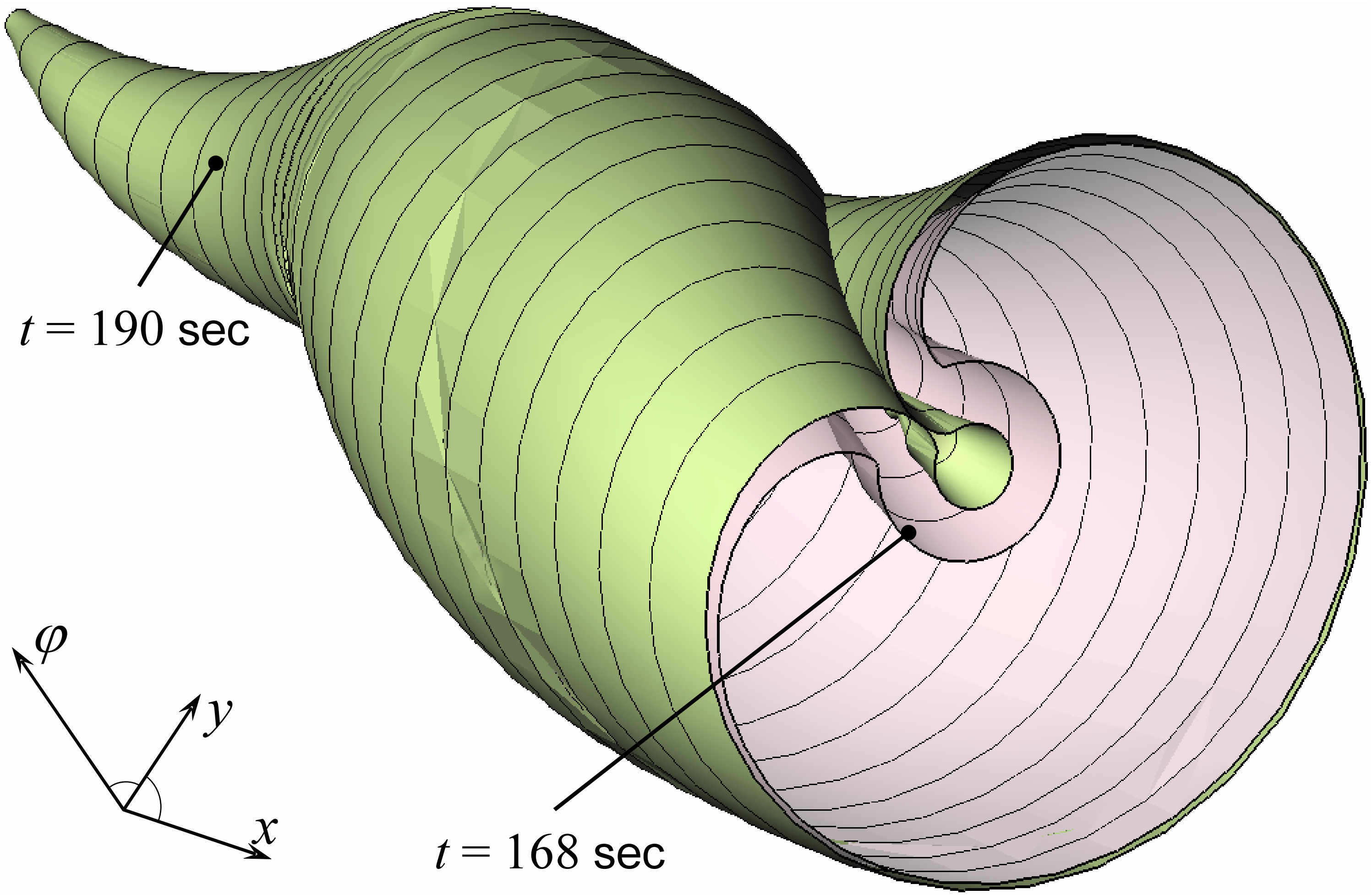


Here, we see how the attainability set is developing in time.

Violation of one-connectedness of the attainability set

$V = 100$ m/sec, $k = 6$ m/sec^2, $t_0 = 0$ sec

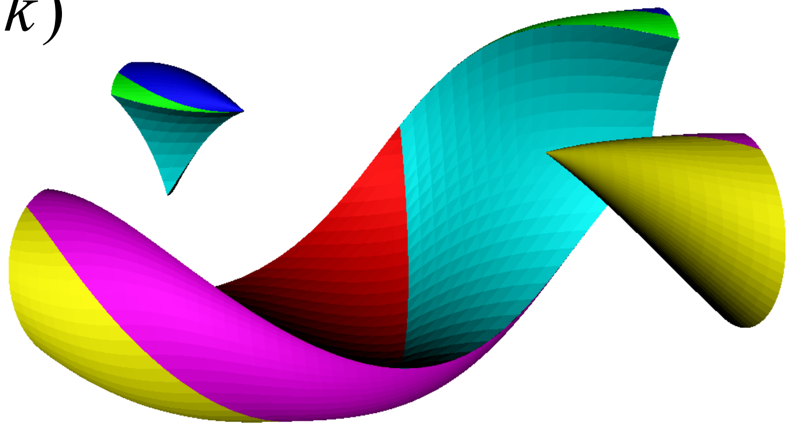
The plane of cross-section corresponds to $\varphi=0$

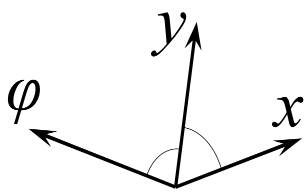


There exist instants, at which the attainability set loses its one-connectedness. In this picture, two sets for two instants are shown. The attainability set corresponding to the instant $t = 168$ sec is one-connected. The instant $t = 190$ sec is a critical one. If the instant t is taken a bit more than $t = 190$ sec, then one-connectedness will be lost. The cross-sections of the attainability sets are implemented for $\varphi=0$.

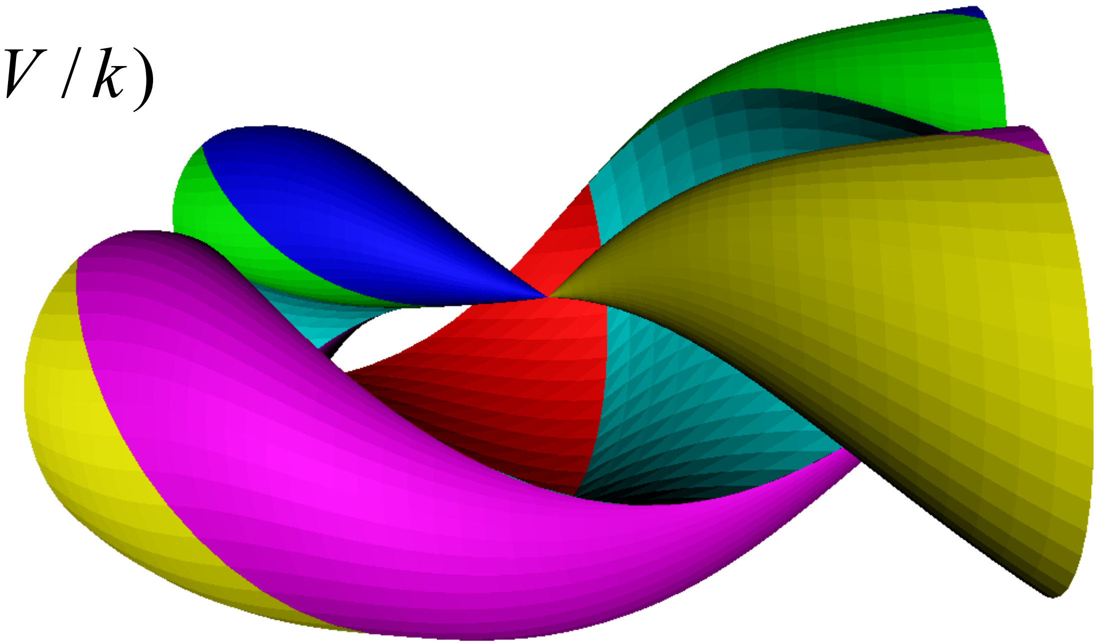
Attainability set (values of φ are identified on modulus 2π)

$t = 1.5\pi(V/k)$





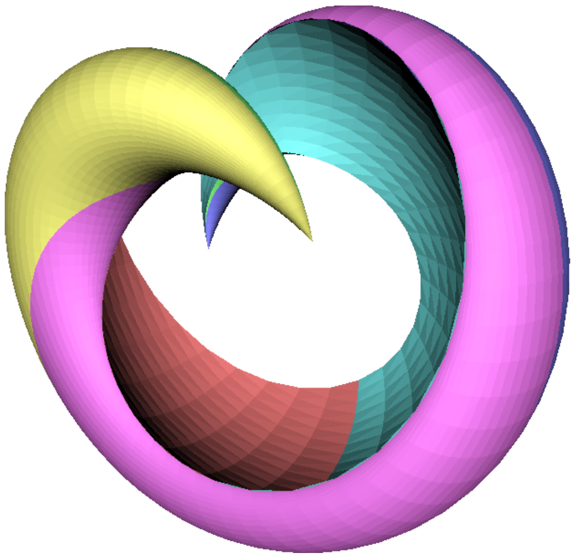
$t = 2\pi(V/k)$

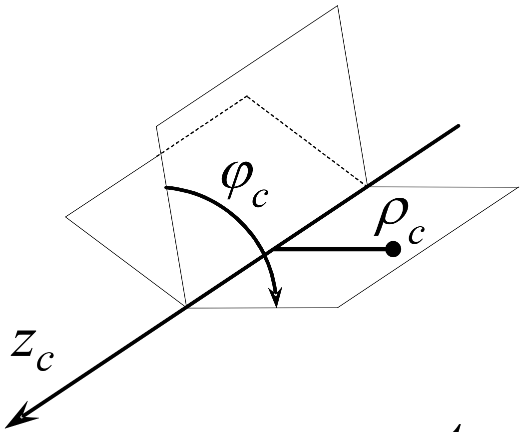


If values of the angle φ are identified on modulus 2π, the attainability sets look like these.

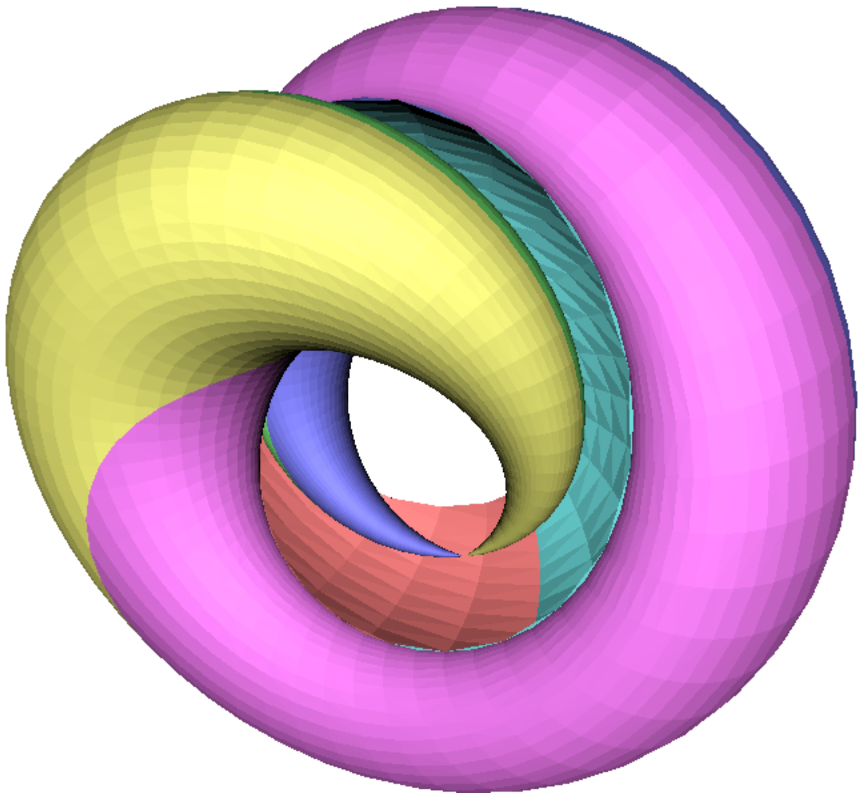
Attainability set in "cylindrical" coordinates

$$t = 1.5\pi(V/k)$$



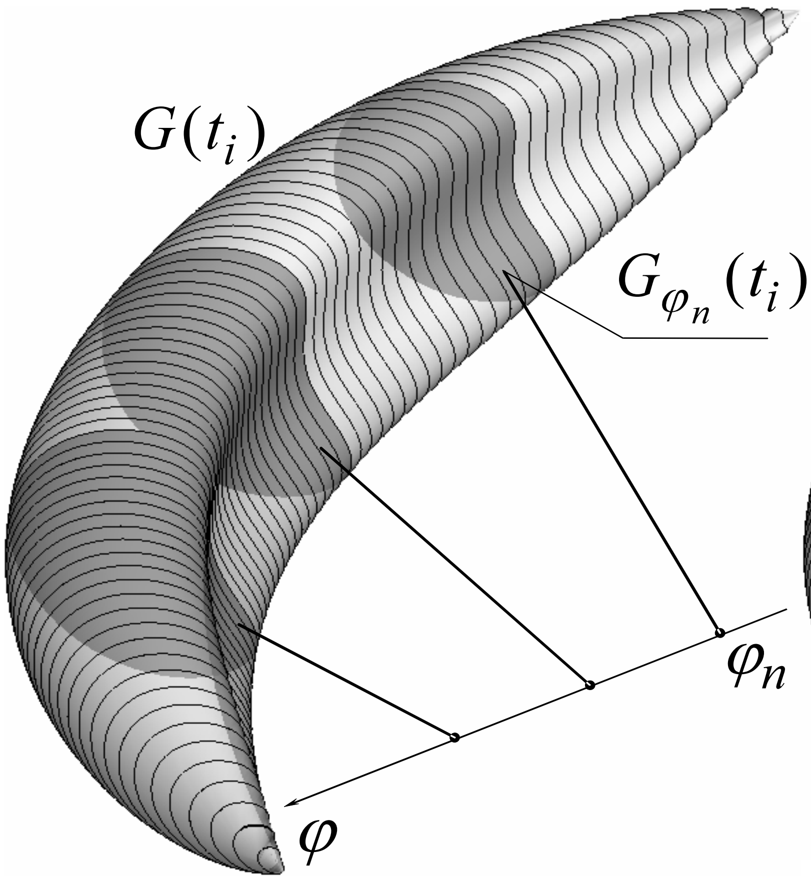


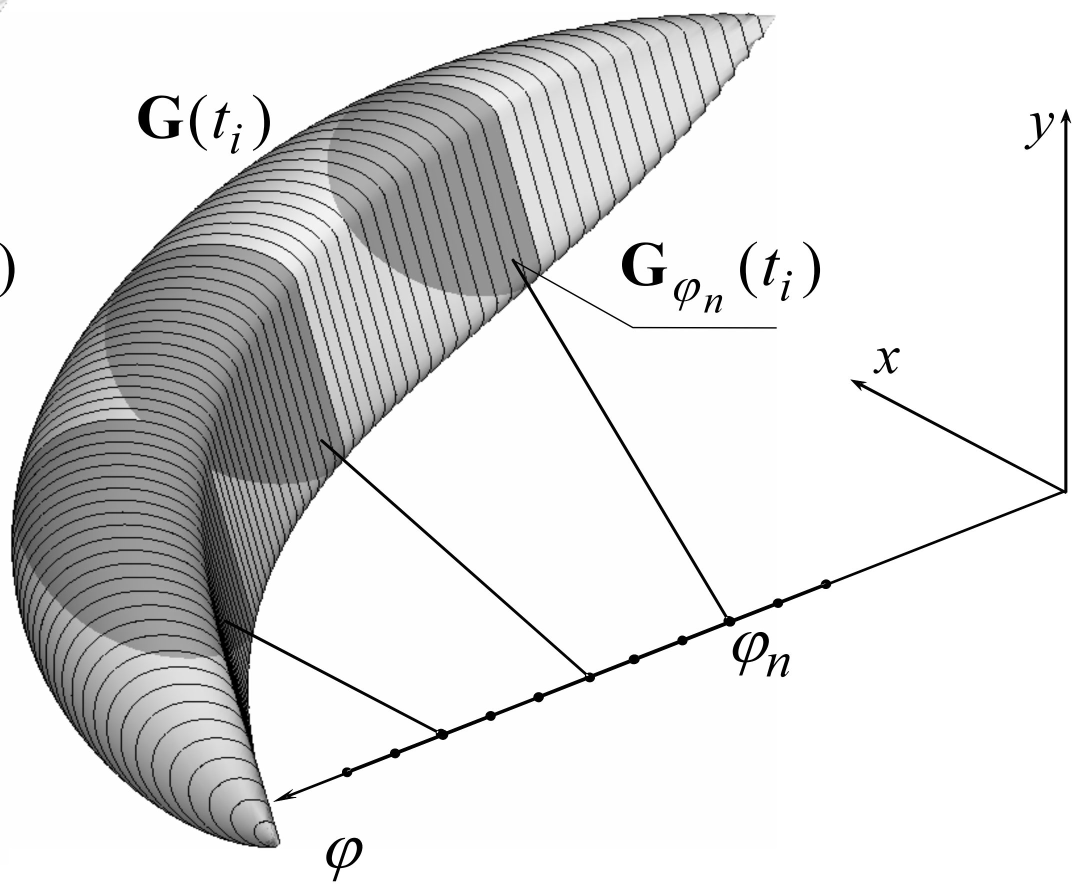
$$t = 2\pi(V/k)$$



Here, the attainability sets are represented in the cylindrical coordinates.

Exact forecast set and its approximation from above

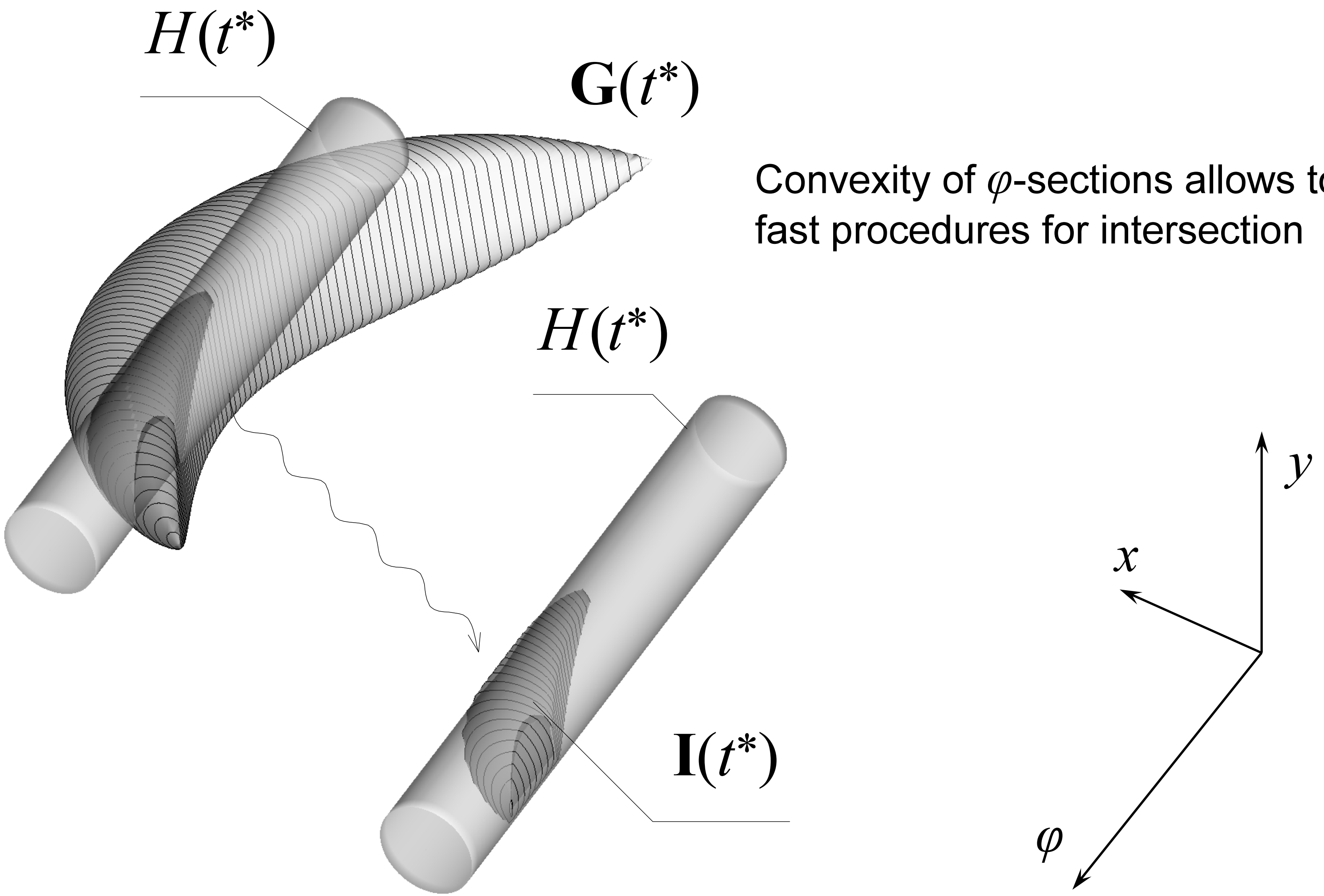




Approximating set is constructed without any information about the exact set

Thus, we see that the attainability sets have complicated structure. In practical constructions of the forecast sets, we apply their approximations from above. Using a grid with a small step in φ, we construct the sets by means of a special algorithm. These sets estimate the true sets from above. Under this, each φ -section is convex in the plane x, y.

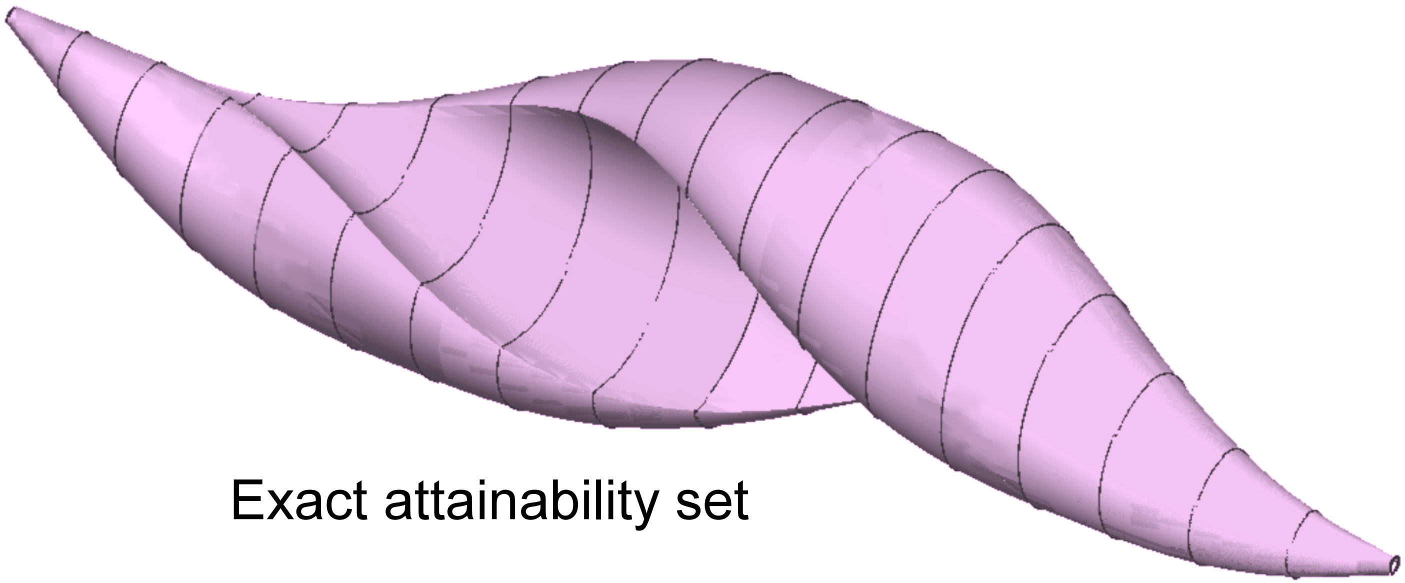
Intersection of the forecast set with a measurement uncertainty set



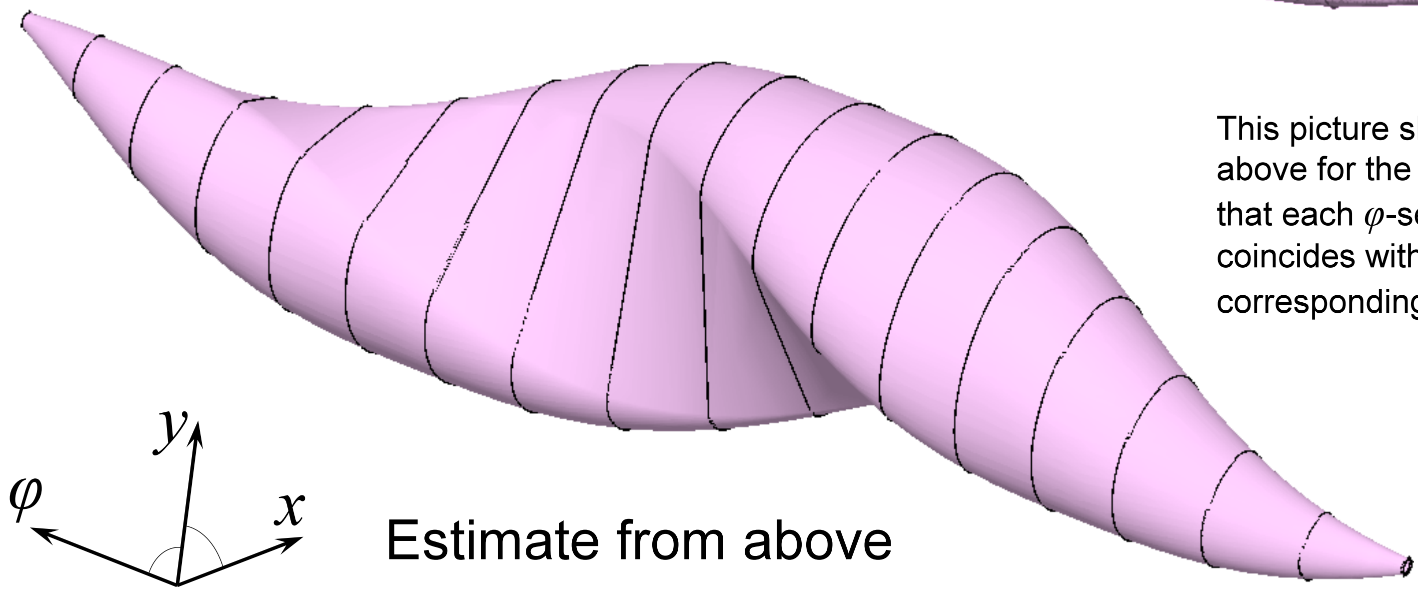
Convexity of φ-sections allows to construct fast procedures for intersection

The forecast sets with convex φ-section is very suitable because the procedure of intersection of such a set with convex cylindrical uncertainty set of measurements is very simple. Recall once more that only geometric coordinates are measured. Uncertainty sets are convex and cylindrical in φ.

Approximation from above for the attainability set
(at the instant $t = 2\pi(V/k)$)



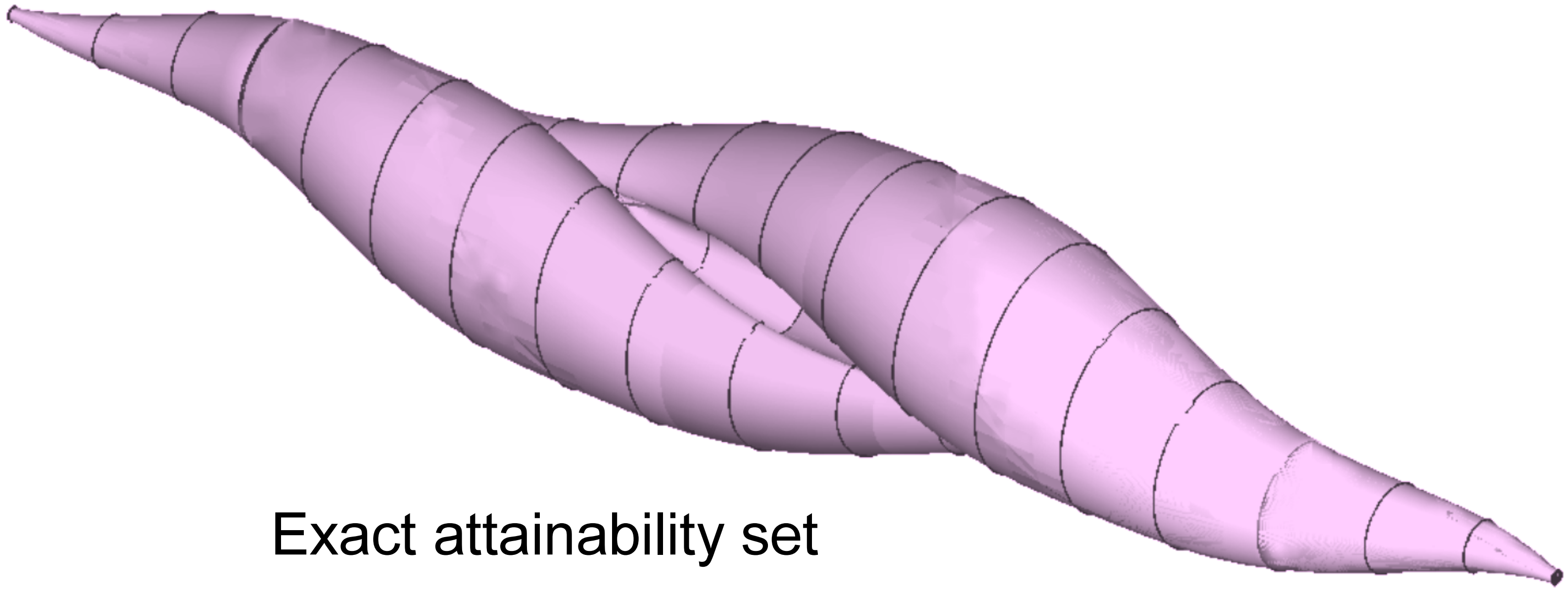
Exact attainability set



Estimate from above

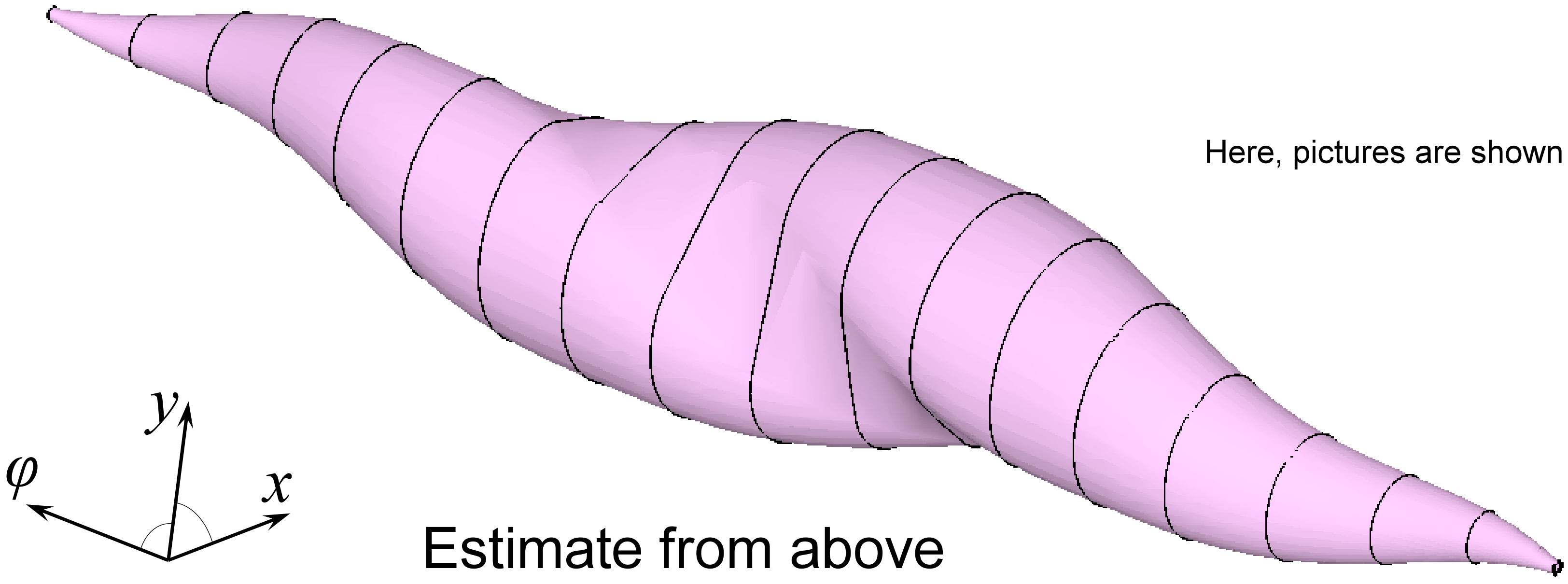
This picture shows the approximation from above for the exact attainability set. We prove that each φ-section of the approximation set coincides with the convex envelope of corresponding φ-section of the true set.

Approximation from above for the attainability set
(at the instant $t = 3\pi(V/k)$)



Exact attainability set

Here, pictures are shown for a larger instant.

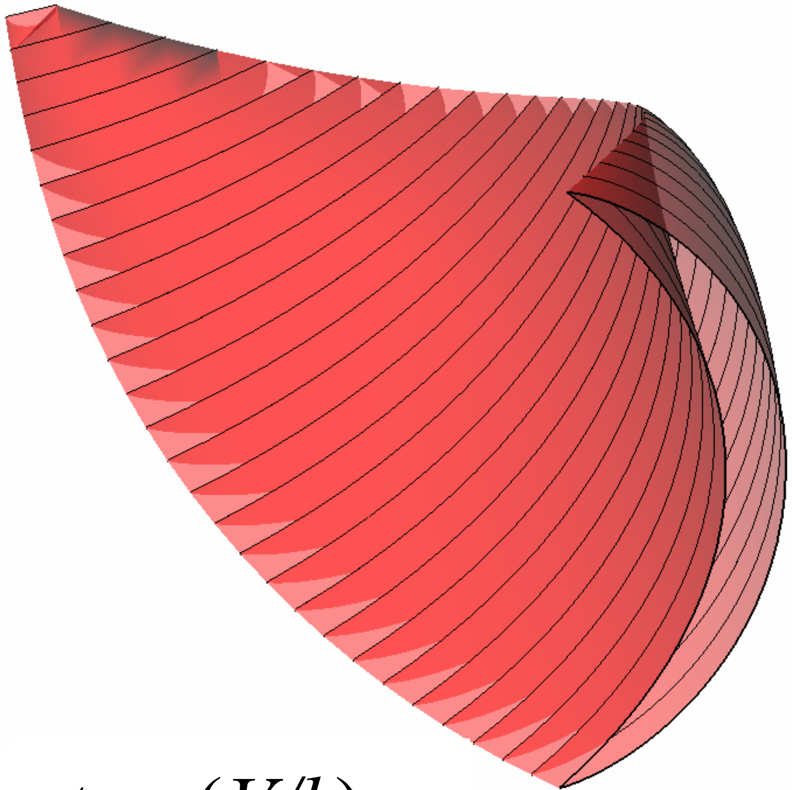


Estimate from above

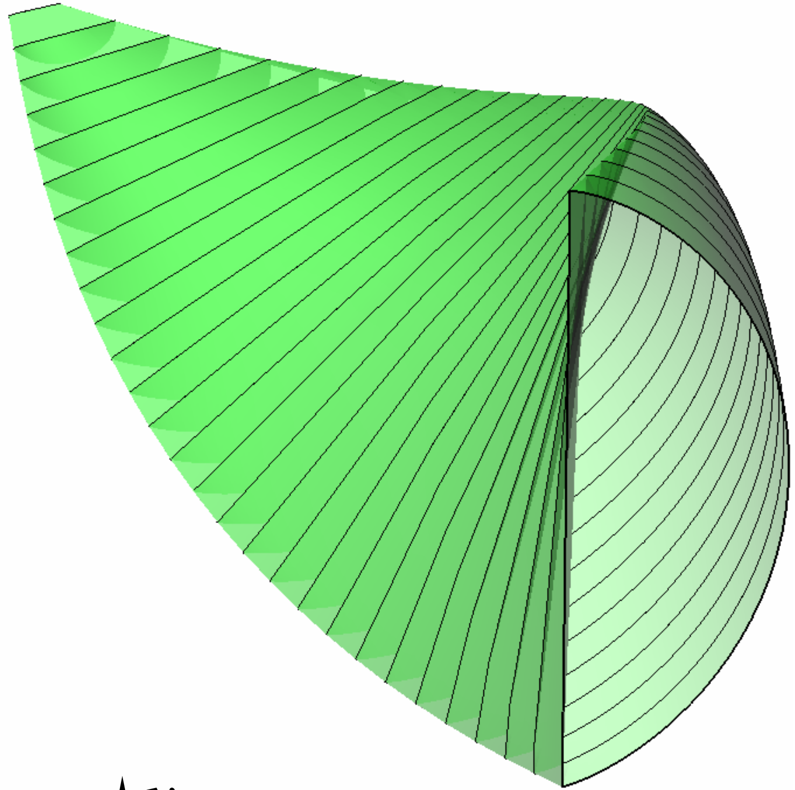
Comparison with exact constructions

Plane of the cross–section corresponds to $\varphi = 0$

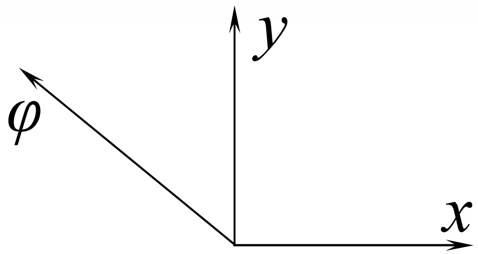
Exact set



Estimate from above

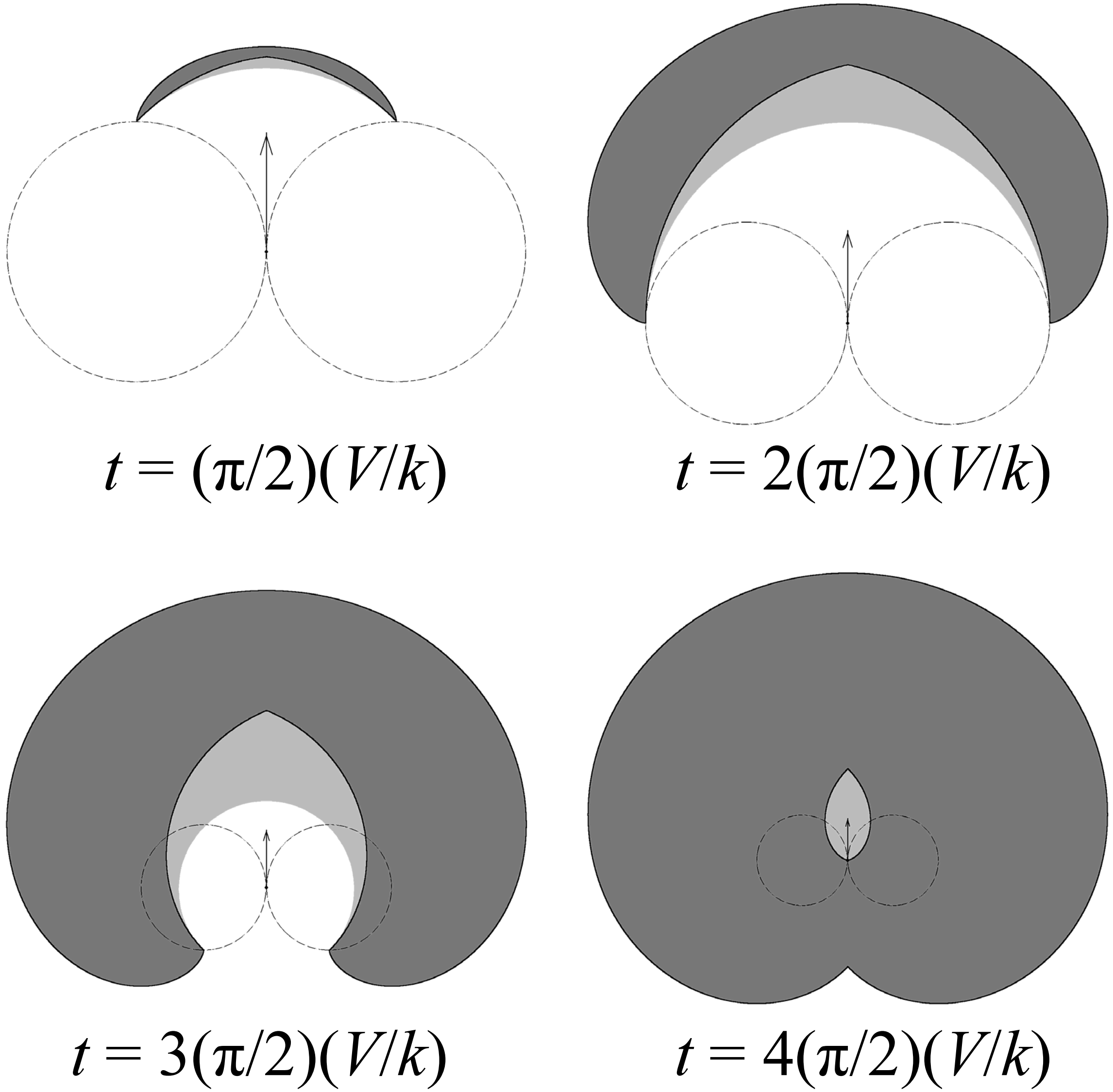


$t = \pi(V/k)$



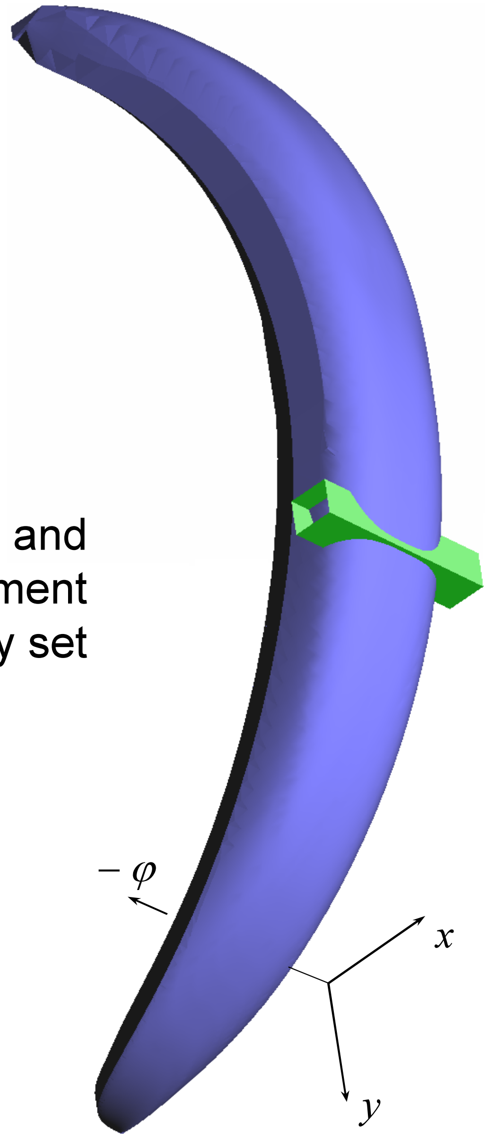
In this picture, we see where the coincidence of boundaries of the exact and approximating sets takes place.

Comparison with exact attainability sets
(projections into the plane *x,y*)

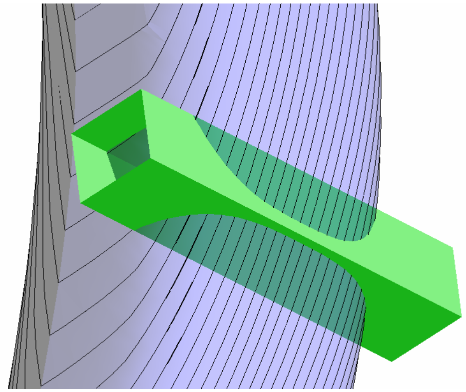


These pictures show the exact attainability set (in dark) and the approximating one (slightly shadowed) as projections into the plane of the geometric coordinates.

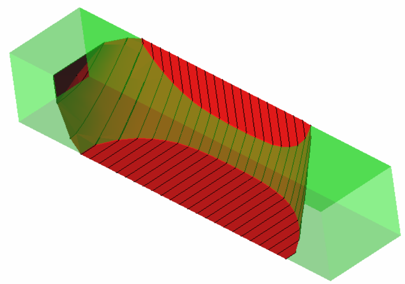
Taking into account a measurement uncertainty set



a) A forecast set and a measurement uncertainty set



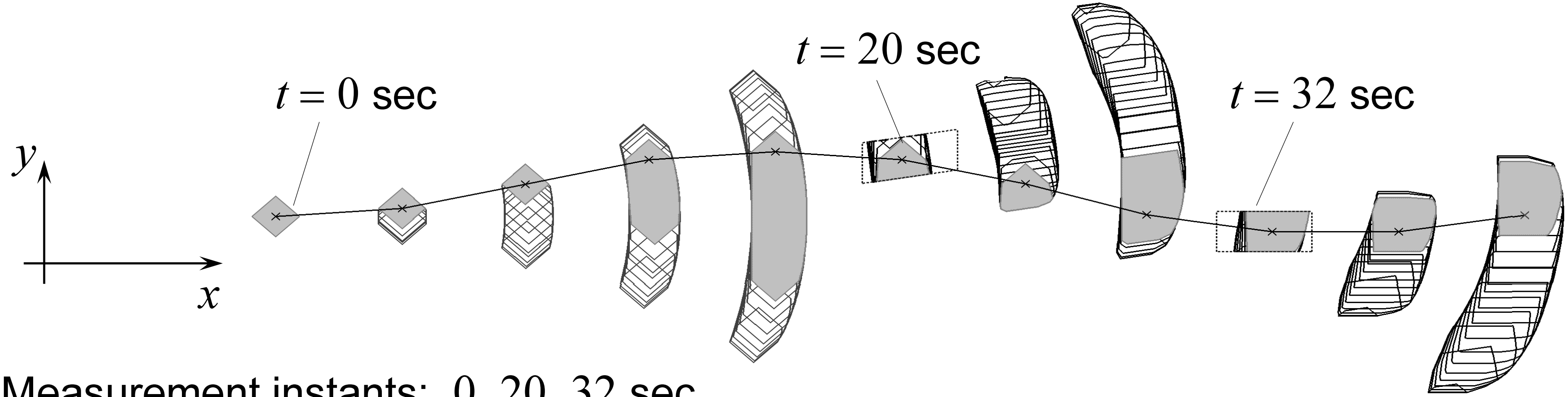
b) The region of intersection of the forecast set and the uncertainty set



c) Result of intersection is an informational set

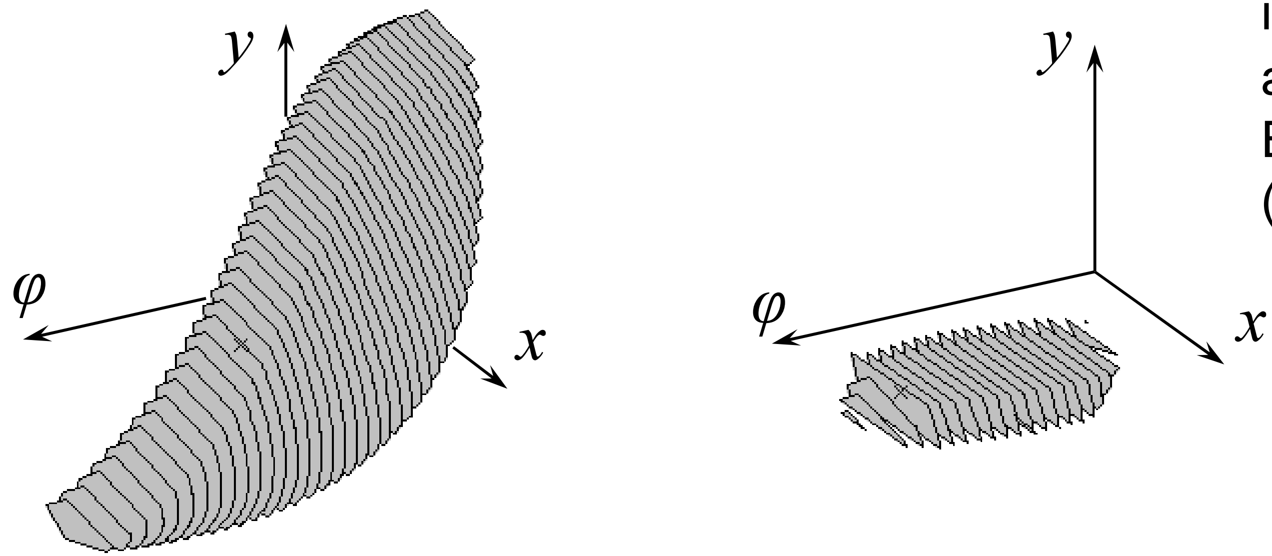
Here, the blue color marks the tree-dimensional forecast set. The green color marks the cylindrical uncertainty set of the measurement. The result of intersection is the current informational set (in red).

Motion of the informational set



Measurement instants: $0, 20, 32$ sec

Structure of the informational set at $t = 20$ sec.
Before the measurement (at left) and after (at right).



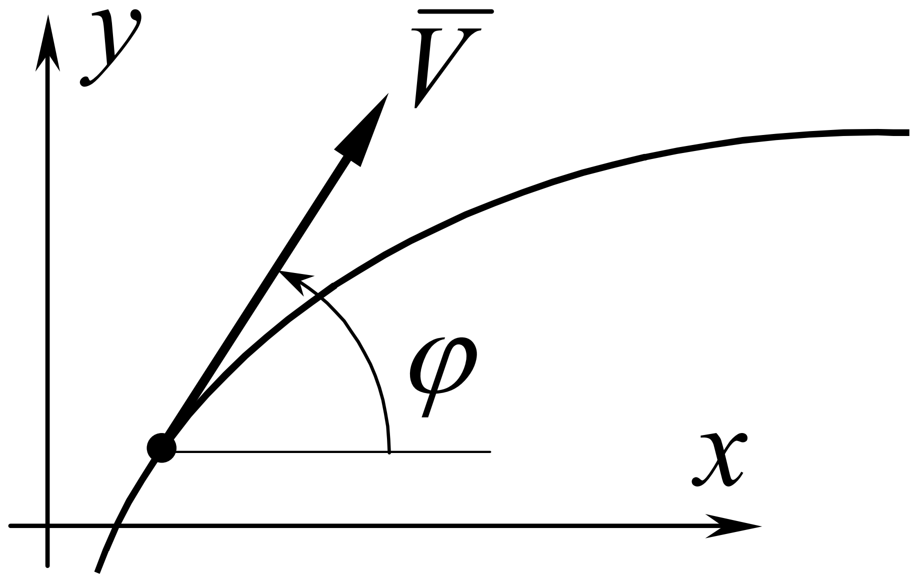
The motion of the informational sets is shown in projections into the plane x, y. The measures come at the instants $t = 0, 20,$ and 32 sec. At $t = 0$ we have the initial set in the form of one layer on φ. Below, the structure of the informational set is drawn before and after the measurement.

Informational sets in a four-dimensional problem

Now let us pass to the four-dimensional problem.

Dynamics of motion

$$\dot{x} = V\cos\varphi,$$
$$\dot{y} = V\sin\varphi,$$
$$\dot{\varphi} = ku/V,$$
$$\dot{V} = w;$$

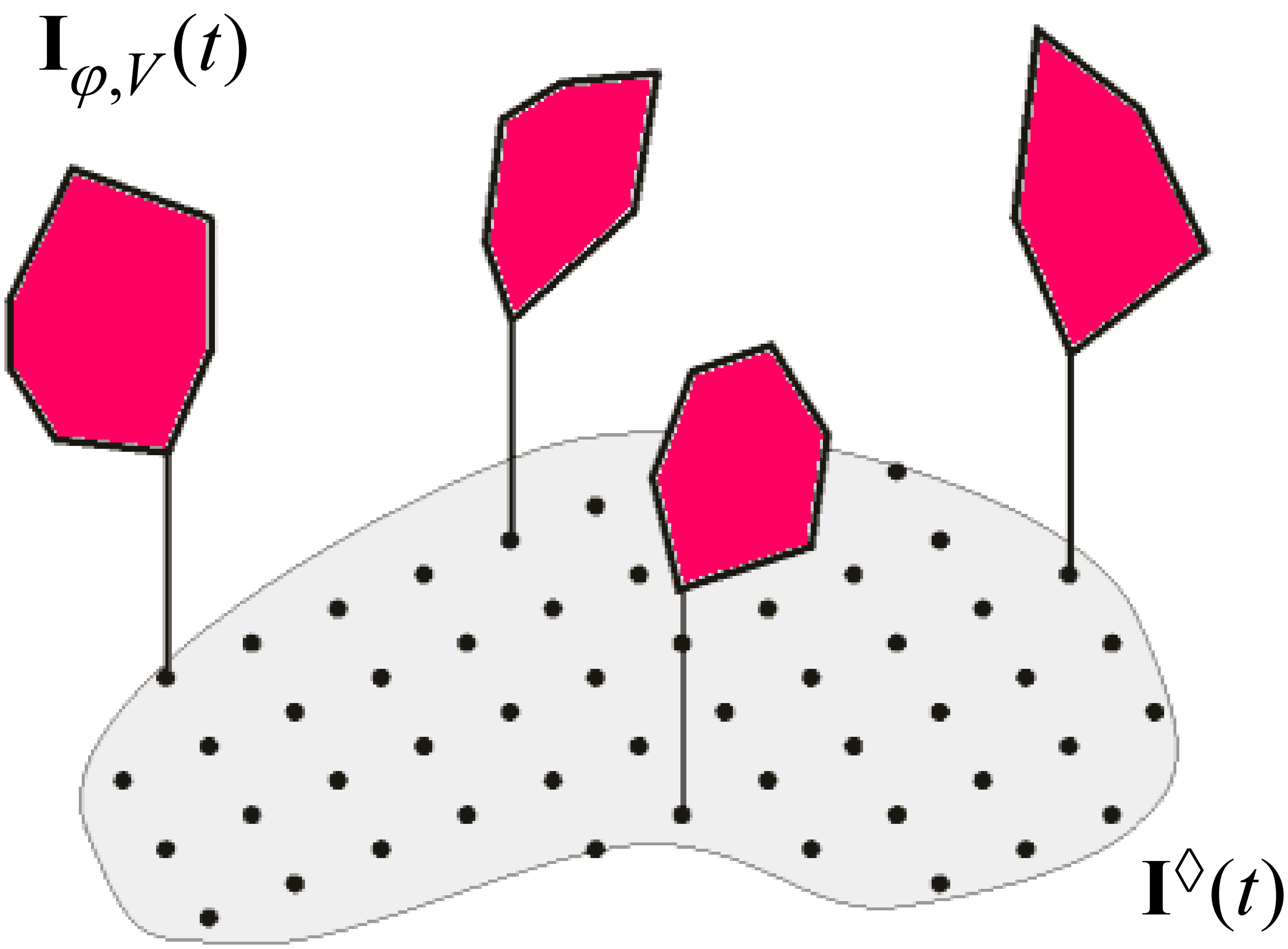


$$|u| \le 1, \quad \mu_1 \le w \le \mu_2, \quad \mu_1 < 0, \quad \mu_2 > 0,$$

$$k = \text{const} > 0, \quad V \ge \text{const} > 0$$

We add the fourth equation to dynamics. The controls u and w are constrained geometrically.

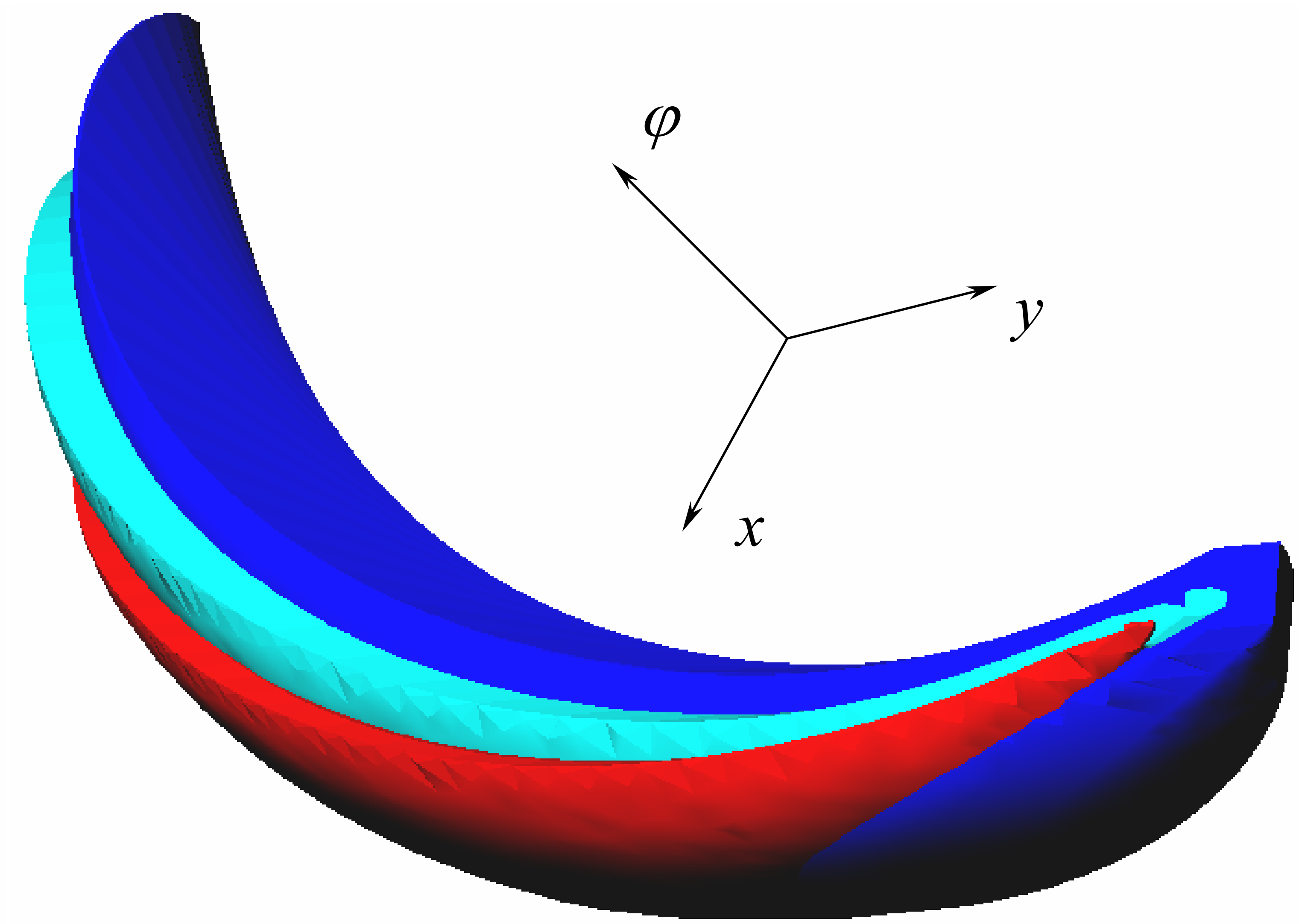
Grid and polygons



Numerical presentation of informational sets

When constructing the forecast and informational sets, we apply a grid in the coordinates φ and V. A convex polygon corresponds to each node of this grid.

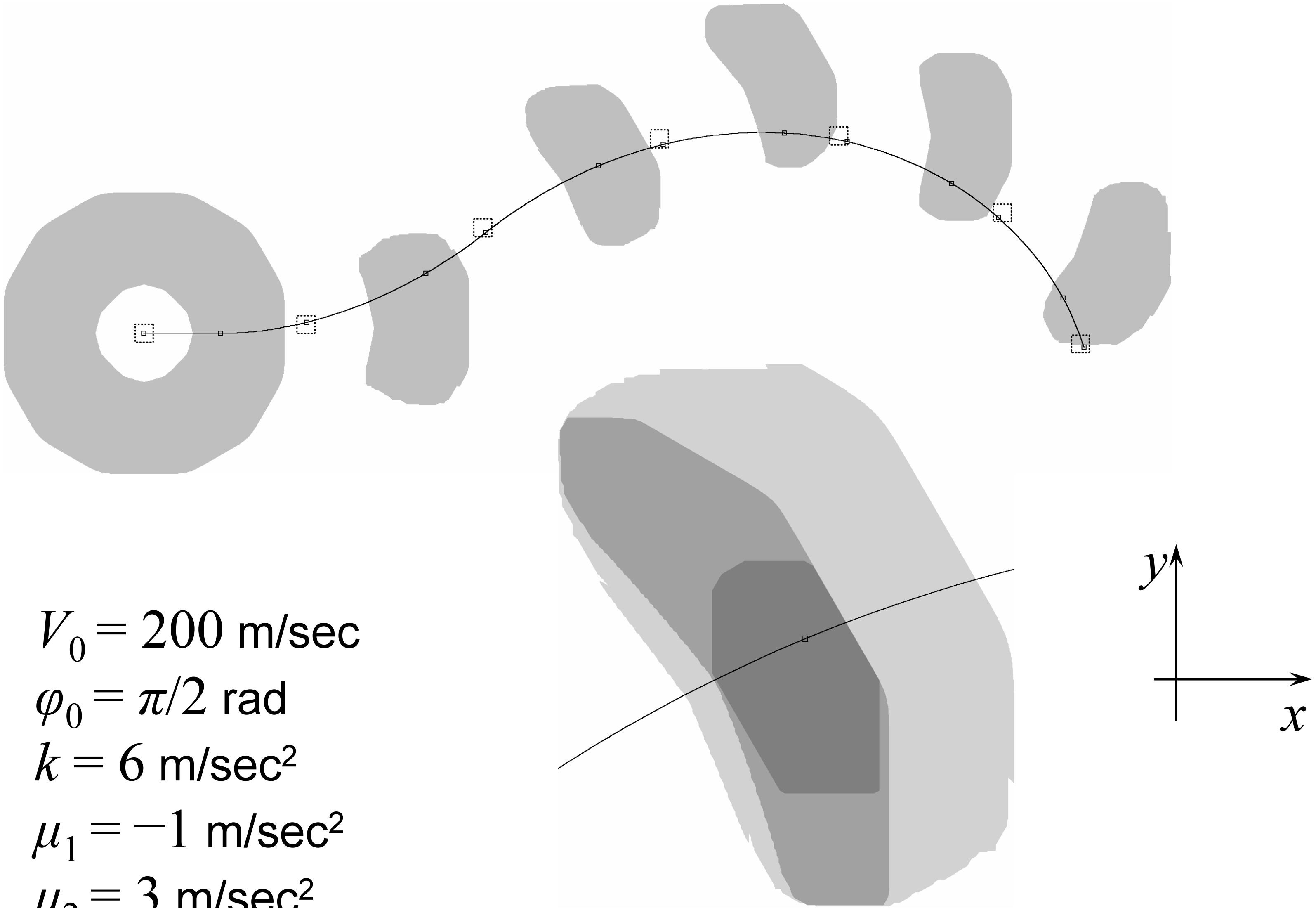
Informational set under varying velocity



Three layers in *V* are shown

A three-dimensional set in the space x, y, and φ corresponds to each node in V. Here, three such sets are presented for three nodes in V.

Motion and structure of an informational set



$V_0 = 200$ m/sec
$\varphi_0 = \pi/2$ rad
$k = 6$ m/sec²
$\mu_1 = -1$ m/sec²
$\mu_2 = 3$ m/sec²

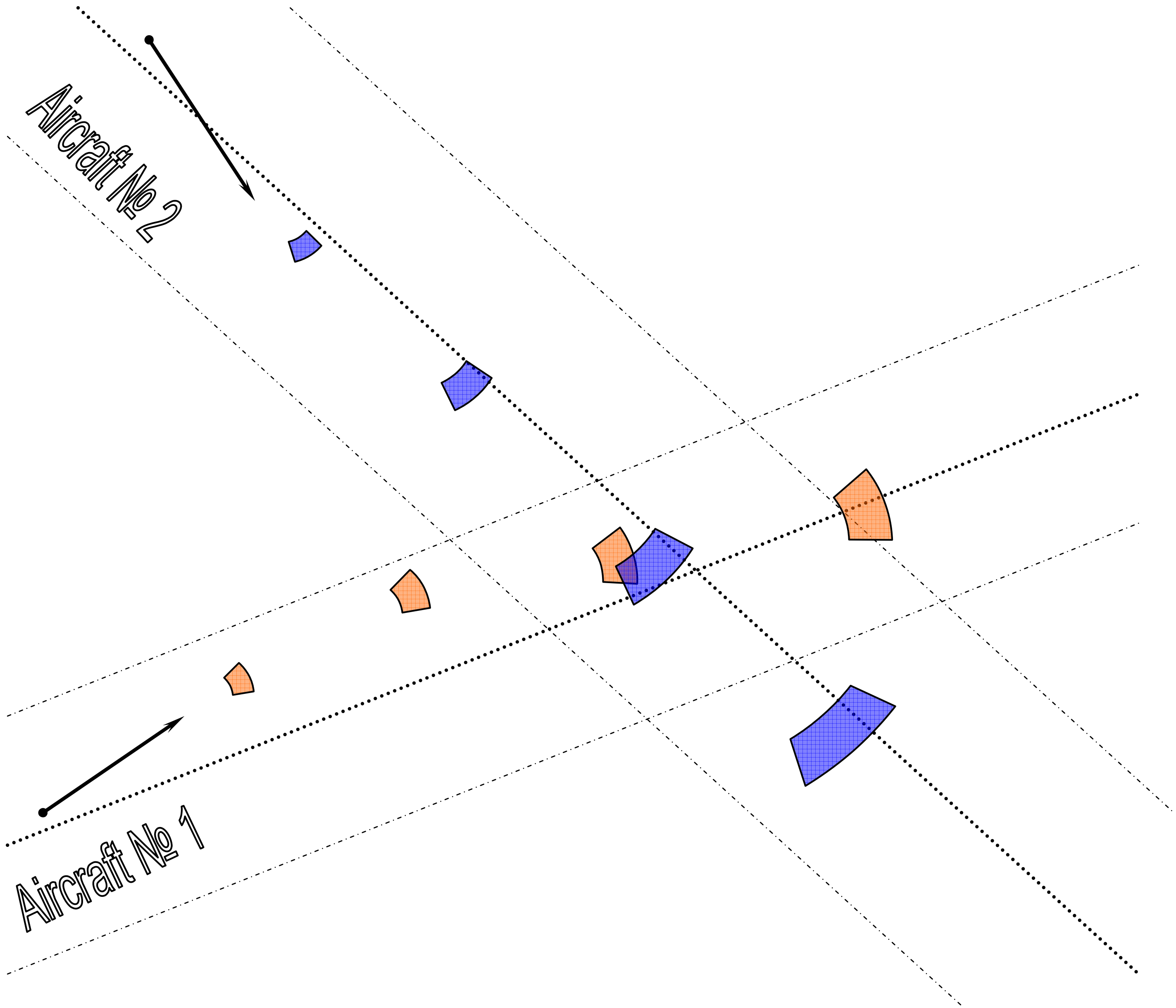
This picture shows a motion of the informational sets in projections into the plane x, y. The uncertainty sets of the measurements are small squares. They are also shown in projections into the plane x, y.

Below, the whole projection of the informational set is marked in light grey; a projection of one layer in V is in middle grey; a polygon corresponding to one node of the φ, V grid is given in dark grey.

Informational sets in a problem of observation of an aircraft moving under an autopilot

Now we consider the third problem, in which an aircraft moves in the horizontal plane under an autopilot. The autopilot's control is known, but there is a disturbance in the dynamics.

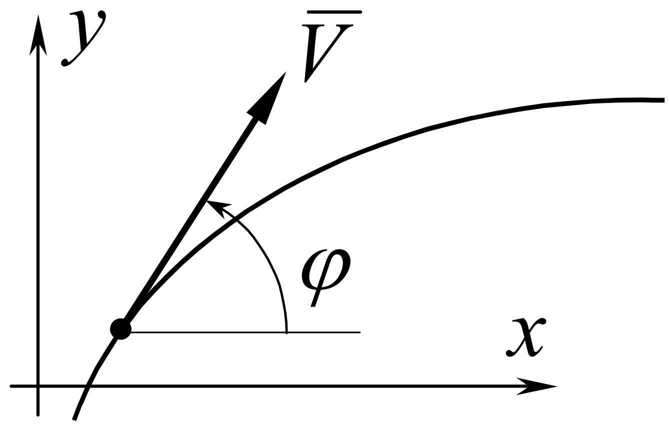
Forecast of aircraft motion along a trace and detecting conflict situation



These investigations can be useful for detecting possible collisions of aircrafts.

Description of dynamics

$$\dot{x} = V\cos\varphi,$$
$$\dot{y} = V\sin\varphi,$$
$$\dot{\varphi} = u(y,\varphi) + v;$$



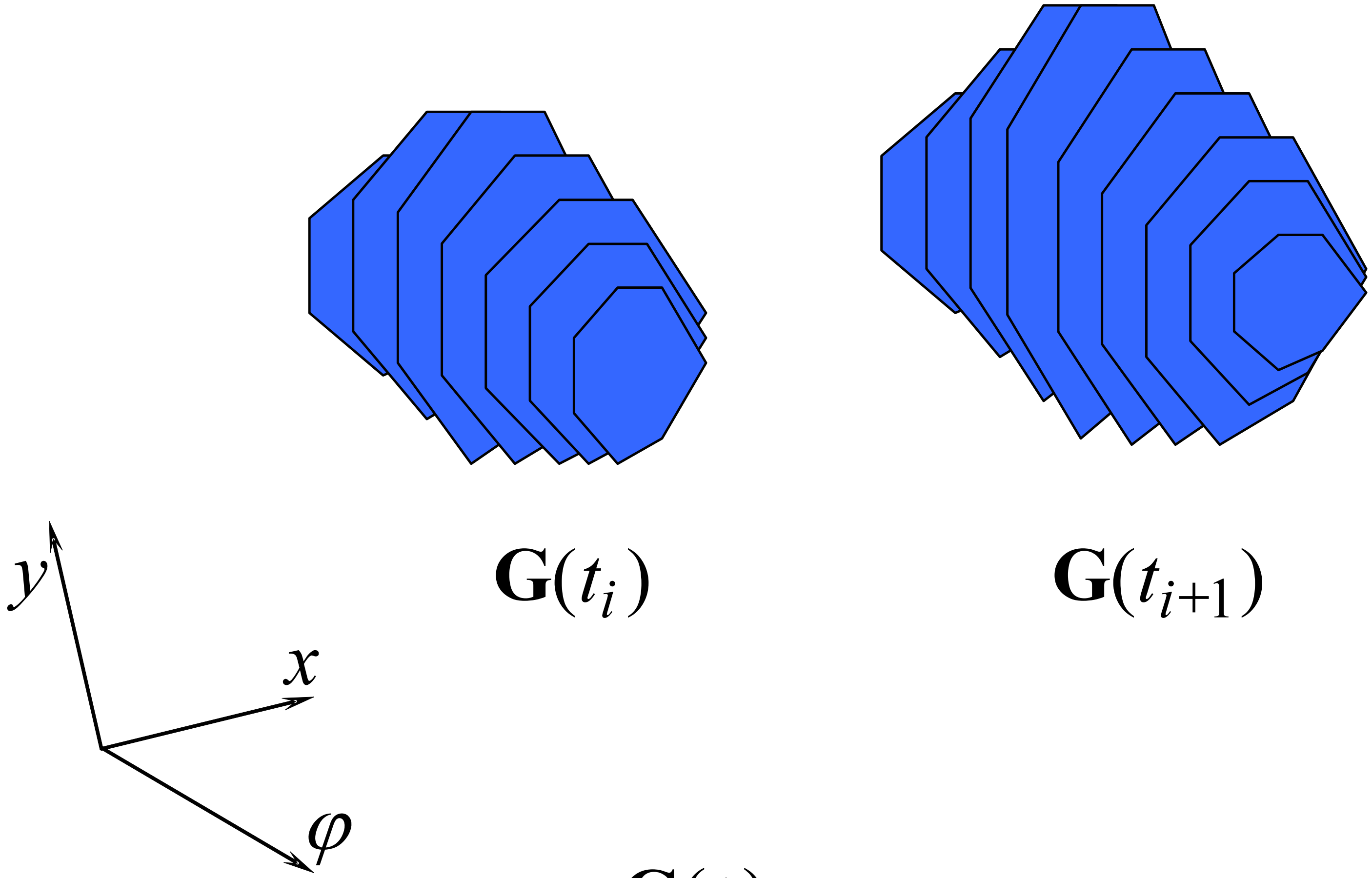
$$u(y,\varphi)=\begin{cases} k_1y+k_2\varphi & \text{if} \quad |k_1y+k_2\varphi|\le M \\ +M & \text{if} \quad k_1y+k_2\varphi > M \\ -M & \text{if} \quad k_1y+k_2\varphi < -M \end{cases}$$

$$V = \text{const} > 0, \quad k_1, k_2 = \text{const} < 0, \quad |v| \le W$$

$u(y,\varphi)$ – autopilot regulator, $\quad v$ – disturbance

The dynamics is three-dimensional. Feedback autopilot control is fixed. There is a disturbance with geometric constraint.

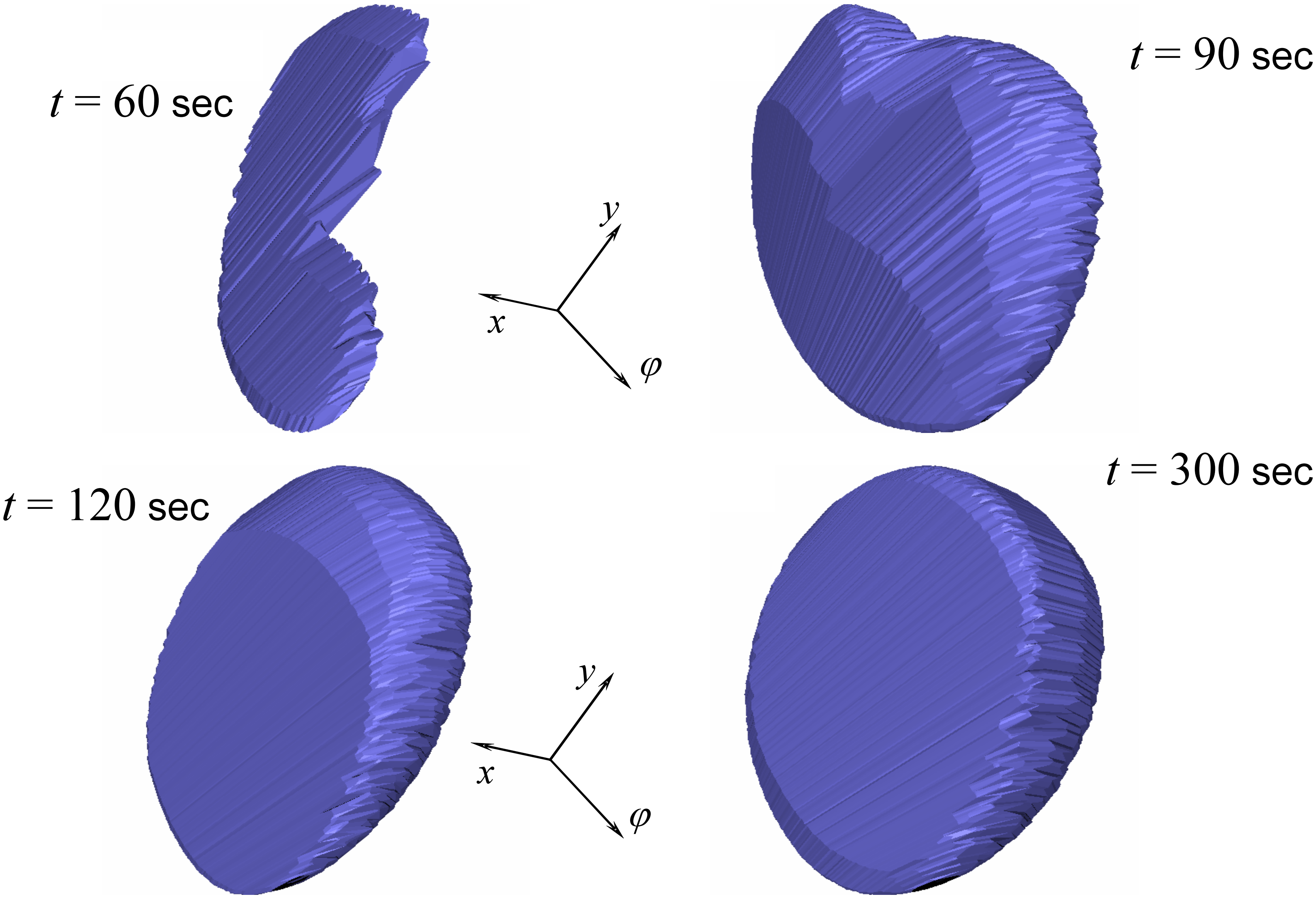
Representation of forecast sets



$\mathbf{G}(t_i)$ — estimate from above of the true attainability set (the forecast set)

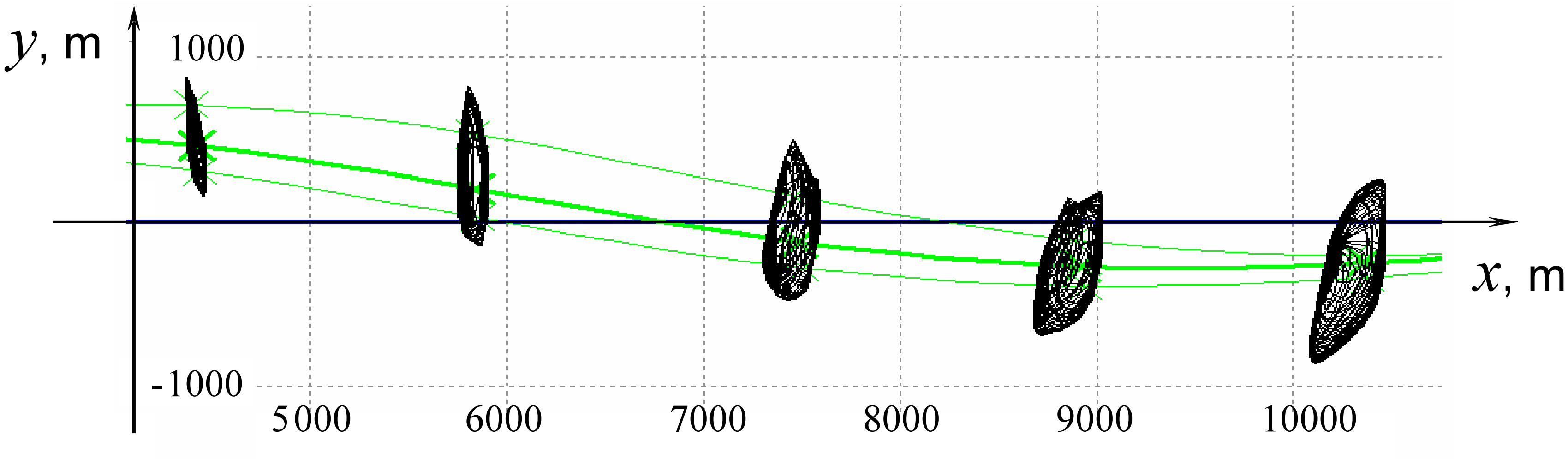
We introduce the φ-grid again and represent the forecast and informational sets as a collections of convex polygons. Each of them corresponds to certain node of the φ-grid.

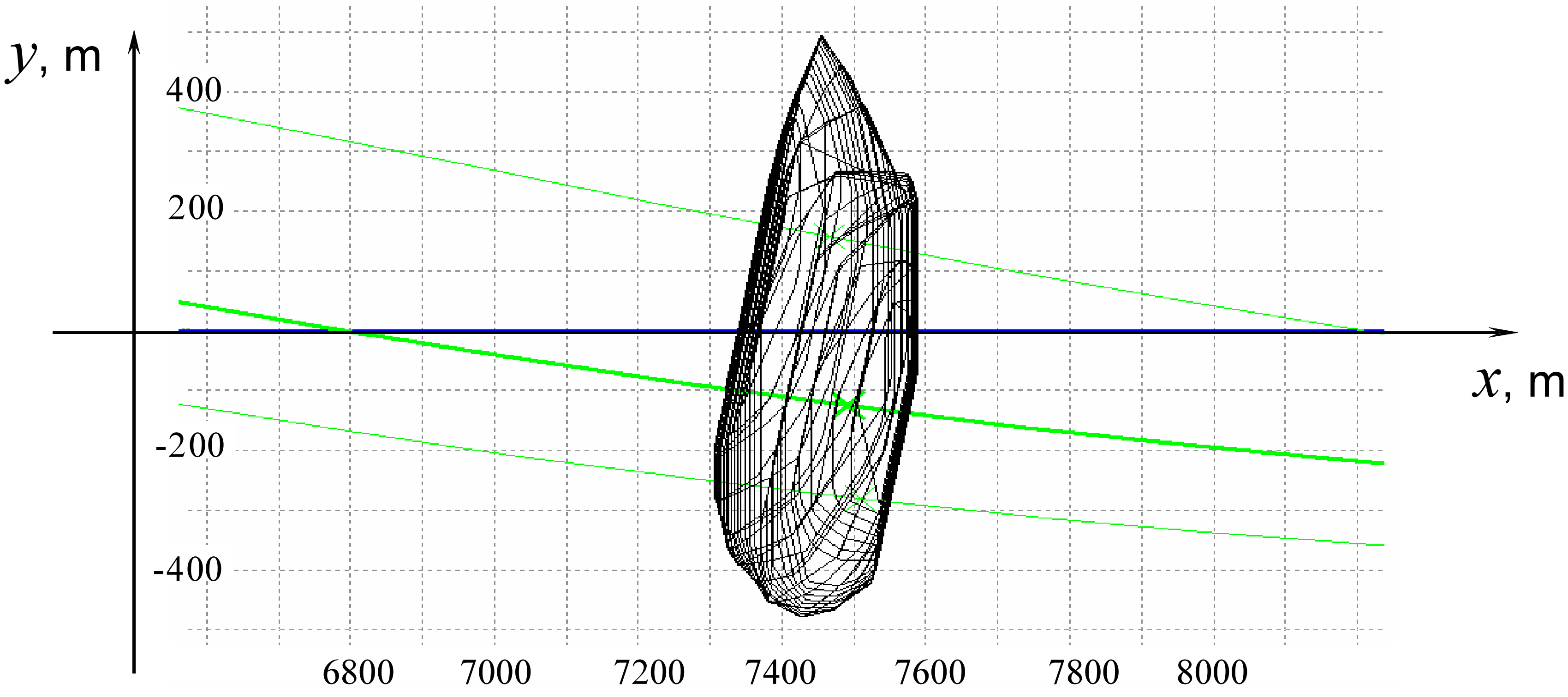
Simulation of the forecast sets



Here, the three-dimensional forecast sets are presented for four instants.

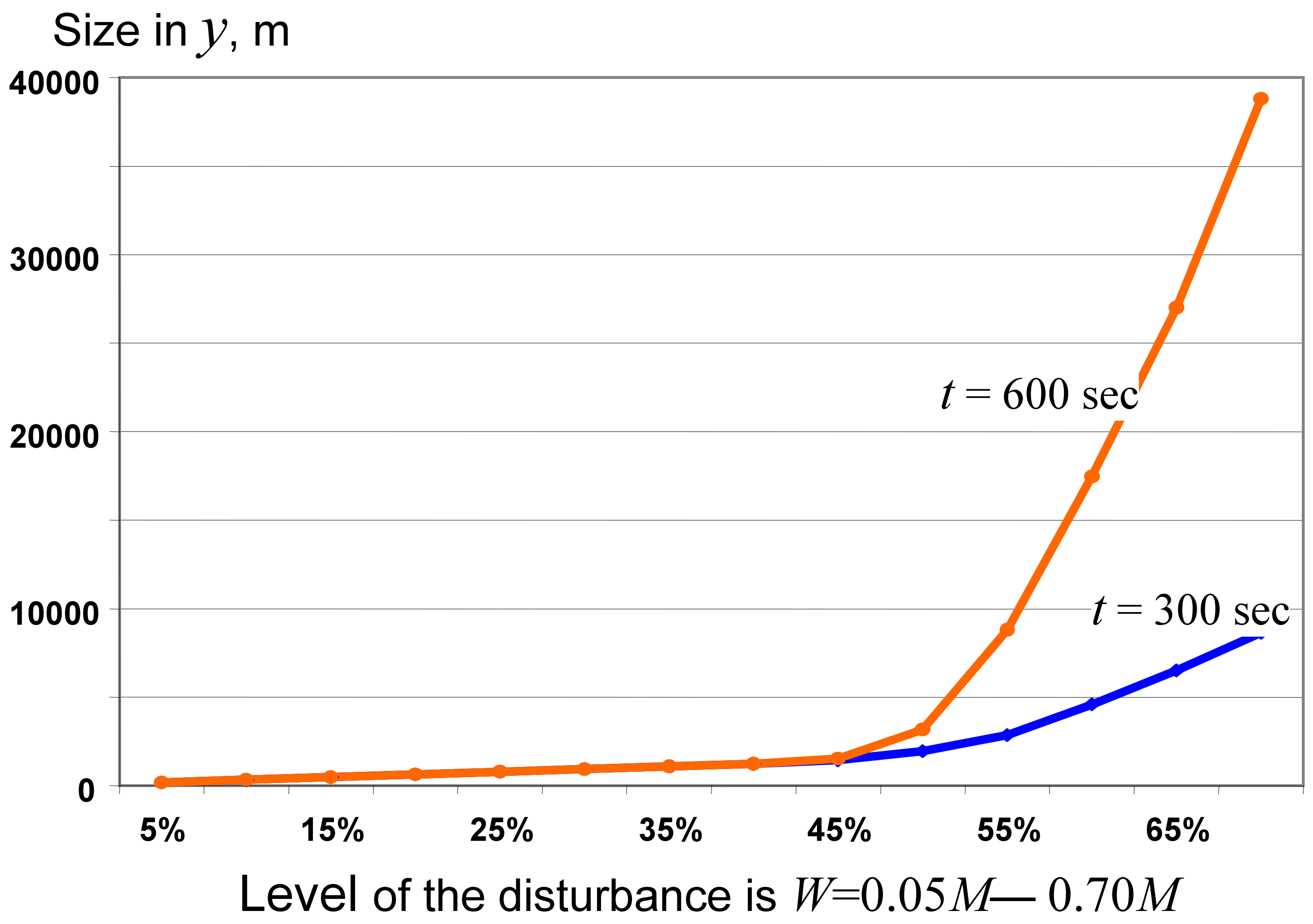
Motion of the forecast sets
(projection into the plane x, y)





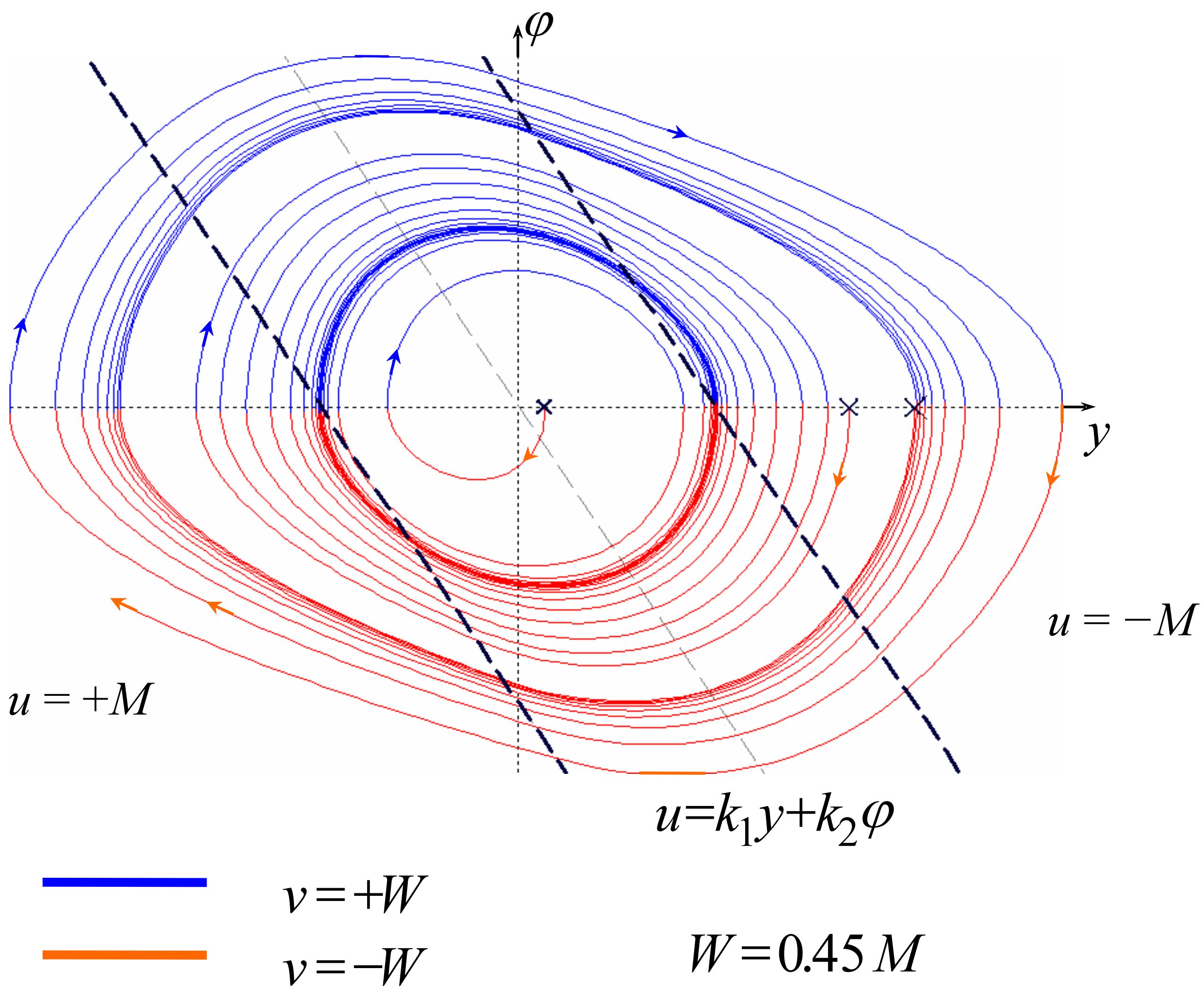
The upper picture illustrates a motion of the forecast sets. The picture below shows the structure of one zoomed set.

Influence of disturbance level on the size of the forecast set



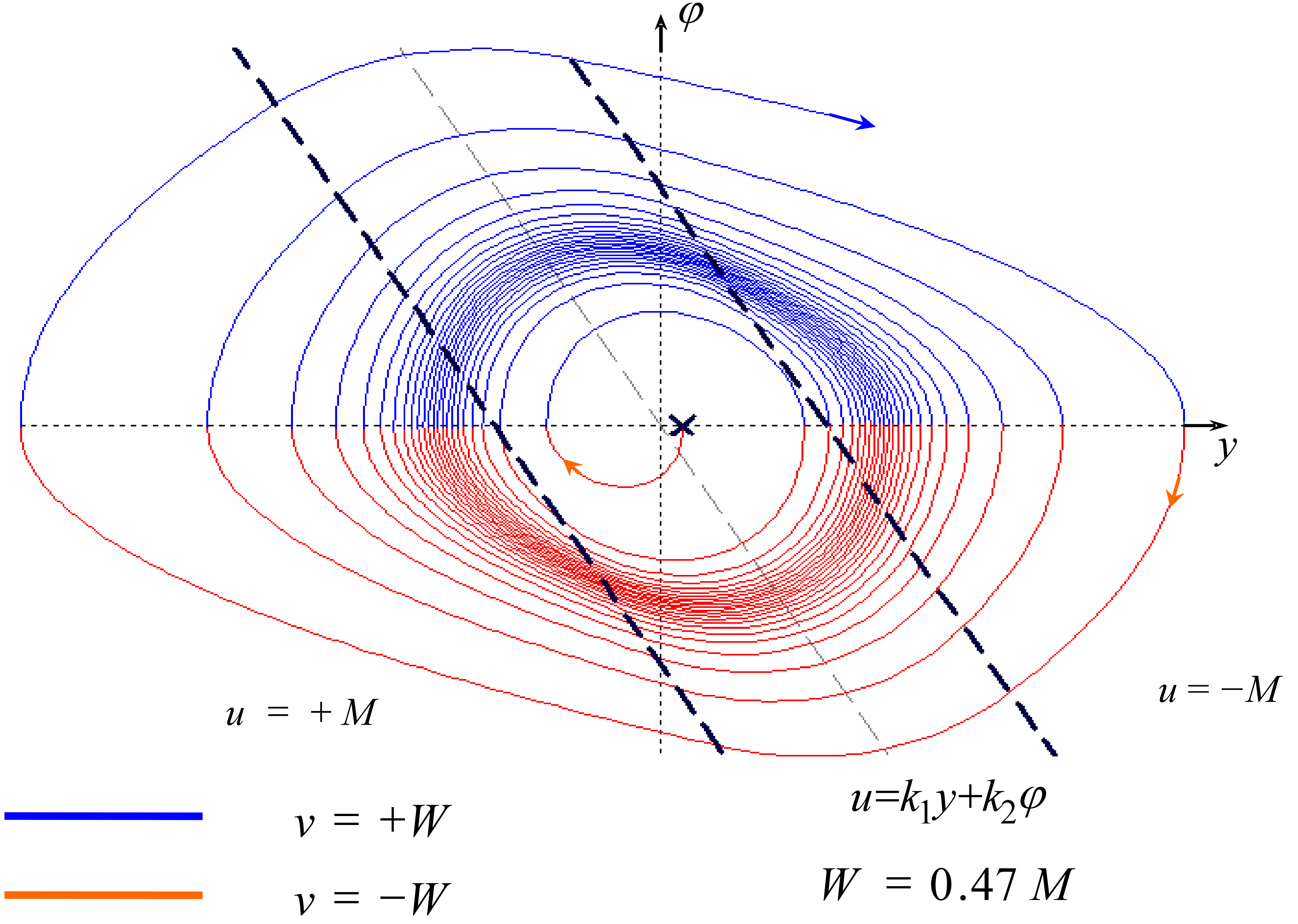
The following fact has been detected. If a level W of the disturbance does not exceed 45% of the level M of the autopilot control, then the size of the forecast set stabilizes in coordinates y and φ. It is illustrated by the curves. One curve corresponds to the instant $t = 300$ sec and the other curve corresponds to $t = 600$ sec. If $W > 0.45\ M$, then the size of forecast sets grows quickly in time.

Stable and unstable limit cycles



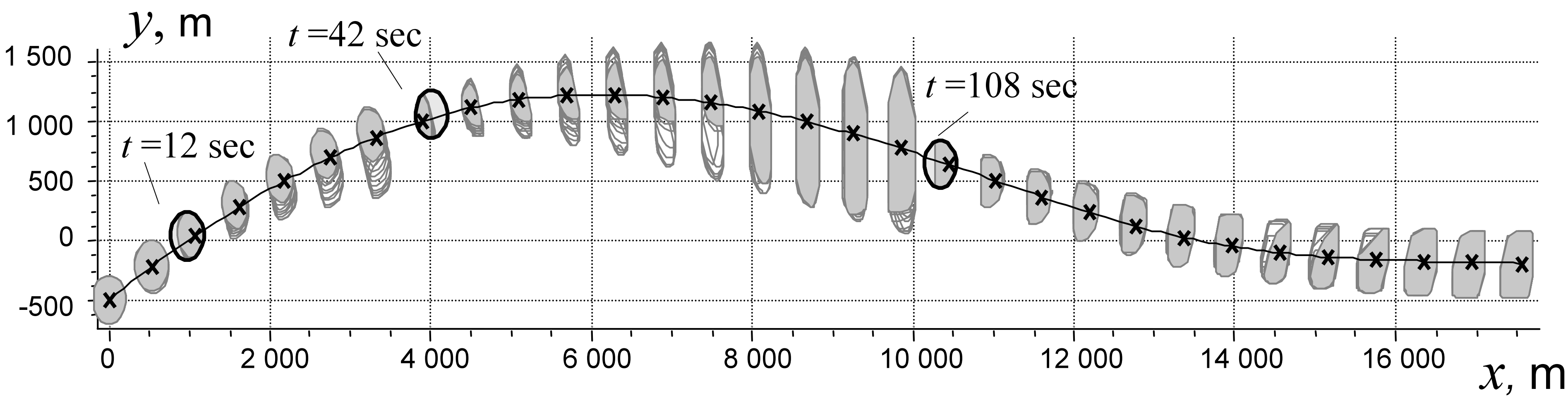
In this picture, the trajectories in the plane y, φ are presented. The disturbance feedback control takes the marginal value $-W$ in the region below the y-axis, and $+W$ above it. We think that this disturbance law is near to the worst one.
When $W \leq 0.45M$, there are two limit cycles: internal one is stable and another one is unstable. The first limit cycle restricts the size of forecast set in coordinate φ, V.

Loss of stability under large level of the disturbance

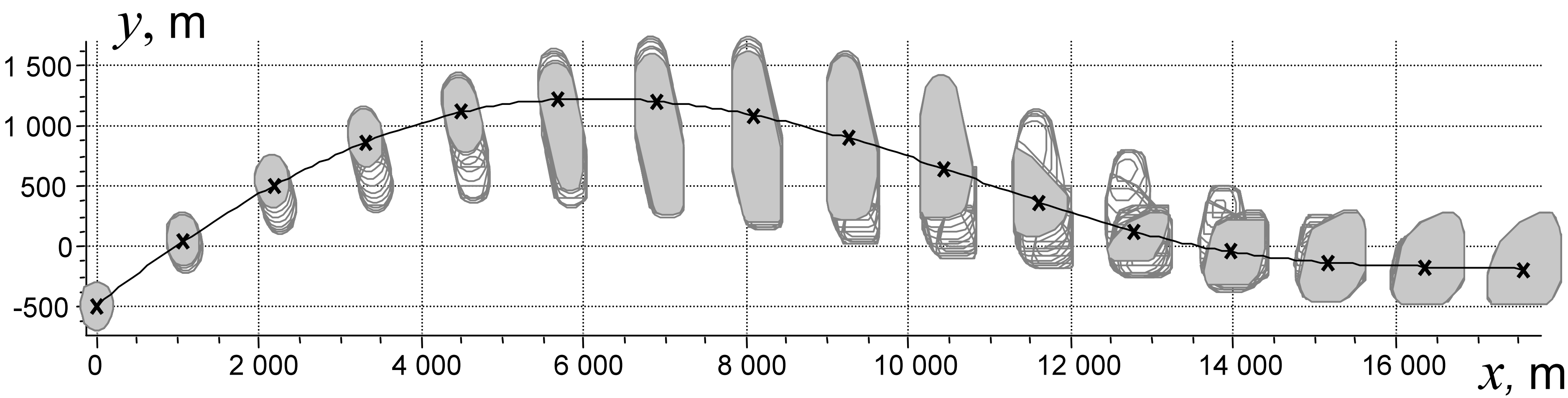


If the level of the disturbance increases, then for $W \approx 0.45M$ the two limit cycles coincide. Under further increasing the disturbance level, there is no any limit cycle and we see an unlimited growth of size of the attainability set in coordinates y, φ.

Motion of the informational sets



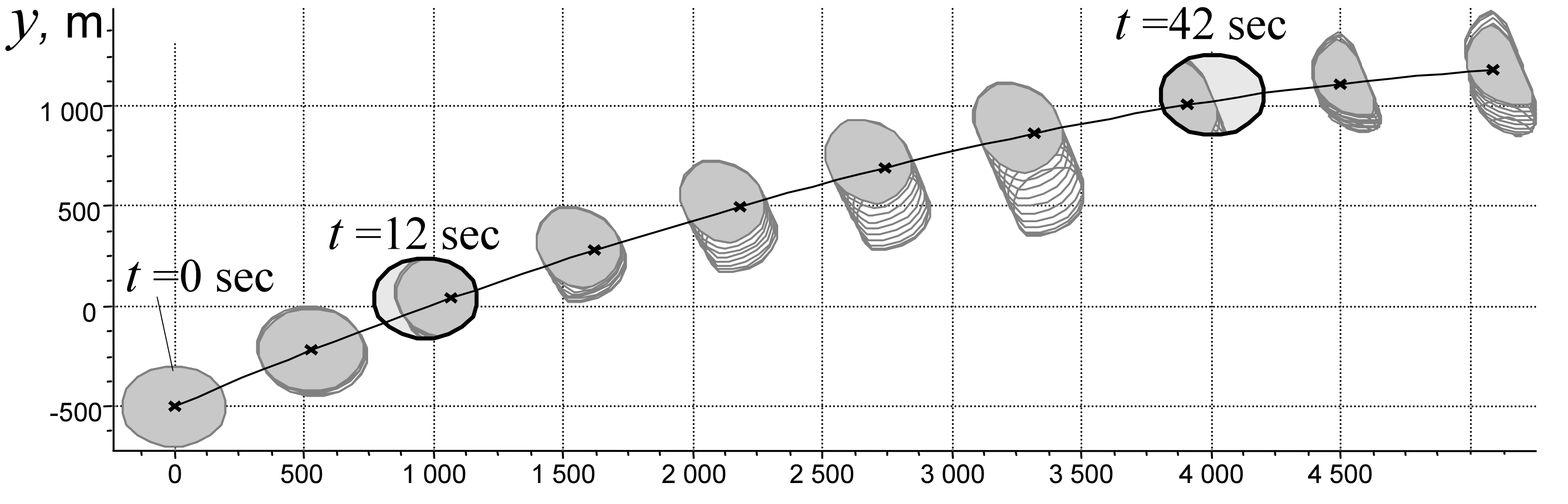
Measurements at the instants $t = 12, 42$, and 108 sec



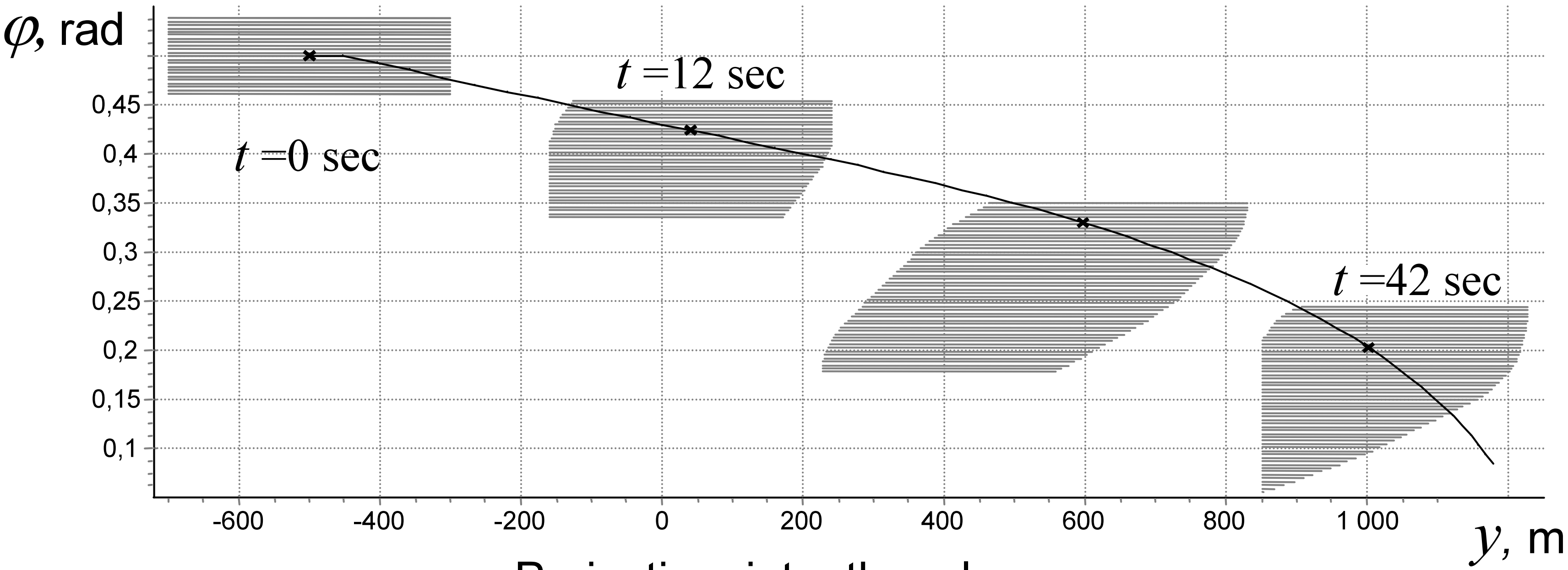
Variant without measurements

Here, two variants of informational set motions are presented. The first one takes into account the measurements. The measurements come at $t = 12, 42$, and 108 sec. The second picture corresponds to a case without measurements. Here, we see the forecast sets only. For both cases, the informational sets are given in projections into the plane x, y.

Motion of the informational sets



Projection into the plane x, y



Projection into the plane y, φ

Here, a zoomed fragment is shown. The zoomed picture below illustrates a motion of the informational set in the plane y, φ.

Authors' publications

1. *V.S.Patsko, S.G.Pyatko, and A.A.Fedotov.* **Three-dimensional reachability set for a nonlinear control system.** In: Journal of Computer and Systems Sciences International, Vol.42, No.3, 2003, pp.320–328. Translated from: Izvestiya Akademii Nauk. Teoriya i Sistemy Upravleniya, No.3, 2003, pp.8–16.

2. *S.I.Kumkov, V.S.Patsko, S.G.Pyatko, V.M.Reshetov, and A.A.Fedotov.* **Informational sets in the problem of observation of aircraft motion in a horizontal plane.** In: Journal of Computer and Systems Sciences International, Vol.42, No.4, 2003, pp.544–554. Translated from: Izvestiya Akademii Nauk. Teoriya i Sistemy Upravleniya, No.4, 2003, pp.51–61.

3. *S.I.Kumkov, V.S.Patsko, S.G.Pyatko, and A.A.Fedotov.* **Informational sets in a problem of observation of aircraft trajectory.** In: Proceedings of the Steklov Institute of Mathematics, Suppl.2, 2000, Moscow: MAIK "Nauka/Interperiodica", Russia, pp.S94–S112.

4. *S.I.Kumkov, V.S.Patsko, S.G.Pyatko, and A.A.Fedotov.* **Four-dimensional informational sets in a problem of aircraft tracking.** In: Proceedings of the 11th IFAC Workshop "Control Applications of Optimization" (CAO 2000), July 3–6, 2000, St.Petersburg, Russia, Vol.2, V.Zakharov (Ed.), Pergamon Press, Oxford, 2000, pp.573–578.

5. *S.A.Ganebny, A.A.Fedotov.* **Construction and analysis of forecast sets of aircraft motion in the horizontal plane under autopilot.** In: Proceedings of the XXIV Russian School on problems of science and technologies, Vol.2, Russian Academy of Sciences, Moscow, 2004, pp.236–246. (in Russian)

Here, some publications are indicated where the results of the talk are partially presented.

Научное издание

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Информационные множества в модельных задачах наблюдения за движением самолета

Альбом иллюстраций к докладу на секции "Control and Estimation under Set-Membership Uncertainty" 16-го международного конгресса IFAC, Прага, июль 2005

Ответственный за выпуск А.Г.Иванов

620219, Екатеринбург, ГСП-384, ул.С.Ковалевской, 16,
Институт математики и механики УрО РАН

Тираж 50 экз. Заказ №52.

Переплет изготовлен в типографии "Уральский центр академического обслуживания"
620219, г.Екатеринбург, ул.С.Ковалевской, 18