

Two encounters with robots

Applied Probability Day for Larry Shepp

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- ▶ Reeds, Shepp, “Optimal paths for a car that goes both forwards and backwards”, *Pacific Journal of Mathematics*, 1990.

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and **A. A. Markov**, *Kharkov Math. J.*, 1889.

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Larry's Bridge Game

- ▶ Multiplayer interactive on-line bridge game. Master program connected with 4 player programs, each of which fronts for a human or a robot.
- ▶ Goal for AT&T: make money
- ▶ Goal for Larry: learn more about bridge
- ▶ Another goal for Larry: have fun programming the robots
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- ▶ Lesson: Don't spring-clean your computer files

LAS's theory of bridge robotics

- ▶ There is a finite number of bridge game situations
- ▶ They can be handled, piecewise, by a system of local point-counting rules, each covering a range of cases
- ▶ Instances of bad play tell when a better local rule is needed

Outcomes

- ▶ Expert bridge playing colleagues said it stank
- ▶ Code grew ¹

files	lines of code	stage
6	1500	1984
20	20000	1986
50	40000	1988

- ▶ We held a duplicate tournament:

team	wins
1986	51
1988	49

¹Fibs: exact numbers lost in spring cleaning

Robot Cart “Blanche”

Ingemar Cox (then AT&T, later NEC, now UCL) asked Larry about motion planning for his robot tricycle Blanche. How best to go from initial position and orientation to final position and orientation in \mathbf{R}^2 , when the path has a bounded radius of curvature? Blanche could reverse, so Y-turns were possible.

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Unicycle idealization:

$$\begin{aligned}t &\mapsto (X(t), Y(t), \theta(t), s(t)) \\(X'(t), Y'(t)) &= s(t)(\cos \theta(t), \sin \theta(t)) \\s(t) &= \pm 1 \\|\theta'(t)| &\leq 1,\end{aligned}$$

with specified starting and finishing values for (X, Y, θ) .
(Here t is arc length, (X, Y) position in the plane, θ orientation, and s is forwards/reverse speed.)

Dubins, 1961

For curves obeying

$$\begin{aligned}t &\mapsto (X(t), Y(t), \theta(t)) \\(X'(t), Y'(t)) &= (\cos \theta(t), \sin \theta(t)) \\|\theta'(t)| &\leq 1,\end{aligned}$$

that is, for cars that have no reverse gear, you can't beat paths of form **CCC** and **CSC**.

That is, curve of form LRL or RLR, or of form LSL, LSR, RSL, or RSR, where L and R stand for left-turning and right-turning unit circle arcs, and S for a straight line segment, all C^1 spliced together.

Does a similar result hold for cars that can also go backwards?

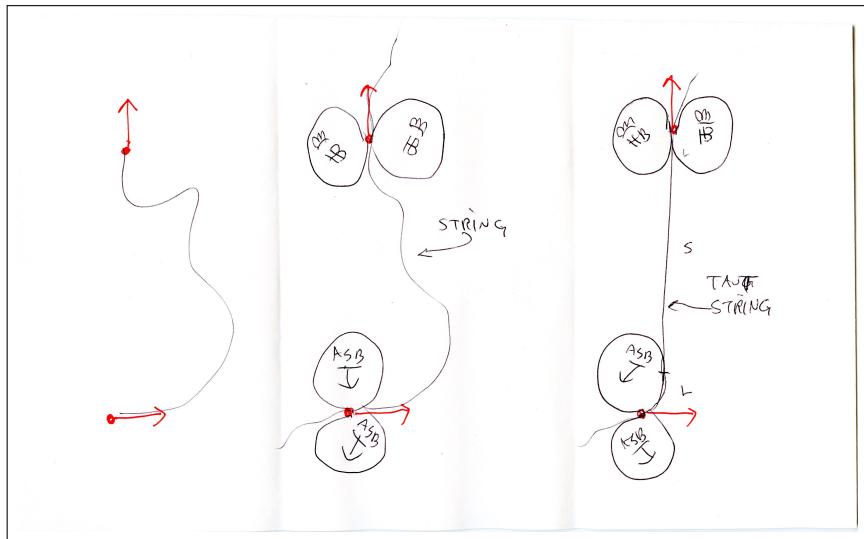
Control theory

Both the Cox car problem and the Dubins car problem look like applications of “control theory”, and the results in both cases seem to asserting that in these cases, “bang-bang” control is optimal. Buzz word: *Pontryagin maximum principle*.

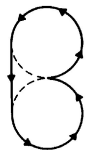
Indeed, both the Dubins and Cox cars are covered by control theory, but Shepp and I were too ignorant to know how to apply it.

This annoys control theorists: if you are a control theory expert our results fall out instantly. Larry and I felt ashamed at our ignorance, but also secretly proud that our method requires much less prior knowledge. But: they can handle the \mathbf{R}^3 case and we couldn't.

"Just pull the string tight" (B. F. "Tex" Logan)

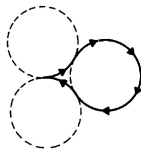


Examples



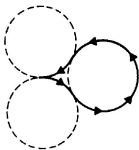
$$\ell \frac{3\pi}{2} S_2 \ell \frac{3\pi}{2}$$

FIGURE A



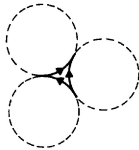
$$\ell \frac{\pi}{3} r \frac{5\pi}{3} \ell \frac{\pi}{3}$$

FIGURE B



$$r \frac{\pi}{3} \ell \frac{5\pi}{3} r \frac{\pi}{3}$$

FIGURE C



$$\ell^+ \frac{\pi}{3} r^- \frac{\pi}{3} \ell^+ \frac{\pi}{3}$$

FIGURE D

Results

For the Cox car, you cannot beat these special forms of paths:

$$C|C|C, \quad CC|C, \quad CSC, \quad CC_u|C_uC, \quad C|C_uC_u|C, \\ C|C_{\pi/2}SC, \quad C|C_{\pi/2}SC_{\pi/2}|C, \quad C|CC, \quad CSC_{\pi/2}|C$$

At most 2 cusps, at most 5 segments. (Where C_u means an arc of length u .)

Suffices to check that paths that are arbitrary concatenations of finitely many C and S segments and cusps cannot beat special form paths.

Discovery method

We grew our list of path types needed, starting from Dubins's $\{CCC, CSC\}$

A path family “word”, such as $CSCS|CS$, is parameterized by the segment durations, and contains a co-dimension 3 set of elements that satisfy the end conditions. Use calculus to rule out many types of subwords in geodesics. What remains are the paths of special type.

By plopping down 100 random points and directions in the plane we had $\binom{100}{3}$ opportunities for triangle law violations. In practice, when our list was insufficient this always worked to suggest how to improve the list.

Oil the squeaky wheel: the underlying principle

Theorem

Every non-empty set S of positive integers has a least element.

Equivalently, the constructive form:

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The sequence $1, 2, 3, \dots$ eventually hits S .

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This was totally silly in the bridge robot case, but worked like a charm in the Cox car case.