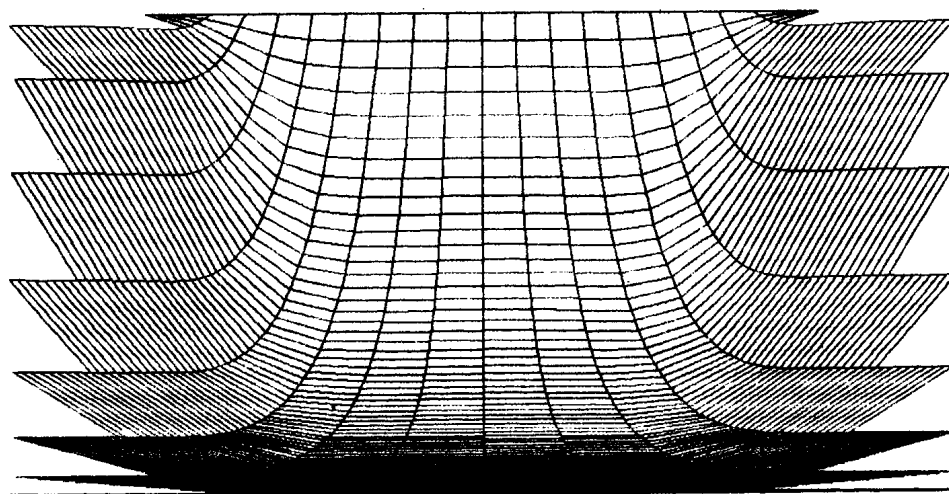


# **TAKE-OFF CONTROL IN A WINDSHEAR**

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Ekaterinburg, 1991**

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## **Abstract**

The problem of aircraft take-off control in the presence of wind disturbances is under investigation. It is assumed that the wind disturbances are caused by a microburst arising due to downward atmospheric flows. A feedback control based on differential game theory methods is designed. The efficiency of the control is compared with results obtained using robust control techniques.

## 1. Introduction

Many aircraft accidents are caused by windshears occurring due to such a meteorological phenomenon as microburst. The microburst appears when a descending air flow hits the earth surface. It is especially dangerous for aircrafts passing the microburst zone during the landing or take-off, since quick changes of the wind velocity take place at relatively low altitudes. In the last years, a lot of papers [1–19] related to the problems of aircraft landing, abort landing, and take-off under various disturbances have been appeared.

In [1–7], aircraft motion in vertical plane is considered. Take-off, landing, and abort landing problems are investigated. Thereby, the aircraft dynamics are described by equations of one and the same type for all these problems.

In papers [1–3,7], the wind velocity field is assumed to be known. In this case, open-loop controls obtained by solving the corresponding optimization problems provide landing, take-off, and abort landing trajectories of satisfactory quality for relatively severe wind disturbances.

As a rule, the wind velocity field is not given in practical problems. Therefore, the computation of feedback controls is quite actual. Feedback controls that use local information about windshear are constructed in [4] on the basis of optimal solutions obtained in [2]. In [5], a feedback control strategy found with the help of robust control theory methods is investigated in connection with the take-off problem. The construction of special Lyapunov functions is behind of these methods. The design of the Lyapunov functions and their utilization require fitting of numerous parameters. The authors of [5] choose parameter values very carefully and arrive at trajectories similar to the ones obtained in [2] by solving open-loop control problems. The design of feedback strategy in [6], where the problem of stabilization of the aircraft climb rate about some nominal value is formulated, is also based on robust control theory.

Diverse variants of the approach based on differential game theory [20] are considered in [8–19] in connection with the problem of aircraft landing. Thereby, the works [12–19] consider a complete aircraft dynamics model which includes equations of lateral and longitudinal (vertical) motion. It should be noted that the game approach does not require any a priori information about the disturbance except for the bounds on the deviations of the wind velocity components from their nominal values.

The present paper is an attempt to obtain acceptable take-off trajectories with the help of differential game theory. The model of aircraft motion employed here is the same as in [1–7]. The use of this model is motivated by the intention to test the efficiency of the game approach and to compare the obtained results with the results of [5].

The method of the construction of feedback game controls is the following. The original non-linear equations are linearized about some nominal trajectory. An auxiliary linear differential game with fixed terminal time and a convex payoff function is formulated. For this differential game, the optimal guaranteeing control implemented by means of a switch surface (see [10,21,22]) is designed using effective computer programs described in [23]. The designed control is then applied to the original non-linear system. Below, the adjective “minmax” will be used to refer to such a kind of control.

When simulating trajectories of the non-linear system, one can use different variants of wind disturbances. In this paper, a microburst model from [5] is employed.

## 2. Non-linear model of aircraft motion

Let us give a short description of the non-linear model borrowed from papers [2,5]. The basic assumptions are the following. The aircraft is considered as a point mass object that moves in vertical plane. The thrust is governed through a prescribed law. The wind flow field is steady-state.

The disturbances are involved into the dynamics through the vertical and horizontal components of the wind velocity vector. The aircraft is controlled via the change of the attack angle.

### 2.1. Basic notation

$V$	is the relative velocity, ft/sec;
$\gamma$	the relative path inclination, rad;
$x$	the horizontal distance, ft;
$h$	the altitude, ft;
$W_x$	the $x$ -component of the wind velocity, ft/sec;
$W_h$	the $h$ -component of the wind velocity, ft/sec;
$\alpha$	the attack angle, rad;
$\delta$	the thrust inclination, rad;
$g$	the acceleration of gravity, ft/sec <sup>2</sup> ;
$m$	the aircraft mass, lb sec <sup>2</sup> /ft;
$T$	the thrust force, lb;
$D$	the drag force, lb;
$L$	the lift force, lb;
$\rho$	the air density, lb sec <sup>2</sup> /ft <sup>4</sup> ;
$S$	the reference surface area, ft <sup>2</sup> .

The non-linear model includes two dynamic equations

$$\begin{aligned} m\dot{V} &= T \cos(\alpha + \delta) - D - mg \sin \gamma - m\dot{W}_x \cos \gamma - m\dot{W}_h \sin \gamma \\ mV\dot{\gamma} &= T \sin(\alpha + \delta) + L - mg \cos \gamma + m\dot{W}_x \sin \gamma - m\dot{W}_h \cos \gamma \end{aligned} \quad (1)$$

and two kinematic relations

$$\begin{aligned} \dot{x} &= V \cos \gamma + W_x \\ \dot{h} &= V \sin \gamma + W_h. \end{aligned} \quad (2)$$

Since the wind flow field is steady-state, the time derivatives of the wind velocity components can be represented as follows:

$$\begin{aligned} \dot{W}_x &= \frac{\partial W_x}{\partial x}(V \cos \gamma + W_x) + \frac{\partial W_x}{\partial h}(V \sin \gamma + W_h) \\ \dot{W}_h &= \frac{\partial W_h}{\partial x}(V \cos \gamma + W_x) + \frac{\partial W_h}{\partial h}(V \sin \gamma + W_h). \end{aligned} \quad (3)$$

The thrust force is of the form:

$$T = A_0 + A_1 + A_2 V^2.$$

The coefficients  $A_0$ ,  $A_1$ , and  $A_2$  depend on the altitude of the runway and on the ambient temperature.

The drag and lift forces are given through the following formulas:

$$D = \frac{1}{2} C_D \rho S V^2, \quad C_D = B_0 + B_1 + B_2 \alpha^2$$

$$L = \frac{1}{2} C_L \rho S V^2, \quad C_L = \begin{cases} C_0 + C_1 \alpha, & \alpha \leq \alpha_{**} \\ C_0 + C_1 \alpha + C_2 (\alpha - \alpha_{**})^2, & \alpha \in [\alpha_{**}, \alpha_*]. \end{cases}$$

The coefficients  $B_0$ ,  $B_1$ ,  $B_2$ ,  $C_0$ ,  $C_1$ , and  $C_2$  depend on the flap setting and undercarriage position;  $\alpha_*$  and  $\alpha_{**}$  are given constants.

The sole control parameter is the attack angle  $\alpha$ . The objective of the control is to avoid the collision of the aircraft subjected to wind disturbances with the earth surface.

For numerical values of the above mentioned parameters which correspond to Boeing-727 see [2,5].

## 2.2. Microburst model

The microburst model is given as follows (see [5]):

$$W_x = \begin{cases} -k, & x \leq a \\ -k + 2k(x - a)/(b - a), & a \leq x \leq b \\ k, & x \geq b, \end{cases}$$

$$W_h = \begin{cases} 0, & x \leq a \\ -k(h/h_*)(x - a)/(c - a), & a \leq x \leq c \\ -k(h/h_*)(b - x)/(b - c), & c \leq x \leq b \\ 0, & x \geq b, \end{cases}$$

where  $c = (a + b)/2$ , and  $h_*$  is a fixed constant. Parameter  $k$  defines the microburst intensity. An example of the velocity field corresponding to this model is shown in Fig. 1.

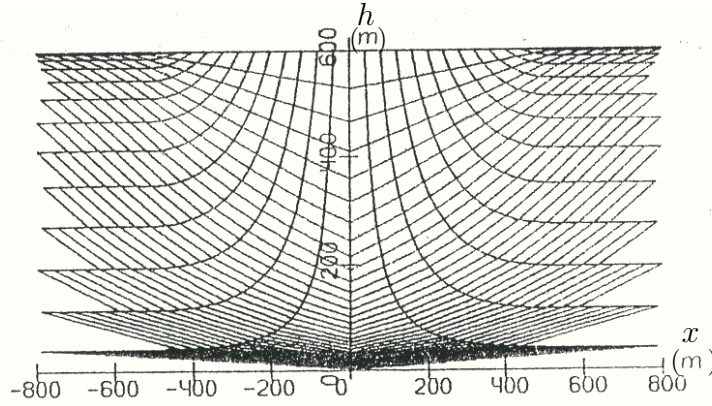


Figure 1: Velocity field associated with the microburst phenomena.

System (1)–(2) uses not only the wind velocity components computed along the aircraft path but also the analytically calculated derivatives of these components with respect to the spatial variables.

## 3. The minmax control

### 3.1. General principles of the design

To construct the minmax control, the same method as for the landing problem described in [13] is used. The aircraft motion with constant relative velocity  $V_0$  along a straight line of slope  $\gamma_0$  to the axis  $x$  is considered as the nominal (desired) trajectory of the aircraft. The nominal values  $W_{x0}$  and  $W_{h0}$  of the wind velocity components are assumed to be known. The nominal value  $\alpha_0$  of the attack angle is calculated using  $V_0$ ,  $\gamma_0$ ,  $W_{x0}$ , and  $W_{h0}$ .

Original non-linear system (1)–(2) is linearized about the nominal values. Thereby,  $\dot{W}_x$  and  $\dot{W}_h$  are replaced as follows:

$$\begin{aligned}\dot{W}_x &= -k_v(W_x - v_1) \\ \dot{W}_h &= -k_v(W_h - v_2).\end{aligned}\tag{4}$$

Equations (4) are being added instead of relations (3) to the linear system obtained. The quantities  $W_x$  and  $W_h$  become state variables, whereas  $v_1$  and  $v_2$  are interpreted as the disturbances (actions of the second player) restricted by the following geometric constraints:

$$|v_1| \leq \nu_1, \quad |v_2| \leq \nu_2.$$

If  $W_x$  and  $W_h$  are bounded at the initial time as follows

$$|W_x| \leq \nu_1, \quad |W_h| \leq \nu_2,$$

they remain within these bounds. Note that relations (4) take into account the inertial behavior of the wind velocity (the coefficient  $k_v$  is chosen according to our guess about this inertia).

For the linear system obtained, an auxiliary differential game with fixed terminal time  $t_f$ , geometric constraints on the control parameter and the disturbance, and convex payoff function depending on two coordinates of the state vector at the time instant  $t_f$  is stated. The first player who governs the control parameter minimizes the payoff function at the time instant  $t_f$ . The second player who is responsible for the disturbance has the opposite objective.

Applying the above mentioned numerical methods to this auxiliary differential game, one can find a switch surface and an optimal guaranteeing control of the first player based on this surface. The designed minmax control is then employed to simulate trajectories of the original non-linear system under various wind conditions including the microburst based disturbances.

### 3.2. The auxiliary linear differential game

The linear system reads

$$\begin{aligned}\dot{z}_1 &= [(A_1 + 2A_2V_0) \cos(\alpha + \delta)/m - \rho SV_0(B_0 + B_1\alpha_0 + B_2\alpha_0^2)/m]z_1 - \cos \gamma_0 z_2 \\ &\quad + [-(A_0 + A_1V_0 + A_2V_0^2) \sin(\alpha_0 + \delta) - \rho SV_0^2(B_1 + 2B_2\alpha_0)/(2m)]u \\ &\quad + k_v \cos \gamma_0(z_3 - v_1) + k_v \sin \gamma_0(z_4 - v_2), \\ \dot{z}_2 &= [(A_1 + 2A_2V_0) \sin(\alpha + \delta)/(mV_0) + (C_0 + C_1\alpha_0)\rho S/m]z_1 + g \sin \gamma_0/V_0 z_2 \\ &\quad + [(A_0 + A_1V_0 + A_2V_0^2) \cos(\alpha_0 + \delta)/(mV_0) + C_1\rho SV_0/(2m)]u \\ &\quad - k_v \sin \gamma_0(z_3 - v_1)/V_0 + k_v \cos \gamma_0(z_4 - v_2)/V_0, \\ \dot{z}_3 &= -k_v(z_3 - v_1), \\ \dot{z}_4 &= -k_v(z_4 - v_2).\end{aligned}\tag{5}$$



Here  $z_1 = \Delta V$ ,  $z_2 = \Delta\gamma$ ,  $z_3 = \Delta W_x$ ,  $z_4 = \Delta W_h$  are the deviations of the relative velocity, the path inclination, and the wind velocity components from their nominal values  $V_0$ ,  $\gamma_0$ ,  $W_{x0} = 0$ , and  $W_{h0} = 0$ , respectively. The control parameter  $u = \Delta\alpha$  is the deviation of the attack angle from its nominal value  $\alpha_0$ . Parameters  $v_1$  and  $v_2$  are interpreted as the components of the disturbance.

The first and the second equations in (5) are obtained via linearization of non-linear equations (1) about the nominal values. The right hand sides of equations (1) do not depend on the aircraft altitude and the distance passed. The payoff function mentioned above and planned to be introduced later will not depend on these quantities too. Therefore, linearized kinematic equations (2) are not included into the system (5).

The numerical form of equations (5) reads

$$\begin{aligned} \dot{z} &= Az + Bu + Cv, \quad z \in R^4, \quad u \in R^1, \quad v \in R^2 & (6) \\ A &= \begin{pmatrix} -0.023751 & -31.946111 & 0.198515 & 0.024323 \\ 0.000793 & 0.014141 & -0.000088 & 0.000717 \\ 0 & 0 & -0.2 & 0 \\ 0 & 0 & 0 & -0.2 \end{pmatrix} \\ B &= (-16.460542, 0.554554, 0, 0)^T \\ C &= \begin{pmatrix} -0.188515 & 0.000088 & 0.2 & 0 \\ -0.024323 & -0.00071 & 0 & 0.2 \end{pmatrix} \\ z &= (z_1, z_2, z_3, z_4), \quad u = \Delta\alpha, \quad v = (v_1, v_2)^T. \end{aligned}$$

The constraint on the control  $u$  is

$$|u| \leq \mu, \quad \mu = 5.633^\circ. \quad (7)$$

The restrictions on the disturbance are chosen as follows:

$$\begin{aligned} |v_1| &\leq \nu_1, \quad \nu_1 = 50 \text{ ft/sec} \\ |v_2| &\leq \nu_2, \quad \nu_2 = 7 \text{ ft/sec.} \end{aligned} \quad (8)$$

To introduce the payoff function depending on  $z_1 = \Delta V$  and  $z_2 = \Delta\gamma$ , consider the convex quadrangle  $M$  with the apexes  $(-30, 0)$ ,  $(-0.9, 0.02)$ ,  $(10, 0)$   $(0.9, -0.02)$ . Put

$$\varphi(z_1, z_2) = \min\{c > 0 : (z_1, z_2) \in cM\}. \quad (9)$$

Fix the time  $t_f$ . The objective of the first player in the auxiliary differential game (6)–(9) is to minimize the function  $\varphi$  at the time instant  $t_f$ , the objective of the second player is opposite. Since all equations are autonomous, no physical sense is attached to  $t_f$ .

It is evident that there are many ways to introduce the auxiliary differential game. For example, one can define some other payoff function using the same state variables, or one can add the components  $\Delta h$  and  $\Delta \dot{h}$  to the state vector and consider a payoff function that depends on these variables. The auxiliary differential game described above was chosen among other variants according to the simulation results.

### 3.3. Optimal strategy of the first player in the linear differential game

It is shown in the papers [21,22] that the optimal strategy of the first player in a linear differential game with fixed terminal time and a convex payoff function depending on two components of the state vector is constructed by means of a switch surface in the space of variables  $t, y_1, y_2$  of an equivalent two-dimensional differential game. In our case, the vector  $y = (y_1, y_2)$  is expressed through the state vector  $z$  of the system (6) as  $y(t) = X(t_f, t)z(t)$ , where  $X(t_f, t)$  is the matrix composed of the first and second rows of the Cauchy matrix of the system  $\dot{x} = Ax$ . The optimal control assumes one of the extremal values ( $\mu$  or  $-\mu$ ) depending on to which side of the switch surface the state vector lies. On the switch surface, any value from the interval  $[-\mu, \mu]$  is appropriate.

Since a discrete control scheme is used, one needs to compute time cross-sections of the switch surface for a given collection of time instants only. The time cross-sections of the switch surface are called switch lines. The algorithm for the computation of switch lines is based on the processing of level sets of the value function.

Let  $\Pi(t_i)$  be the switch line at  $t_i$ , and the direction of the vector  $B(t_i) = X(t_f, t_i)B$  defines the positive side of the switch line, whereas the opposite direction defines the negative side of it. If the point  $y(t_i) = X(t_f, t_i)z(t_i)$  lies to the positive side of  $\Pi(t_i)$ , then  $u = -\mu$  in the next step of the discrete control scheme. If  $y(t_i)$  lies to the negative side of  $\Pi(t_i)$ , then  $u = \mu$ .

## 4. The usage of switch lines in the original non-linear system

In the auxiliary differential game,  $t_f = 15$  is chosen, and the collection of switch lines  $\Pi(t_i)$  for the time instants  $t_i = i\Delta$ ,  $\Delta = 0.1$ ,  $i = \overline{0, 150}$  is constructed. When simulating trajectories of the non-linear system, the constructed switch lines are being used in the following two ways.

**Scheme 1.** Let  $x(t)$  be the distance along the  $x$ -axis covered during the time  $t$ ,  $V_{x0} = V_0 \cos \gamma_0$  the  $x$ -projection of the nominal relative velocity. Then  $t' = x(t)/V_{x0}$  is the nominal time of travelling the distance  $x(t)$ .

Fix  $t_* \in (0, t_f)$  and consider the interval  $[t_*, t_f]$ . At the time  $t$ , the switch line  $\Pi(t_i)$  with

$$i = [t_*/\Delta] + \text{mod}([t'/\Delta], [(t_f - t_*)/\Delta])$$

is taken. Here,  $[ ]$  denotes the integer part of a number, and  $\text{mod}(a, b)$  is the remainder of division of  $a$  by  $b$ . Then the vectors  $y(t) = X(t_f, t_i)z(t)$  and  $B(t_i) = X(t_f, t_i)B$  are being computed. Here,

$$z(t) = (V(t) - V_0, \gamma(t) - \gamma_0, W_x(t) - W_{x0}, W_h(t) - W_{h0})^T.$$

To find the control  $u$ , the location of  $y(t)$  with respect to the line  $\Pi(t_i)$  is analyzed using the vector  $B(t_i)$ .

This variant of using the switch lines can be interpreted as follows. The  $t$ -axis is sampled by the instants  $t_f - t_*$ ,  $2(t_f - t_*)$ , ..., and the values of the function  $\varphi$  defined on

trajectories of the non-linear system are intended to be minimized at these time instants. Thus, trajectories of the non-linear system are forced to track the nominal trajectory at the given times but not continuously.

**Scheme 2.** A drawback of scheme 1 is possible destabilization of the system because of the multiple repetition of the same time interval  $[t_*, t_f]$  in the control scheme. To diminish this defect, another scheme of control that employs only one switch line of the whole collection is proposed. Namely, some time instant  $t_{i*} \in [0, t_f]$  is fixed. The control  $u$  for the current state  $z(t)$  is being found from the analysis to which side of the line  $\Pi(t_{i*})$  with respect to the vector  $B(t_{i*})$  the point  $y(t) = X(t_f, t_{i*})z(t)$  lies. Thus, it is assumed at every time instant that the termination of the control process will happen in time  $t_f - t_{i*}$ . The simulation described below is done for  $t_f - t_{i*} = 3$ . The value of  $t_{i*}$  is heuristically optimized through the analysis of simulation results. The idea of such a control scheme has been obtained due to discussions with V.M. Kein and A.I. Krasov.

The peculiarity of minmax controls is possible rapid switching from one extremal value to the other (e.g. in the case where the point  $y(t)$  is close to the switch surface). To reduce the number of switches, the following rule for the computation of the control is used. Let  $d$  be the distance from the point  $y(t)$  to the switch surface in the direction of the vector  $B(t_i)$ . Put

$$u^\varepsilon = \begin{cases} u, & d > \varepsilon \\ \frac{ud}{\varepsilon}, & d \leq \varepsilon \end{cases}$$

Here,  $u$  is the value computed using control scheme 1 or 2, and  $\varepsilon$  is a fixed positive real. When simulating trajectories of the original non-linear system,  $\alpha = \alpha_0 + u^\varepsilon$  is substituted.

## 5. Simulation results

The initial data are the same as in [5], that is  $x = 0, h = 50$  ft,  $V = 276.8$  ft/sec,  $\gamma = 6.989^\circ$ . The microburst center is located at the beginning of the path, i.e.  $a = 3000$  ft,  $b = 4300$  ft. The intensity  $k = 50$ .

For comparison, the robust control strategy from [5] was recovered and utilized with the same as in [5] values of parameters.

Figure 2 shows the angle of attack (degrees) and the altitude (feet) versus time (seconds). The unmarked curves correspond to control scheme 1, the curves marked with diamonds represent results related to the robust control strategy. One can see that the latter technique gives a smoother evolution of the attack angle.

Figure 3 presents results related to control scheme 2 (unmarked curves). As expected, the usage of a single switch line reduces jumps in the attack angle so that the minmax strategy becomes conquerable to the robust control (see the diamonded curves).

In Figure 4, simulation results for the case where the microburst center is shifted to the right (i.e.  $a = 2300$  ft,  $b = 6300$  ft, and  $k = 40$ ) are presented. The unmarked curves correspond to control scheme 2, the curves marked with diamonds are related to the robust control.

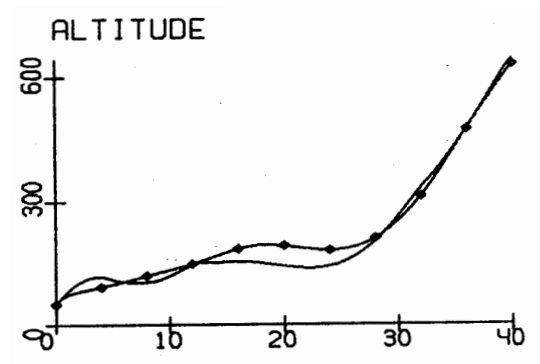
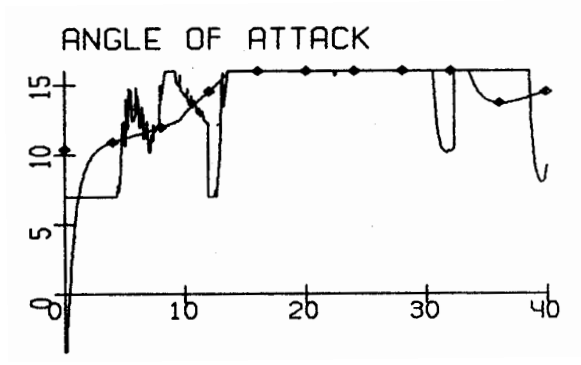


Figure 2: Control scheme 1. The microburst location is:  $a = 300$  ft,  $b = 4300$  ft, the intensity  $k = 50$ .

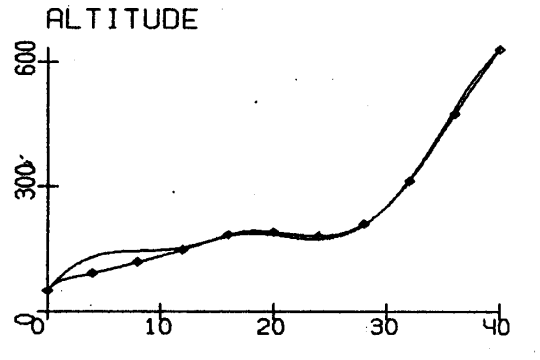
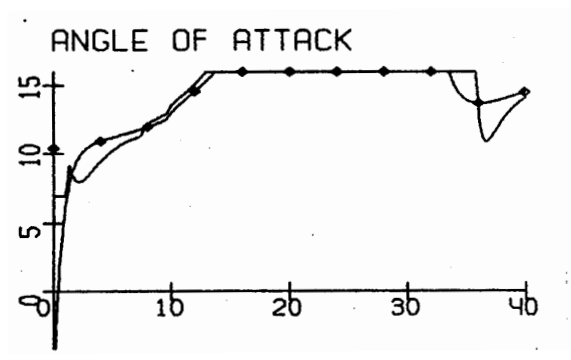


Figure 3: Control scheme 2. The microburst location is:  $a = 300$  ft,  $b = 4300$  ft, the intensity  $k = 50$ .

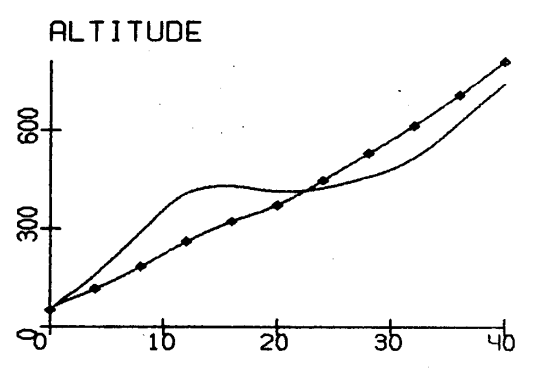
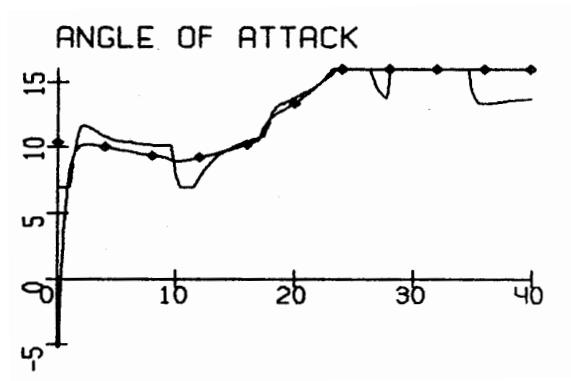


Figure 4: Control scheme 2. The microburst location is:  $a = 2300$  ft,  $b = 6300$  ft, the intensity  $k = 40$ .

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## Управление на взлете при сдвиге ветра

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В препринте исследуется задача управления самолетом на взлете в условиях сдвига ветра, обусловленного явлением микровзрыва. Движение происходит в вертикальной плоскости. Нелинейные дифференциальные уравнения, численные данные и модель микровзрыва ветра взяты из работ А.Мiele. Управляющим воздействием является угол атаки.

Построение управления обратной связи на основе численных методов теории дифференциальных игр включает линеаризацию нелинейной системы относительно выбранного номинального движения, постановку вспомогательной линейной дифференциальной игры с фиксированным моментом окончания, нахождение линий переключения, определяющих оптимальный синтез во вспомогательной дифференциальной игре, задание правила использования построенных линий переключения в исходной нелинейной системе. Предлагаемый способ управления не сложен по реализации и не требует точной информации о ветровом возмущении.

Результаты моделирования движения самолета на взлете в силу предложенного способа управления сравниваются с результатами зарубежных авторов, полученными на основе использования методов теории робастного управления.