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Fixed Types of Motion in Aircraft Trajectory Recovering ⋆

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Abstract: We describe an algorithm for recovering a trajectory of an aircraft that is based on the construction of a bundle of approximating trajectories. Each of them is a possible version of the real aircraft motion. The specific feature of the algorithm is the approximation of measurements by means of a fixed set of motion patterns. A procedure of detecting the motion type determines the most probable motion pattern, and then the weight of the corresponding approximating trajectory in the final estimate of the aircraft current position increases. Such a design improves the accuracy of the coordinate determination at the stages of steady motion. The results of application of the algorithm to some model data are presented.

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1. INTRODUCTION

We consider the task of on-line aircraft trajectory reconstruction according to incoming radar measurements: after arrival of the next radar measurement, the algorithm must immediately yield an estimate of the aircraft position. The main difficulty in the problem is that the object moves nonstationary, i.e., it performs maneuvers whose characteristics and duration are unknown to the observer. Here, long-term sections are possible where the motion type is constant, on which the aircraft trajectory is well approximated by one simple model. In such sections, it is important that the recovery algorithm should yield accuracy close to the accuracy of algorithms specially designed for this particular type of motion. In addition, the robustness to "outliers" in measurements is important for work in real situations. There is a large number of failures in real data that can lead to a loss of the trajectory by the recovery algorithm.

In spite of the existing solutions (see Bar-Shalom et al. (2004), Konovalov (2014)), a lot of publications constantly appear on various aspects of this problem; see, for example, Bortnikov (2014). They consider the trajectory processing not only for the aircraft motion, but also for other objects; various mathematical methods are used, including those involving the use of multiple motion models (see, for example, Gutorov (2017)).

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The effectiveness of this approach lies primarily in the timely detection of the motion type (the localization of maneuvers).

In the present paper, one of the possible solutions of this problem is considered. The results of processing typical model data are presented.

2. GENERAL ALGORITHM DESCRIPTION

The basic structure of the algorithm is a set (bundle) of "most probable" aircraft trajectories, which is constructed taking into account the dynamics of the aircraft and possible outliers of measurements. The endpoints of the trajectories in the bundle are used to construct an estimate of the aircraft position. This estimate is issued as a result of the algorithm at the current time.

We assume that the aircraft moves in a horizontal plane according to the standard model of the simplest airplane motion in Akhmedov et al. (2004) (x and z are the coordinates on the plane, φ is the path angle, and v is the speed):

$$\begin{cases} \dot{x} = v\cos\varphi, \\ \dot{z} = v\sin\varphi, \\ \dot{\varphi} = u/v, \\ \dot{v} = w. \end{cases}$$

In the case of constant tangential w and transversal u controls, these equations can be integrated analytically, see Bedin et al. (2010). Each trajectory of the bundle corresponds to the dynamics and piecewise constant controls w and u. It is assumed that the duration of the sections of constancy cannot be less than a certain given constraint.

At the beginning of the algorithm work, the bundle startup procedure is executed with several first measurements.

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Further, the main cycle is running where each iteration is connected with receiving a new measurement.

The track bundle is recalculated using measurements from a sliding time window of a fixed duration ending with the last measurement. The recalculation is started at each newly arrived measurement. A bundle of trajectories is formed with a view to support the maximum representativeness of various variants of motion.

For each trajectory of the bundle, the "accordance with measurements" criterion is calculated taking into account the distance between the trajectory and the measurements, as well as additional penalties.

Several criteria with different properties are used. The following properties are common to all criteria:

- the smaller value of the criterion corresponds to the trajectory that is closer to the measurements;
- if the trajectory passes exactly through the measurements, the value of the criterion is zero.

Additionally, the penalties are charged:

- for exiting the limitation on the maximum absolute value of the transversal and tangential controls;
- if the duration of a constant control section is less than the prescribed value;
- if the duration of two adjoining constant control sections is less than the prescribed value;
- if the value of the aircraft velocity module is too small or too large;
- if the motion type does not correspond to the type determined by the motion detector.

3. BASIC PROCEDURES OF ALGORITHM

Track extension and track trimming. At this stage, the predicted position of all tracks is calculated at the time of the newly arrived measurement. The last section of constant controls is extended until the moment of the current measurement. On the other hand, the tracks are shortened in time from the old measurements, so that the total duration of the track does not exceed the preset window length.

New measurement branching. Branching is a procedure where, for each trajectory, possible variants of its extension are constructed with altered (with respect to the original trajectory) controls. There is a continuous "gluing" of the branch with the parent trajectory at an intermediate point. Choosing different branch points on the initial trajectory and different control values in the section after the branch, we obtain different variants of the trajectory. Only a few of all the possible options will be left in the bundle. The best values of the accordance criterion select the trajectories to be kept.

One of the variants of the branching is the trajectory that hits exactly the point of the last measurement. To construct this trajectory, we use the solution of an auxiliary problem of hitting a point described in Bedin et al. (2010). Other options are also used: a branch with zero control and branches that hit random points near the last measurement. A branch with zero control is taken with intention to improve the approximation of measurements in areas where the aircraft finishes its maneuver and starts to move uniformly along a straight line.

At the same stage, special trajectories are formed, namely, "OLS straight line" and "OLS circle", which are calculated without using any trajectory of the bundle as the parent path. The root-mean-square deviation of the constructed trajectory from the measurements is minimized. The "OLS straight line" assumes constant tangential acceleration and zero transversal acceleration. The "OLS circle" is constructed with zero tangential acceleration and constant transversal acceleration.

Motion Type Detecting The main algorithm forms the evaluation of the tangential and transversal accelerations. Each of them is analyzed separately by a special algorithm, which we shall later call the "detector". The purpose of the detector is to discover that the input signal is close to constant, or vice versa, to discover its sudden change after a period of constancy.

Consider the detector in more details. Let a certain function $u(t)$ be measured at discrete time instants t_i; its value at the time t_i is denoted u_i. We denote by t_f the current instant for which the analysis is carried out; the index f for other values means that they correspond to this time.

The detector has two modes: the mode for searching the constancy of $u(t)$ and the mode for searching the end of the $u(t)$ constancy section.

Preliminary bundle pruning. At this stage, the trajectories that are poorly aligned with the available measurements and with physical limitations are deleted.

Selective optimization. Optimization means the variation of the values of controls and switching times between the sections of constant control. A direct search method for finding the minimum of a multidimensional function is used. The optimization procedure applied to all the trajectories leads to poor results due to the "thinning" of the bundle and the loss of multi-hypotheses. Therefore, optimization is carried out only over a small number of trajectories with the best value of the criterion.

Duplicate tracks deleting. A matrix of mutual distances between the bundle trajectories is created. In this case, if two coincident tracks or very close tracks are detected in the process of creating the distance matrix, then the track with the less good criterion is removed from the bundle.

Calculation of the current position of the aircraft. At each instant, when the measurement is arrived, the algorithm must produce an estimate of the aircraft position as an output. We use the averaging of the positions at this time for the trajectories of the bundle.

The estimation using the same criterion as in the basic procedures does not always yield good results. In the described version, the evaluation of the current position is generated using weights derived from other criteria of quality.

Not all available trajectories of the bundle are involved in the estimation, but only those for which the value of the main criterion of accordance is small. For each trajectory, its weight is calculated. Depending on the detected type of current motion, the weight of the "OLS" trajectories can be forcibly increased.

Grouping and pruning. The objective of this procedure is to reduce the number of tracks in the bundle while maintaining the representativeness of different hypotheses about the aircraft motion. A pair of trajectories with a minimum distance is determined in the matrix of mutual distances between tracks of the bundle and the trajectory of this pair with the worst criterion is removed from the bundle. Then we again look for a pair of trajectories with the minimum distance, etc. The procedure continues until the number of trajectories is less than a prescribed number.

4. OLS STRAIGHT LINE

Consider the problem of constructing an optimal straight-line motion with a constant tangential acceleration. We should construct the trajectory closest to the measurements in the sense of mean-square deviation. This problem is not linear due to the constraint that the acceleration have to be parallel to the velocity. Therefore, we construct a quasi-optimal solution with three linear subproblems.

(1) First, we solve the linear problem of construction of an optimal motion along a straight line with a constant velocity:
$$Z \to x_0, z_0, v_x, v_z$$
(Z is the set of measurements, x_0, z_0 are the coordinates of the initial point, v_x, v_z are the projections of the velocity on the coordinate axes).

(2) Then we find the longitudinal acceleration for the straight line with the fixed direction obtained in the previous step:
$$Z, \frac{v_x}{v_z} \to x_0, z_0, v_0, w$$
(v_0 is the magnitude of the initial velocity of motion along a straight line, w is the magnitude of the tangential acceleration).

(3) For a given ratio between magnitudes of the initial velocity and the longitudinal acceleration, we correct the direction of the straight line:
$$Z, v_0, w \to x_0, z_0, \alpha_x, \alpha_z$$
Using it, we obtain the projections of the initial velocity ($v_{x0} = v_0\alpha_x$, $v_{z0} = v_0\alpha_z$) and a new value of tangential acceleration ($w := w\sqrt{\alpha_x^2 + \alpha_z^2}$).

Each of the problems above is linear and has an analytic solution.

If at the stage (2) we get $|w| \le w_*$ (w_* is a given parameter of the algorithm), we decide that the true motion occurs without acceleration. In this case, we use the straight line from stage (1) as the solution.

5. OLS CIRCLE

We consider the problem of approximating a set of measurements in the plane
$$Z = \{Z_i\}_{i=1}^n\,, \quad Z_i = \begin{bmatrix}\tilde{x}_i\\ \tilde{z}_i\end{bmatrix} \in \mathbb{R}^2$$
by a circular motion $\xi(t) = [x(t), z(t)]$ with constant velocity. We assume that the parameters of the motion corresponds the time t_0, and the measurement instants are denoted by $\{t_i\}$, $i = 1, \ldots, n$. The parameters of the circular motion are:

- R is the radius;
- $\xi_c = [x_i\ z_i]^\mathsf{T} \in \mathbb{R}^2$ is the center of the circle;
- φ_0 is the angle between axis x and a line connecting points $\xi(t_0)$ and ξ_c;
- v is the speed.

Thereafter, instead of the parentheses to denote the time reference, we use subscripts, for example:
$$\xi_i := \xi(t_i) = [x_i\ z_i]^\mathsf{T}.$$

The approximation we make is in the sense of a mean-square criterion with weights $\{w_i\}_{i=1}^n$:
$$J(\xi(\cdot)) = \frac{\sum\limits_{i=1}^n w_i\|Z_i - \xi_i\|^2}{\sum\limits_{i=1}^n w_i}, \quad w_i \ge 0\,.$$

Here and below, we assume that $\sum_{i=1}^n w_i = 1$, without loss generality.

The position $\xi_i = [x_i\ z_i]^\mathsf{T}$ is described by the following equations:
$$\begin{cases} x_i = x_c + R\cos\varphi_i\,,\\ z_i = z_c + R\sin\varphi_i\,,\end{cases} \qquad \varphi_i = \varphi_0 + \frac{v}{R}(t_i - t_0)\,.$$
Here φ_i are the angular positions of the point ξ_i with respect to the center of the circle. Since the velocity v enters the equations only in the combination v/R, it is convenient to introduce a new parameter $\omega = v/R$ which means the angular velocity of motion along the circle. Make some useful designations:
$$\Delta\varphi_i = \omega(t_i - t_0)\,,$$
$$\begin{bmatrix}x_i\\ z_i\end{bmatrix} = \begin{bmatrix}x_c\\ z_c\end{bmatrix} + R\begin{bmatrix}\cos\varphi_0 & -\sin\varphi_0\\ \sin\varphi_0 & \cos\varphi_0\end{bmatrix}\begin{bmatrix}\cos\Delta\varphi_i\\ \sin\Delta\varphi_i\end{bmatrix},$$
$$\xi_i = \xi_c + RS(\varphi_0)r(\Delta\varphi_i)\,. \tag{1}$$
Here we introduce new symbols for the orthogonal rotation matrix S and the direction vector of unit length r (formulas are given for an arbitrary angle ψ):
$$S(\psi) = \begin{bmatrix}\cos\psi & -\sin\psi\\ \sin\psi & \cos\psi\end{bmatrix}, \qquad r(\psi) = \begin{bmatrix}\cos\psi\\ \sin\psi\end{bmatrix}.$$
It is useful to note that
$$S(\varphi_0)r(\Delta\varphi_i) = S(\Delta\varphi_i)r(\varphi_0) \tag{2}$$
and equation (1) can be rewritten as
$$\xi_i = \xi_c + RS(\Delta\varphi_i)r(\varphi_0)\,.$$

Thus, the problem has the form
$$J = J(\xi_c, R, \varphi_0, \omega) = \sum_{i=1}^n w_i\|Z_i - \xi_i\|^2 \to \min_{\xi_c, R, \varphi_0, \omega} \tag{3}$$
provided that ξ_i meets (1).

5.1 Partial Analytic Minimization of Functional J

The expression of ξ_i is linear with respect to the center ξ_c of the circle, so the partial minimization in problem

(3) with respect to the variable ξ_c only can be written in analitical form:

$$\frac{\partial}{\partial \xi_c} J = -2 \sum_{i=1}^{n} w_i (Z_i - \xi_i)^{\mathsf{T}}$$

$$= -2 \sum_{i=1}^{n} w_i \big(Z_i - \xi_c - RS(\varphi_0) r(\Delta\varphi_i)\big)^{\mathsf{T}} = 0$$

$$\Leftrightarrow \ \hat{\xi}_c = \sum_{i=1}^{n} w_i Z_i - RS(\varphi_0) \sum_{i=1}^{n} w_i r(\Delta\varphi_i) \,. \qquad (4)$$

Here $\hat{\xi}_c$ is the optimal value of the parameter ξ_c. Let's introduce some notations:

$$\hat{Z} = \sum_{i=1}^{n} w_i Z_i \,, \qquad \hat{r} = \sum_{i=1}^{n} w_i r(\Delta\varphi_i) \,.$$

Next, we substitute expression (4) for $\hat{\xi}_c$ into the functional J and obtain a modified functional J_{ξ_c}:

$$J_{\xi_c} = \min_{\xi_c} J$$

$$= \sum_{i=1}^{n} w_i \Big(\Big(Z_i - \hat{Z}\Big) - RS(\varphi_0) \big(r(\Delta\varphi_i) - \hat{r}\big) \Big)^2$$

$$= \sum_{i=1}^{n} w_i \left(Z_i - \hat{Z}\right)^2$$

$$- 2R \left(\sum_{i=1}^{n} w_i \left(Z_i - \hat{Z}\right)^{\mathsf{T}} S(\Delta\varphi_i) \right) r(\varphi_0) + R^2 \left(1 - \hat{r}^2\right) \,.$$

Here we use equality (2). Let's make a new notation:

$$\hat{d} = \sum_{i=1}^{n} w_i S(\Delta\varphi_i)^{\mathsf{T}} \Big(Z_i - \hat{Z}\Big) = \sum_{i=1}^{n} w_i S(-\Delta\varphi_i) \Big(Z_i - \hat{Z}\Big) \,.$$

We obtain the final formula for the partial optimized functional

$$J_{\xi_c} = \min_{\xi_c} J$$

$$= \sum_{i=1}^{n} w_i \left(Z_i - \hat{Z}\right)^2 - 2R\,\hat{d}^{\mathsf{T}} r(\varphi_0) + R^2 \left(1 - \hat{r}^2\right) \,, \qquad (5)$$

and a new problem

$$J_{\xi_c} \to \min_{R, \varphi_0, \omega}$$

In the new problem, the partial optimization in R can be done analytically too:

$$\frac{\partial}{\partial R} J_{\xi_c} = -2\hat{d}^{\mathsf{T}} r(\varphi_0) + 2R \left(1 - \hat{r}^2\right) = 0$$

$$\Leftrightarrow \ \hat{R} = \frac{1}{(1 - \hat{r}^2)}\, \hat{d}^{\mathsf{T}} r(\varphi_0) \,. \qquad (6)$$

Optimization for φ_0 is also analytical: it is sufficient to note that the choice of the angle φ_0 is equivalent to the choice of the unit vector $r(\varphi_0)$. The latter enters functional (5) linearly. The minimum, therefore, is achieved when

$$r(\varphi_0) \upuparrows \hat{d} \ \Leftrightarrow \ r(\varphi_0) = \frac{\hat{d}}{\|\,\hat{d}\,\|} \,. \qquad (7)$$

Substituting the optimal $r(\varphi_0)$ from (7) into the expression for the optimal $\hat{R}$, we obtain the final formula for it:

$$\hat{R} = \frac{\|\,\hat{d}\,\|}{(1 - \hat{r}^2)} \,.$$

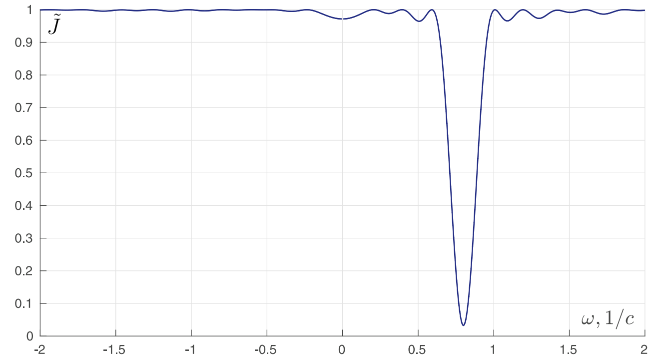

Fig. 1. An example of the typical behavior of the functional $\tilde{J}$

We also get a new functional optimized over R, φ_0, after substituting (6), (7) into (5), and a new problem as follows:

$$J_{\xi_c, R, \varphi} = \min_{R, \varphi_0} J_{\xi_c} = \min_{\xi_c, R, \varphi_0} J$$

$$= \sum_{i=1}^{n} w_i \left(Z_i - \hat{z}\right)^2 - \frac{\hat{d}^2}{(1 - \hat{r}^2)} \to \min_{\omega} \,, \qquad (8)$$

Functional (8) depends on one parameter ω only, so the entire problem reduces to a one-dimensional minimization. To normalize the functional, we make some transformations. Let us introduce "root-mean-squared" deviation of measurements:

$$\sigma_Z = \sqrt{\sum_{i=1}^{n} w_i \left(Z_i - \hat{z}\right)^2}$$

and consider scaled vectors

$$Z_i' = \frac{1}{\sigma_Z} Z_i \,, \qquad \hat{Z}' = \frac{1}{\sigma_Z} \hat{z} \,.$$

Then

$$J_{\xi_c, R, \varphi} = \sigma_Z^2 - \frac{\sigma_Z^2\, \hat{d}'^2}{(1 - \hat{r}^2)},$$

$$\hat{d}' = \sum_{i=1}^{n} w_i\, S(-\Delta\varphi_i) \left(Z_i' - \hat{Z}'\right),$$

and we can consider a "dimensionless" problem:

$$\tilde{J} = 1 - \frac{\hat{d}'^2}{(1 - \hat{r}^2)} \to \min_{\omega} \qquad (9)$$

Everywhere, except for individual values of ω, inequalities $0 \le \hat{d}'^2 \le 1$, $0 < \hat{r}^2 \le 1$ are satisfied. As a consequence, the functional $\tilde{J}$ has values in the range $[0, 1]$. For some points where $\|\hat{r}\| = 1$, the functional $\tilde{J}$ is not defined. For example, such a point is the value $\omega = 0$, corresponding to the motion along a straight line.

5.2 Optimization of Functional $\tilde{J}$

Figure 1 presents a typical plot of the functional $\tilde{J}$ with a narrow "well" near the true value of ω, where the functional is close to 0. For ω, which are very different from the true value, the value is close to 1. A large number of local minima is specific.

The functional $\tilde{J}$ is rather complicated for optimization. The well near the true value can be very narrow, and for a good work of the minimum search algorithm we need a correct initial approximation. Also $\tilde{J}$ becomes a convex function only near the floor of the well, so one cannot rely on the second derivative until the final steps of the procedure. For the sake of numerical optimization, the analytical expressions of the derivatives of (9) are provided here. The derivatives of the functions $\hat{d}'$, $\hat{r}$ with respect to ω will be denoted by dot. To improve readability, the variables in the formula below are renamed as $d := \hat{d}'$, $r := \hat{r}$.

$$\frac{d}{d\omega}\tilde{J} = -2\frac{d^{\mathsf{T}}\dot{d}}{(1-\hat{r}^2)} - 2\frac{d^2 r^{\mathsf{T}}\dot{r}}{(1-\hat{r}^2)^2}\,,$$

$$\frac{d^2}{d\omega^2}\tilde{J} = -2\frac{\left(\dot{d}^2 + d^{\mathsf{T}}\ddot{d}\right)}{(1-\hat{r}^2)} - 8\frac{\left(d^{\mathsf{T}}\dot{d}\right)\left(r^{\mathsf{T}}\dot{r}\right)}{(1-\hat{r}^2)^2}$$

$$-2\frac{d^2\left(\dot{r}^2 + r^{\mathsf{T}}\ddot{r}\right)}{(1-\hat{r}^2)^2} - 8\frac{d^2\left(r^{\mathsf{T}}\dot{r}\right)^2}{(1-\hat{r}^2)^3}\,,$$

$$\dot{r} = \sum_{i=1}^{n} w_i(t_i - t_0) S(\pi/2)\, r(\Delta\varphi_i)\,,$$

$$\ddot{r} = -\sum_{i=1}^{n} w_i(t_i - t_0)^2\, r(\Delta\varphi_i)\,,$$

$$\dot{d} = \sum_{i=1}^{n} w_i(t_i - t_0) S(-\Delta\varphi_i - \pi/2)\,(Z_i' - \hat{z}')\,,$$

$$\ddot{d} = -\sum_{i=1}^{n} w_i(t_i - t_0)^2 S(-\Delta\varphi_i)\,(Z_i' - \hat{z}')\,.$$

In practice, such an algorithm showed a good work: in the case of local concavity we make a step of simple gradient descent with magnitude $\delta_\omega = 0.005$ 1/s^2, and in the case of local convexity we make a step of the Newton's method

$$\begin{cases} \omega_{k+1} = \omega_k - \delta_\omega \dfrac{d}{d\omega}\tilde{J}(\omega_k)\,, & \dfrac{d^2}{d\omega^2}\tilde{J}(\omega_k) \le 0\,, \\ \omega_{k+1} = \omega_k - \dfrac{d}{d\omega}\tilde{J}(\omega_k)/\dfrac{d^2}{d\omega^2}\tilde{J}(\omega_k), & \dfrac{d^2}{d\omega^2}\tilde{J}(\omega_k) > 0\,. \end{cases}$$

5.3 Initial Approximation for ω

To find the initial approximation, the following simple algorithm is used. We construct difference vectors

$$Z_{i+1,i} = Z_{i+1} - Z_i$$

and calculate their direction angles ψ_i with the axis x. In the absence of measurement errors, the following equation is true for uniform motion along the circle

$$\psi_i = \psi_0 + \omega\,\tau_i\,, \quad \tau_i = \left(\frac{t_i + t_{i+1}}{2} - t_0\right).$$

Then we can obtain the approximate value $\breve{\omega}$ by mean-square approximation of the observations ψ_i:

$$\breve{\omega} = \frac{\sum\limits_{i=1}^{n-1}\tau_i^2 \sum\limits_{i=1}^{n-1}\psi_i - \sum\limits_{i=1}^{n-1}\tau_i \sum\limits_{i=1}^{n-1}\tau_i\psi_i}{(n-1)\sum\limits_{i=1}^{n-1}\tau_i^2 - \left(\sum\limits_{i=1}^{n-1}\tau_i\right)^2}\,. \tag{10}$$

For correct work of the algorithm, it is necessary to remove jumps in the values ψ_i, which can arise because standard procedures like `atan2` return value in the range $(-\pi, \pi]$.

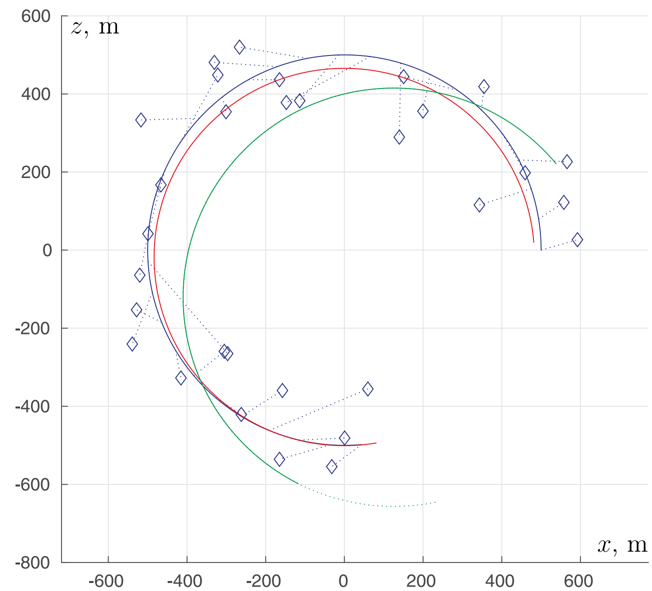

Fig. 2. Example of approximation of measurements by a circle

5.4 Example of Algorithm Work

An example of the algorithm approximation is shown in Fig. 2. The solid blue arc corresponds to the true movement. Blue diamonds are measurements, a dotted line connects each of them to the true position at the same time instant. The red solid line shows the approximation of the circular motion with angular velocity $\hat{\omega}$ evaluated by minimization of functional (9), and the remaining parameters are calculated by analytical formulas (4), (7), and (6). The initial approximation $\breve{\omega}$ for the optimization procedure was obtained using formula (10). The green solid line depicts a circle with ω exactly equal to $\breve{\omega}$.

The true movement have the following parameters: $R = 500$ m, $v = 80$ m/s, $\omega = v/R = 0.16$ 1/s, $\xi_c = [0\ 0]^{\mathsf{T}}$ m, $\varphi_0 = 0$ rad. The root-mean-square deviation of random measurement errors is 100 m.

6. MODELING RESULTS

An ideal model trajectory is formed that consists of steady motion sections and transition sections for which the maximum tracking errors are defined in the standards SUR.ET1.ST01.1000-STD-01-01. For this trajectory, 100 model tracks of measurements with a mean square deviation of 70 m were generated. For each of them, a trajectory was restored using the described algorithm. The graphs of the root-mean-square deviation of the restored positions from the true motion are constructed. The time plot of the tangential deviation is depicted in Fig. 3 (thick solid black line). Also, a similar graph for the Interacting Multiple Model (IMM) algorithm, see Bar-Shalom et al. (2004), Konovalov (2014), is given (red line). The dashed blue line shows the graph of root-mean-square deviation of measurements.

In the case of processing trajectories with outliers (rare large deviations that are not be drawn from regular distribution), the advantage of the proposed algorithm becomes more evident. For example, see Fig. 4, which shows the

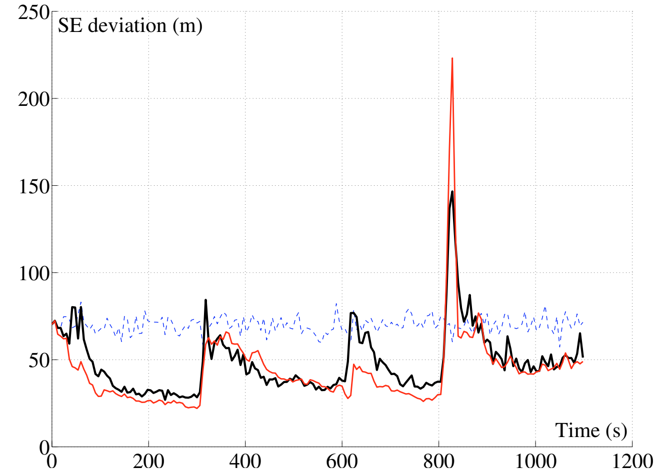

Fig. 3. Graph of the root-mean-square longitudinal deviation σ as a function of time. There are no outliers in the measurements. The thick solid black line shows the results by the described algorithm. The red line is the results by the Interacting Multiple Model (IMM) method. The dashed blue line is the track of measurements.

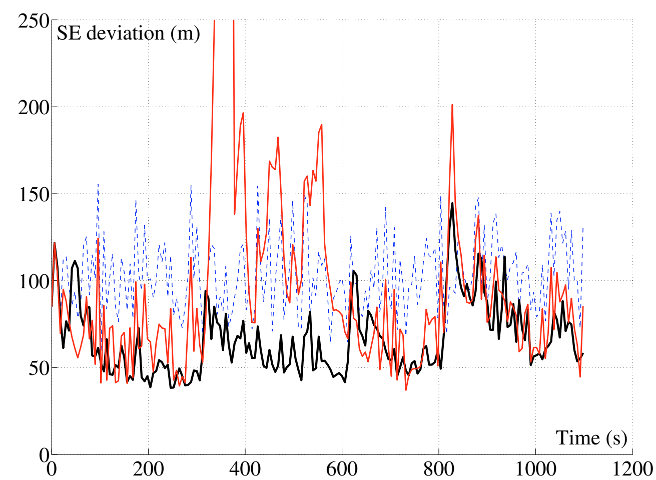

Fig. 4. Graph of the root-mean-square longitudinal deviation σ as a function of time. The case of the presence of outliers in the measurements. The thick solid black line shows the results by the described algorithm. The red line is the results by the Interacting Multiple Model (IMM) method. The blue dashed line is the track of measurements.

tangential deviation in the case of the presence of outliers in measurements. With a probability of 1/20, the measurement is an outlier. The outlier error is increased by a factor of five compared to the regular level.

7. CONCLUSION

The algorithm for recovering an aircraft trajectory is created. It is based on the calculation of the bundle of trajectories, each of which represents a hypothesis about the true motion of an aircraft. Among the assumptions on the aircraft motion, there are the assumption about the motion along a straight line and the assumption of motion along a circle. The trajectories corresponding to these assumptions are processed in the algorithm in a special way. This allowed to improve the algorithm accuracy on long straight line and circle sections, which are often occurred in the real aircraft trajectories.

The constructed algorithm has a large number of parameters. At present, the values of almost all the parameters are established from empirical considerations. Therefore, it is planned to use the methods of machine learning to find the optimal values of these parameters.

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