To memory of Lawrence Shepp

Valerii Patsko
Institute of Mathematics and Mechanics, Ekaterinburg, Russia

Varvara Turova
Technical University of Munich, Germany

Working on probability theory and mathematical statistics, far away from optimal control theory, in 1990, Larry Shepp surprisingly published a joint paper with Jim Reeds on a model of control motion in the plane. In this model, an object going forward can stop immediately and continue to move in the opposite direction. The model became very popular and received the name „Reeds and Shepp´s car“. Now it is intensively used in theoretical robotics and mathematical control theory.

We also use this model in works related to differential games.

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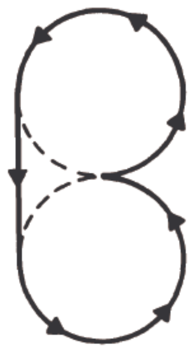
OPTIMAL PATHS FOR A CAR THAT GOES BOTH FORWARDS AND BACKWARDS

J. A. REEDS AND L. A. SHEPP

The path taken by a car with a given minimum turning radius has a lower bound on its radius of curvature at each point, but the path has cusps if the car shifts into or out of reverse gear. What is the shortest such path a car can travel between two points if its starting and ending directions are specified? One need consider only paths with at most 2 cusps or reversals. We give a set of paths which is *sufficient* in the sense that it always contains a shortest path and *small* in the sense that there are at most 68, but usually many fewer paths in the set for any pair of endpoints and directions. We give these paths by explicit formula. Calculating the length of each of these paths and selecting the (not necessarily unique) path with smallest length yields a simple algorithm for a shortest path in each case. These optimal paths or geodesics may be described as follows: If C is an arc of a circle of the minimal turning radius and S is a line segment, then it is sufficient to consider only certain paths of the form $CCSCC$ where arcs and segments fit smoothly, one or more of the arcs or segments may vanish, and where reversals, or equivalently cusps, between arcs or segments are allowed. This contrasts with the case when cusps are not allowed, where Dubins (1957) has shown that paths of the form CCC and CSC suffice.



$\ell_{\frac{3\pi}{2}}\, S_2\, \ell_{\frac{3\pi}{2}}$

FIGURE A

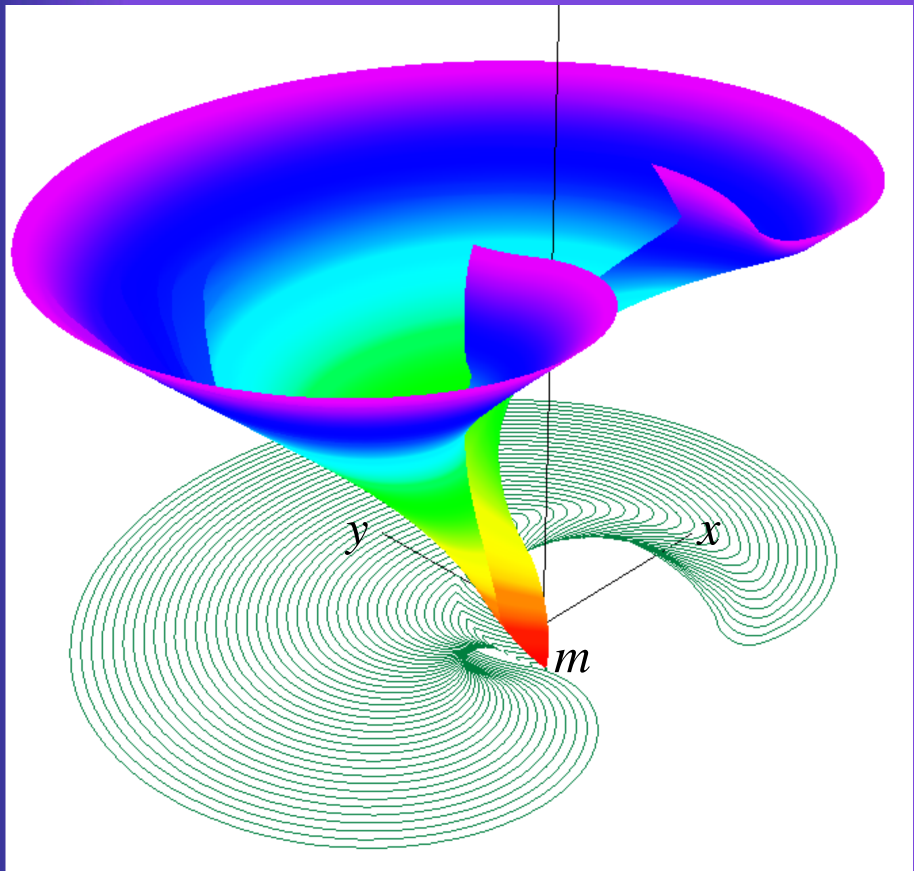
1. Introduction. We want to find a shortest path in the plane with specified initial and final points and directions and with the further constraint that at each point the radius of curvature should be ≥ 1. This problem arose in a simple model for a robot cart which moves under computer control. The cart can shift into reverse and so the path is allowed to have cusps.

In an elegant paper, Lester Dubins (1957) solved the problem when the car cannot reverse and cusps are not allowed. Even in this case it is apparently impossible to give an explicit formula for the shortest path. Instead Dubins gives a sufficient set of paths, i.e. a set which always contains what he called a *geodesic*, or optimal path. His sufficient set is so small that there are at most 6 contenders in the set for each case of specified endpoint conditions, and it is a simple matter to find the shortest of these 6, which gives an algorithm for the solution. He showed that any geodesic can be described by one of 6 words: $\ell r \ell$, From August 1966 to January 1967 Larry Shepp had a fellowship in the Kolmogorov group at the Moscow State University. Larry knew that we work on differential games and described his stay in Russia with playful words (letter to us, 14.03.2009) :

„By the way, I knew Pontryagin very well. I lived in Moscow from August, 1966 to January, 1967. I speak Russian which I studied only as an adult. I was forced to leave early due to a pursuit game I got into with the KGB playing the role of the pursuer and me playing the role of the evader: I arrived at the US Embassy just ahead of their attempts to grab me. The entire game took place in front of the embassy. The game was a variant where the evader has to get into the door rather than just avoid the tag for a maximal time. Clearly neither of us was using an optimal solution; I survived for nearly 50 years, so the evader seems to have won - I ran faster in those days than I do today, and they were not expecting me, apparently, or chose not to catch me due to other criteria entering. "

Since 1989, Larry has visited Russia many times. He used to attend mathematical conferences and gained many friends in Russia.

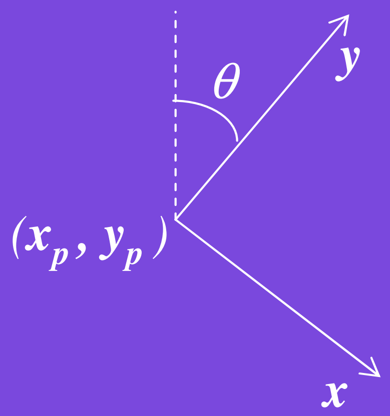
Mathematical flower to memory of Larry Shepp



$$\dot{x}_p = w \sin \theta$$

$$\dot{y}_p = w \cos \theta$$

$$\dot{\theta} = u, \ \ |u| \leq 1, \ \ a \leq w \leq 1$$



Graph of the minimum-time function for $a = 0.8$ and terminal point m

$a = 1$: Dubin´s car

$a = -1$: Reeds-Shepp´s car