28-30 MAY 2018

25th ANNIVERSARY SAINT PETERSBURG INTERNATIONAL CONFERENCE ON INTEGRATED NAVIGATION SYSTEMS

STATE RESEARCH CENTER OF THE RUSSIAN FEDERATION
CONCERN **CSRI ELEKTROPRIBOR**, JSC

STATE RESEARCH CENTER OF THE RUSSIAN FEDERATION
CONCERN CSRI ELEKTROPRIBOR, JSC

25th ANNIVERSARY SAINT PETERSBURG INTERNATIONAL CONFERENCE ON INTEGRATED NAVIGATION SYSTEMS

P R O C E E D I N G S

28 – 30 May 2018

Saint Petersburg, Russia

CO-SPONSORED BY:

- **RUSSIAN FOUNDATION FOR BASIC RESEARCH**
- **INTERNATIONAL PUBLIC ASSOCIATION – ACADEMY OF NAVIGATION AND MOTION CONTROL (ANMC)**
- **NATIONAL RESEARCH UNIVERSITY ITMO, RUSSIA**
- **AMERICAN INSTITUTE OF AERONAUTICS AND ASTRONAUTICS (AIAA)**
- **INSTITUTE OF ELECTRICAL AND ELECTRONICS ENGINEERS – AEROSPACE AND ELECTRONIC SYSTEMS SOCIETY (IEEE AESS)**
- **INSTITUT FRANÇAIS DE NAVIGATION (IFN)**
- **DEUTSCHE GESELLSCHAFT FUER ORTUNG UND NAVIGATION (DGON)**
  GERMAN INSTITUTE OF NAVIGATION
- **JOURNAL GYROSCOPY AND NAVIGATION**

In the present publication the plenary and poster papers of the 25th Anniversary Saint Petersburg International Conference on Integrated Navigation Systems (28 – 30 May, 2018) are presented.

*The poster papers marked with *.*

Editor-in-Chief

Academician of RAS
Vladimir G. Peshekhonov

PROBLEM OF MULTILATERATION WITH SEVERAL SIGNAL TRANSMISSION INSTANTS*

D. Bedin
Krasovskii Institute of Mathematics and Mechanics, UB RAS, Ekaterinburg, Russia
bedin@imm.uran.ru

***Abstract*—The multilateration problem is described in the case when measurements from several consecutive instants of signal transmission are processed together. For the corresponding observation equations, the Cramer–Rao Lower Bound is computed. The solution is based on minimization of a nonlinear functional, and we proposed an effective numerical method for this optimization task. The results of work of the algorithm on simulation data corresponding to some real location and characteristics of the receiving stations are presented.**

Keywords—multilateration; nonlinear least squares; Cramer–Rao lower bound; Levenberg–Marquardt algorithm

I. Statement of the Problem

The problem of multilateration (MLAT) is as follows: at some unknown time instant t the object (aircraft) under observation, which is at location r, transmits a radio signal (for example, it can be a reply of the airborne transponder to the secondary radar request). This signal is received by several stations (there are m stations) with known coordinates $\{r_i\}_{i=1}^{m}$. The receiving station i records the time of arrival (TOA) t_i of this signal, but there is a random measurement error w_i. We can write the following observation equation (c is the speed of light):

$$\begin{cases} t_i = t + \frac{1}{c}\|r - r_i\| + w_i, \\ \quad i = 1,\dots,m. \end{cases} \tag{1}$$

It is necessary to produce an estimate of the object position $\hat{r}$ so that the estimation error would be as less as possible. We assume the estimation error as mean squared error (MSE) $E(\hat{r}-r)^2$.

Note that in the literature (for example, in [1]) the problem of multilateration is often considered for the case when the measurements are not times of arrival (TOA) $\{t_i\}_{i=1}^{m}$, but the time differences of arrival (TDOA) $\{t_i - t_j\}_{i,j\in P}$ for some pairs of receivers P. However, taking into account the features of architecture of the system, where a practical application is possible, we are interested in the statement with measurements of the times of arrival. For this case, the mathematical statement of the multilateration problem almost completely coincides with the statement of the problem of positioning in the global positioning system (GPS). But there are differences. So, in the case of GPS, the unknown variable is not the transmission time t, but the time bias between the receiver clock and the satellite clock. Also, the accuracy of time measurement in the receivers and the typical values of the station coordinates r_i are different. In multilateration, the stations are usually located on the Earth surface, and the observed object is usually an aircraft, which is also near the surface (in comparison with the satellites in the case of GPS). As a consequence, all the vectors $r - r_i$ are close to the plane of the local horizon, which makes the task difficult from the view point of numerical methods, and even makes the solution impossible in some cases. The estimation error along the vertical direction is especially large.

Another feature of multilateration is that there is often no reception of a signal at some station (for example, i) due to the shading of the propagation path by obstacles on the Earth surface. In this case, there is no corresponding measurement t_i. If the number of stations that received the signal is less than 4, it is impossible to make the estimate $\hat{r}$ using the remaining measurements. These peculiarities led us to the idea of combining the measurements obtained from several successive signal transmission instants and making a joint estimate. It would be possible to use all the measurements available from the beginning of the observation, and to perform filtering using the model of the moving object [2]. But such a solution does not look universal; for different aircraft, it would be necessary to use different filters with specific settings. The combination of measurements into small batches and using the simplest assumption of straight line and steady speed motion looks as a more useful and straightforward approach. But this easy method can already reduce the effect of lost measurements and increase the accuracy of estimation.

II. Observation Model. Cramer–Rao Lower Bound

The observation model for the problem with several signal transmission instants is based on equations (1). As before, let us denote by t the time, for which it is necessary to provide the estimate $\hat{r}$. Assume that the instants of signal transmission are $t^j = t + \Delta^j$, the number of them is n. The measurement instants and their random errors are denoted by t_i^j and w_i^j, respectively. The following equations are fulfilled

$$\begin{cases} t_i^j = t + \Delta^j + \frac{1}{c}\left\| r + v\Delta^j - r_i \right\| + w_i^j, \\ \quad i \in I^j, \quad j = 1,\ldots,n. \end{cases} \tag{2}$$

Here, I^j is the set of indices of the stations that have received the signal at the time t^j (they can be not all stations, and at each time the set can change); r and v are the position and velocity of the object at the instant t. Next, we shall consider a special case where $t = t^n$, $\Delta^n = 0$, that is, we estimate the position at the time of the last transmission of the signal.

In the observation model (2), there are $7 + n$ variables: $r, v \in R^3$, $t, \Delta^1, \ldots, \Delta^n \in R^1$. Equality $\Delta^n = 0$ removes one of them, but their number still remains significant. Often, in practice, the differences Δ^j of transmission instants are known (for example, in a case of periodic broadcast of the signal) or they can be easily recovered. Let us consider the case of known differences Δ^j. For sake of brevity, we denote the remaining parameters as a vector $\theta = [r^T \quad v^T \quad t]^T$.

For the observation model (2), it is possible to construct the Cramer–Rao lower bound of accuracy [3], [4], which shows potentially achievable accuracy of unbiased estimators. Let us introduce unit vectors of the direction from the location r_i of the station i to the location $r + v\Delta^j$ (the position of the object at the instant t^j)

$$e_i^j = \frac{r + v\Delta^j - r_i}{\left\| r + v\Delta^j - r_i \right\|}.$$

The density $\rho_t(t_i^j)$ of the distribution of the measurements t_i^j is expressed through the density $\rho_w(\cdot)$ of the random variable w_i^j (we assume that they are independent and identically distributed for all i, j) and such a residual

$$f_i^j(\theta) = t_i^j - t - \Delta^j - \frac{1}{c}\left\| r + v\Delta^j - r_i \right\|, \tag{3}$$

$$\rho_t(t_i^j) = \rho_w(f_i^j(\theta)).$$

The Fisher information matrix has the form

$$I(\theta) = E\left(\frac{\partial \ln f_i^j}{\partial \theta}\right)^T \left(\frac{\partial \ln f_i^j}{\partial \theta}\right) =$$

$$= \frac{\kappa}{c^2} \sum_{j=1}^{n} \sum_{i \in I^j} \begin{bmatrix} e_i^j e_i^{jT} & \Delta^j e_i^j e_i^{jT} & c e_i^j \\ \Delta^j e_i^j e_i^{jT} & (\Delta^j)^2 e_i^j e_i^{jT} & c\Delta^j e_i^j \\ c e_i^{jT} & c\Delta^j e_i^{jT} & c^2 \end{bmatrix} = \frac{\kappa}{c^2} D(\theta), \tag{4}$$

where $\kappa = \int \frac{\dot\rho(w)^2}{\rho(w)} dw$ is a constant depending on the properties of the random measurement errors w_i^j only, and the matrix $D(\theta)$ expresses the geometric properties of the observation problem. The Cramer–Rao inequality [3]

$$E(\hat\theta - \theta)(\hat\theta - \theta)^T \geq I(\theta)^{-1} = \kappa^{-1} D(\theta)^{-1} \tag{5}$$

should be considered in the sense of positive semidefiniteness of the matrices difference. The accuracy of an estimate of θ cannot be better than boundary (5) (in the case of unbiased estimates) [3]. The estimation accuracy of a part of the vector θ (for example, r, which is important in our case) is connected with the corresponding part of the matrix $I(\theta)^{-1}$.

Given the characteristics of the multilateration system (location of the receiving stations r_i, characteristics of the errors w_i^j), it is possible to build the accuracy boundary at each position in the observation zone using formula (5). The vectors e_i^j are almost identical for different j in some reasonable range of v and Δ^j. As a consequence, the dependence of matrix (4) on v is very weak and it can be neglected in calculations.

Figure 1 shows the levels of the accuracy lower bound (according to equation (5)) for the horizontal coordinates are shown. They are calculated for specific locations of the receiving stations on the ground (shown by asterisks) and the normally distributed errors w_i^j with zero mean and standard deviation of 1 μs; the aircraft altitude is 2000 m.

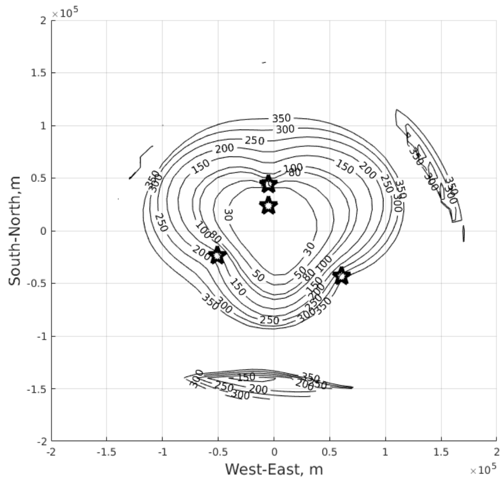

Fig. 1. Levels for the Cramer–Rao lower bound of accuracy (in meters) of estimation for the horizontal coordinates of the aircraft for the specific location of the receiving stations (marked with asterisks)

III. Nonlinear Least Squares Estimate. Minimization of the Functional

The solution of the multilateration problem and the development of estimates of $\hat r$ can be made in different ways, but apparently the most promising is maximum likelihood estimates [1]. In the particular case of normally distributed observation errors w_i^j, such an estimate corresponds to the minimization of mean square of residuals (3)

$$J(\theta)=\sum_{j=1}^{n}\sum_{i\in I^j}(f_i^j(\theta))^2 \qquad (6)$$

This estimate reaches accuracy (5) in the limit (asymptotic efficiency) when the number of measurements increases [3]. Even if the distribution of the errors w_i^j is not normal, the estimate $\hat{\theta}=\arg\min_\theta J(\theta)$ has good properties.

Functional (6) is not convex, so its optimization is complicated. Different scales of variables also make significant difficulties. The gradient descent method used for minimization showed an extremely low rate of convergence, requiring a large number of iterations (about 10^4). And convergence is not always achieved in experiments. In a sufficiently large percentage of cases, the numerical procedure makes fluctuations and "jumps".

Second-order methods based on Newton's method have a very high rate of convergence [4], but their application to the optimization of functional (6) requires regularization. Thus, the classical Newton's method [5] has the form

$$\theta_{k+1}=\theta_k-(\nabla^2 J(\theta_k))^{-1}\nabla J(\theta_k), \qquad (7)$$

where $\nabla^2 J(\theta_k)$ is the matrix of second derivatives calculated at the current approximation point θ_k. However, the main condition of convergence of procedure (7) is positive definiteness of $\nabla^2 J(\theta_k)$, which is not fulfilled in the case of functional (6). To overcome this difficulty Levenberg–Marquardt modification [5] of Newton's method is used, which is constructed as follows. Consider the matrix of the second derivatives

$$\nabla^2 J(\theta)=\sum_{j=1}^{n}\sum_{i\in I^j}\left(\nabla f_i^j(\theta)\nabla f_i^j(\theta)^T+f_i^j(\theta)\nabla^2 f_i^j(\theta)\right).$$

Only the second term in the brackets is responsible for the violation of positive definiteness of $\nabla^2 J(\theta_k)$. But near the optimal point it cannot be large, since the values of the residuals f_i^j are small. Therefore, we can make an approximate matrix Q_λ for which positive definiteness is true

$$Q_\lambda=\sum_{j=1}^{n}\sum_{i\in I^j}\left(\nabla f_i^j(\theta)\nabla f_i^j(\theta)^T+\lambda I\right)\ge\nabla^2 J(\theta),\quad Q_\lambda>0,$$

and which can be used instead of $\nabla^2 J(\theta_k)$ in (7):

$$\theta_{k+1}=\theta_k-Q_\lambda^{-1}\nabla J(\theta_k). \qquad (8)$$

Method (8), directly applied to the optimization, shows a good rate of convergence with the constant λ of the order of 10^{-3} in some experiments, but often stops far from the minimum of functional (6). This situation has been overcome by dynamic changing the constant of regularization λ. If the functional J ceases decreasing, but the value of J remains large, then the constant increases $\lambda := 2\lambda$. If the step length is small, but there is a stable decrease of the functional on several iterations, an attempt is made, on the contrary, to decrease λ.

With such a modification, the Levenberg–Marquardt algorithm has shown a good performance on simulated data in terms of recovery of the horizontal components of the coordinates of r. In Fig. 2, the levels of accuracy of the method are shown for the same configuration of the receiving stations as in Fig. 1. The empirical standard deviation $\hat{\sigma}(r)=\left(\frac{1}{N}\sum_{k=1}^{N}(\hat{r}-r)^2\right)^{1/2}$ over N realizations is taken as the accuracy estimate. The comparison of Figs. 2 and 1 shows that the accuracy of the estimate obtained by minimizing the functional (6) using the method (8) is comparable to the Cramer-Rao lower bound of accuracy.

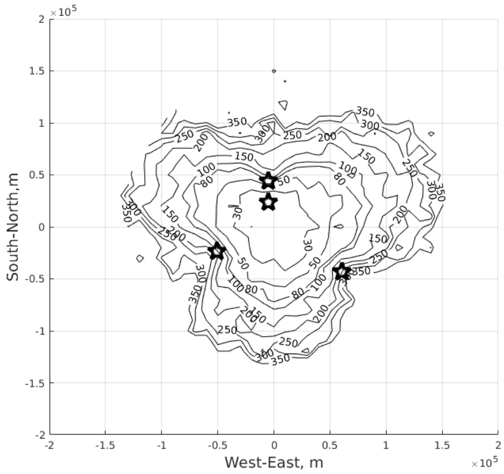

Fig. 2. Levels for the root mean squared deviation of estimation for the horizontal coordinates of the aircraft for the specific location of the receiving stations (marked with asterisks)

ACKNOWLEDGMENT

This research was supported by the Presidium of the Russian Academy of Sciences, Program no. 30 “Theory and Technologies of Multi-level Decentralized Group Control under Confrontation and Cooperation”.

We are thankful to NITA, LLC (“New Information Technologies in Aviation”, St. Petersburg, Russia) for the formulation of the problem, providing the data, and fruitful discussions about the results.

REFERENCES

[1] I.A. Mantilla-Gaviria, M. Leonardi, G. Galati, J.V. Balbastre-Tejedor, Localization algorithms for multilateration (MLAT) systems in airport surface surveillance // Signal Image and Video Processing, 2015. Vol. 9. P. 1549–1558.

[2] Y. Bar-Shalom, X.R. Li, T. Kirubarajan, Estimation with applications to tracking and navigation: Theory Algorithms and Software. John Wiley & Sons, 2004.

[3] A.A. Borovkov, Matematicheskaya statistika (Statistics). Moscow: Nauka, 1984.

[4] V. Chernyak, Using potential accuracy of object localisation with multilateration systems // Proceedings of ESAV'08, September 3–5, Capri, Italy.

[5] N.S. Bakhvalov, N.P. Zhidkov, G.M. Kobel’kov, Chislennye metody (Numerical methods), Moscow: Nauka, 1987.