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AIRCRAFT LANDING CONTROL IN THE PRESENCE OF WINDSHEAR

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Traditional laws of aircraft control in the landing operate unsatisfactorily when rapid wind changes occur. For this reason, new ways of aircraft landing control are investigated recently [1–6]. This paper deals with the minimax approach based on the differential game theory methods [7, 8]. Applications of differential game theory to the landing problem were considered in [1–3, 9–13].

1. Introduction

There are many papers [3–6, 9, 14, 15] devoted to the investigation of the aircraft motion during take-off and landing with rapid change of wind velocity (windshear). Physical conditions leading to windshear, its mathematical models and aircraft control are analyzed.

This paper deals with the landing problem for middle-sized aircraft in the conditions of wind disturbance. We consider the aircraft motion along the glide path till the moment of passing the runway (RW) threshold. The information we get about the wind is supposed to be minimal. Namely, we suppose that only the deviation boundary of the wind velocity from its nominal value and the nominal value are known. Any other information about the windshear zone and the internal distribution of wind velocity is absent. So, the problem that appears naturally is to find the minimax closed-loop control which can cope with arbitrary variation of the wind velocity in noted boundaries.

Using methods of the antagonistic differential game theory [7, 8] we obtain the minimax solution for auxiliary linear problems. Further, this solution is applied for computer simulation of the motions in a complete nonlinear system. Simulation results deal with the case when the windshear is stipulated by aircraft flight through microburst zone. The microburst is caused by falling mass of air which hits the ground and gives vortex. The mathematical model of the microburst is taken from [14].

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2. Nonlinear system of aircraft landing motion

The aircraft motion during landing is described by a differential equations system of 12th order. The state vector includes three coordinates x, y, z of mass center in the coordinate system connected with the RW surface (Fig. 1), angles of pitch ϑ, yaw ψ, bank γ and corresponding linear and angular velocities. These equations are specified, for example, in [16, 17].

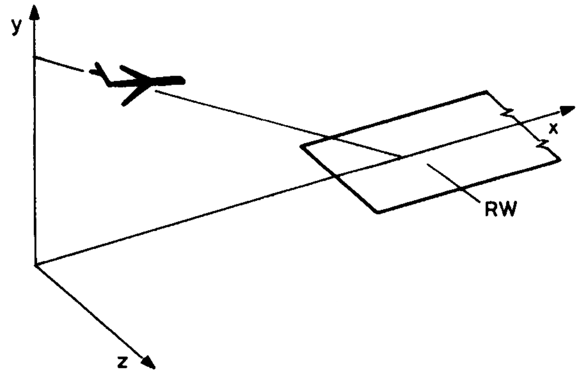

Fig. 1. Coordinate system

The control factors are deviations of the elevator δ_e, the rudder δ_r, the ailerones δ_a and change of thrust force P. Equations of servo and engine dynamics are supplemented to the main system of aircraft motion. So, the noted factors enter in the broadened state vector and new control parameters are the scheduled (commanded) deviations δ_{es}, δ_{rs}, δ_{as}, δ_{ps}. Every parameter has upper and lower restrictions. As a result we get the complete system of differential equations which we write down in vector form as

$$\dot{\xi}=f(\xi,\,\delta_s,\,W)\,. \tag{2.1}$$

Here $\delta_s=(\delta_{es},\,\delta_{ps},\,\delta_{rs},\,\delta_{as})^T$ is the control vector, $W=(W_x,\,W_y,\,W_z)^T$ is the disturbance vector, consisting of three wind components along the x, y, z axes.

3. Minimax control law

Nominal aircraft motion during landing till the moment of passing RW threshold is a uniform motion (without rotation) along the descending rectilinear glide path.

Control problem is to bring real motion near enough to the nominal motion in the presence of wind disturbance. It is also desirable that the performance of the control law would not demand any accurate and detailed information about the wind. We assume that for the deviations of wind velocity components W_x, W_y, W_z from the nominal values W_{x0}, W_{y0}, W_{z0} only approximate characteristics of restrictions are known. The nominal values are assumed to be known as well. To solve the problem we apply minimax approach using the methods of differential games.

Effective computer programs have been created [9, 12, 18–20] to find optimal control laws (strategies) in linear differential games with fixed terminal moment and convex payment function, depending on two coordinates of the state vector. System (2.1) is nonlinear. However, we can linearize it, solve auxiliary differential games and use the results to the initial nonlinear system. So, having the nominal values W_{x0}, W_{y0} and W_{z0}, the glide path inclination, the nominal relative velocity, we calculate values of the state variables, corresponding to the nominal motion of system (2.1). The linearization of system (2.1) with respect to the nominal motion gives a linear controllable system, desintegrating to two subsystems of vertical (longitudinal) and lateral motions. The state vector of the vertical motion (VM) subsystem consists of deviations Δx, Δy and some quantities which determine these deviations. The state vector of lateral motion (LM) subsystem consists of deviation Δz and quantities which determine Δz.

For each of the subsystems we consider an auxiliary differential game with fixed terminal time T, geometric restrictions on control variables and wind disturbances and with convex payment function depending on two state vector coordinate at the moment T. In the VM subsystem such coordinates are Δy and $\Delta \dot{y}$, in the LM system these coordinates are Δz and $\Delta \dot{z}$. The first player chooses the control variables to minimize the payment function. The second player, choosing the wind disturbances, maximizes the payment function. It is not necessary to give the moment T any physical meaning in auxiliary problems.

Variables δ_{ps} and δ_{rs} in system (2.1) are intended for the relative velocity and sideslip angle stabilization near the nominal values. So, it is not natural to find closed-loop laws for δ_{ps} and δ_{rs} using the solution of the auxiliary problems mentioned above. Therefore, formulating the auxiliary problems, we assume that the thrust force is a constant and is equal to its nominal value (i.e. $\Delta\delta_{ps} \equiv 0$) and the variation of $\Delta\delta_{rs}$ satisfies a linear differential equation corresponding to traditional rudder control. With that we omit the restriction on $\Delta\delta_{rs}$. As a result we obtain the only control factor $\Delta\delta_{es}$ in the VM subsystem and the control factor $\Delta\delta_{as}$ in the LM subsystem.

To take into account the inertial character of wind velocity variations along the motion, we suppose that variables ΔW_x, ΔW_y, ΔW_z satisfy additional linear differential equations, for example,

$$\Delta \dot{W}_x = k_1(\Delta F_x - \Delta W_x)$$

$$\Delta \dot{F}_x = k_2(w_x - \Delta F_x). \tag{3.1}$$

Here w_x is a new independent variable, constants k_1 and k_2 determine the inertial character of ΔW_x. Similar equations are considered for ΔW_y and ΔW_z. Variables w_x, w_y, w_z are interpreted as new disturbance factors. We add the equations for ΔW_x and ΔW_y to the VM subsystem and the equations for ΔW_z to the LM subsystem.

Solving the auxiliary problems on computer, we find optimal laws for $\Delta\delta_{es}$ and $\Delta\delta_{as}$. These laws are realized by means of sets K_{es} and K_{as} of switch lines [9, 13, 18, 19]. Both sets are defined on a collection of moments τ_i of reverse time counting off the terminal moment T. The sets K_{es} and K_{as} give the desired control laws for components δ_{es} and δ_{as} to initial system (2.1). Using these laws we prognose the time remained till the moment of passing the RW threshold. Depending on the time prognose, certain switch lines are used to choose values δ_{es} and δ_{as}. Simulating motions of system (2.1), we assume that the control factors δ_{ps} and δ_{rs} are obtained by means of control laws accepted nowadays.

So, speaking about the minimax control, we mean the way of finding control factors δ_{es} and δ_{as} from the auxiliary linear differential games. The factors δ_{ps} and δ_{rs} are constructed by traditional methods.

4. The auxiliary linear differential games

The linear VM system is

$$\dot{\mathbf{x}} = A_*\mathbf{x} + B_*u + C_*v, \qquad \mathbf{x} \in R^{11}, \tag{4.1}$$

$$A_* = \left[\begin{array}{cccccccccccc}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.050 & 0 & -0.097 & -2.642 & 0 & 0.063 & 0.050 & 0 & 0.097 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.241 & 0 & -0.639 & 45.278 & 0 & 1.448 & -0.241 & 0 & 0.638 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.007 & -0.501 & -0.526 & -0.383 & 0 & 0 & -0.007 & 0 \\
 & & & & & & -4 & 0 & 0 & 0 & 0 \\
\multicolumn{6}{c}{} & 0 & -0.5 & 0.50 & 0 & 0 \\
\multicolumn{6}{c}{0} & 0 & 0 & -3 & 0 & 0 \\
 & & & & & & 0 & 0 & 0 & -0.5 & 0.5 \\
 & & & & & & 0 & 0 & 0 & 0 & -3
\end{array}\right]$$

$$B_* = (0, 0, 0, 0, 0, 0, 4, 0, 0, 0, 0)^T, \qquad C_* = \begin{pmatrix} 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0 \\ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3 \end{pmatrix}^T,$$

$$\mathbf{x} = (x_1, x_2, \ldots, x_{11})^T, \qquad u = \Delta\delta_{es}, \qquad v = (w_x, w_y)^T.$$

Here $x_1 = \Delta x$, $x_3 = \Delta y$ are deviations from nominal motion in x and y, respectively; $x_5 = \Delta \vartheta$ is a deviation of pitch angle. The coordinate x_7 is a deviation of the elevator from its trim position. By means of variables x_8, x_9 we describe a variation of ΔW_x. The corresponding equations coincide with (3.1), $x_8 = \Delta W_x$. Similarly, variables x_{10}, x_{11} are used for description of ΔW_y variation ($x_{10} = \Delta W_y$). The control factor of the first player is the scheduled deviation $\Delta\delta_{es}$ of the elevator. The parameters w_x, w_y are used to obtain wind disturbances and belong to the second player.

The restrictions are the following:

$$|\Delta\delta_{es}| \leqq 10 \deg \frac{\pi}{180}, \qquad |w_x| \leqq 10 \text{ m sec}^{-1}, \qquad |w_y| \leqq 5 \text{ m sec}^{-1}.$$

Introduce a function φ_* which depends on coordinates $x_3 = \Delta y$ and $x_4 = \Delta \dot{y}$. Let M_* be a convex hexagon on the plane x_3, x_4 with apexes $(-3, 0)$, $(-3, 1)$, $(0, 1)$, $(3, 0)$, $(3, -1)$, $(0, -1)$. Suppose

$$\varphi_*(x_3, x_4) = \min \{c \geqq 0 : (x_3, x_4)^T \in cM_*\}.$$

Consider an antagonistic differential game with dynamics (4.1), fixed terminal moment T and payment φ_*. The first player tries to minimize values of the function φ_* at the moment T. The aim of the second player is opposite. The set M_* can be considered as a tolerance for deviations $x_3 = \Delta y$ and $x_4 = \Delta \dot{y}$ at the moment T. The function φ_* indicates a deviation from the tolerance. The optimal strategy of the first player in game (4.1) will be used to define δ_{es} in system (2.1).

The linear LM system is

$$\dot{\mathbf{x}} = A^*\mathbf{x} + B^*u + C^*v, \qquad \mathbf{x} \in R^{11}, \tag{4.2}$$

$$A^* = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.077 & -5.555 & 0 & 9.272 & 0 & -1.485 & 0 & 0.077 & 0 & 0 \\
0 & 0 & 0 & 1.001 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.013 & -0.933 & -0.259 & -0.088 & -0.030 & -0.246 & -0.046 & 0.012 & 0 & 0 \\
0 & 0 & 0 & -0.051 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.033 & -2.386 & -0.953 & -0.226 & -1.459 & -0.233 & -0.689 & 0.033 & 0 & 0 \\
 & & & & & & -4 & 0 & 0 & 0 & 4 \\
 & & & & & & 0 & -4 & 0 & 0 & 0 \\
 & & & 0 & & & 0 & 0 & -0.5 & 0.5 & 0 \\
 & & & & & & 0 & 0 & 0 & -3 & 0 \\
0 & -0.058 & -4.202 & -0.365 & -0.397 & -0.136 & -1.105 & -0.207 & 0.058 & 0 & -0.4
\end{bmatrix}$$

$$B^* = (0, 0, 0, 0, 0, 0, 0, 4, 0, 0, 0)^T, \qquad C^* = (0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0)^T,$$

$$\mathbf{x} = (x_1, x_2, \ldots, x_{11})^T, \qquad u = \Delta\delta_{as}, \qquad v = w_z.$$

Here $x_1 = \Delta z$ is a deviation in z from the nominal motion; $x_3 = \Delta\psi$ and $x_5 = \Delta\gamma$ are deviations of yaw and bank angles. The coordinates x_7, x_8 are deviations of the rudder and ailerons ($\Delta\delta_r$ and $\Delta\delta_a$); $x_{11} = \Delta\delta_{rs}$. By means of variables x_9, x_{10} we describe variation of ΔW_z $(x_9 = \Delta W_z)$. The control factor of the first player is the scheduled deviation $\Delta\delta_{as}$ of the ailerons. The parameter w_z is used to obtain wind disturbance and belongs to the second player.

Restrictions are

$$|\Delta\delta_{as}| \leqq 10 \deg \frac{\pi}{180}, \qquad |w_z| \leqq 10 \text{ m sec}^{-1}.$$

Introduce a function φ^* depending on coordinates $x_1 = \Delta z$ and $x_2 = \Delta\dot{z}$. Let M^* be a convex hexagon on the plane x_1, x_2 with apexes $(-6, 0)$, $(-6, 1.5)$, $(0, 1.5)$, $(6, 0)$, $(6, -1.5)$, $(0, -1.5)$. Suppose

$$\varphi^*(x_1, x_2) = \min\{c \geqq 0 : (x_1, x_2)^T \in cM^*\}.$$

Consider an antagonistic differential game with dynamics (4.2), fixed terminal moment T and payment φ^*. The first player endeavours to minimize values of the function φ^* at the moment T. The aim of the second player is opposite. The set M^* can be considered as a tolerance for deviations $x_1 = \Delta z$, $x_2 = \Delta\dot{z}$ at the moment T. The function φ^* indicates a deviation from the tolerance. The optimal strategy of the first player in game (4.2) will be used to define δ_{as} in system (2.1).

In systems (4.1) and (4.2) the dimension of linear variables is the meter. Angles are measured in radians and time is in seconds.

For the calculation of coefficients in systems (4.1) and (4.2) we used the following data: the glide path inclination $\Theta = -2.66$ deg, the nominal relative velocity $\tilde{V}_0 = 72.2 \text{ m sec}^{-1}$, the nominal wind components $W_{x0} = -5 \text{ m sec}^{-1}$, $W_{y0} = W_{z0} = 0$.

5. Optimal first player strategy in the linear differential game

The main features characteristic to differential games (4.1), (4.2) are the following. Each of the games has a fixed stopping instant and a convex payment function which depends on two coordinates of the phase vector. Besides, the control factor of the first player is scalar. These features simplify the specifying of the optimal first player strategy. The strategy is realized by means of the switch surface in the space t, y_1, y_2 of equivalent [7, 8] second-order game. The relation between vectors $\mathbf{y} = (y_1, y_2)^T$ and $\mathbf{x}$ is described by the formula $\mathbf{y}(t) = X_*(T, t)\mathbf{x}(t)$ $(\mathbf{y}(t) = X^*(T, t)\mathbf{x}(t))$ where $X_*(T, t)$ $(X^*(T, t))$ is a matrix composed of the third and fourth (the first and second) rows of the fundamental Cauchy matrix for the homogeneous part of system (4.1) ((4.2)).

On one side from switch surface the optimal control takes an extremal value of one sign, on the other side the optimal value is opposite. The mathematical proof of the optimality of such a control law (by means of switch surface) and the analysis of its stability are given in [18, 19]. Corresponding computational procedures are stated in [9].

The switch surface is realized on computer as a set of its sections on the given collection of the time moments. These sections are called the switch lines. The switch lines $\Pi_*(\tau)$ for problem (4.1) are shown in Fig. 2. The lines have been built for the reverse time moments $\tau = 7$, 11, 15. Let $\mathbf{x}(t_i)$ be a state of system (4.1) at a moment t_i. If the point $\mathbf{y}(t_i) = X_*(T, t_i)\mathbf{x}(t_i)$ lies in the direction of vector $D_*(t_i) = X_*(T, t_i)B_*$ with respect to the switch line $\Pi_*(\tau_i)$, corresponding to the moment $\tau_i = T - t_i$, then $\Delta\delta_{es} = -10$ on the next step of the discrete scheme of control. We put $\Delta\delta_{es} = +10$ in the opposite situation of the point $\mathbf{y}(t_i)$ with respect to the line $\Pi_*(\tau_i)$. Similarly, the choice of optimal control factor $\Delta\delta_{as}$ in system (4.2) is made with the help of switch lines $\Pi^*(\tau)$ and vector $D^*(t) = X^*(T, t)B^*$.

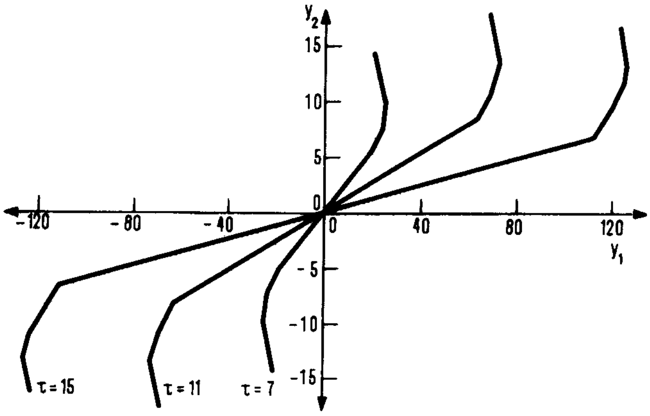

Fig. 2. Switch lines

6. Microburst model

Simulating system (2.1) motions, we suppose that the wind disturbance is caused by the aircraft flight through the microburst zone. The microburst model we used has been taken from [14]. Below we give an outline of this model.

The microburst is idealized as a three-dimensional axially symmetric vortex field. In this field we distinguish the thoroidal region ("core") where the wind velocity, being zero in the center, increases linearly along the radius to the frontier of the core. Outside the core the vortex field is determined by the stream function. Differentiation of this function gives radial and vertical components of the wind velocity. The radial one is resolved into two components, the first of which is parallel and the second is orthogonal to the RW axis. The microburst is given by three parameters: $\mathscr{V}$ is the modulus of the wind velocity vector in the central microburst part, $\mathscr{H}$ is the altitude of the central part, and $\mathscr{R}$ is the radius of the vortex. The core radius is equal to 0.8 $\mathscr{H}$. The disposition of the microburst with respect to the glide path is determined by two coordinates of its center in the horizontal plane.

The microburst we use for simulation has the following parameters: $\mathscr{V} = 6 \text{ m sec}^{-1}$, $\mathscr{H} = 700$ m, $\mathscr{R} = 1200$ m. A computed picture of the wind velocity field in the vertical plane, passing through the microburst center, is shown in Fig. 3.

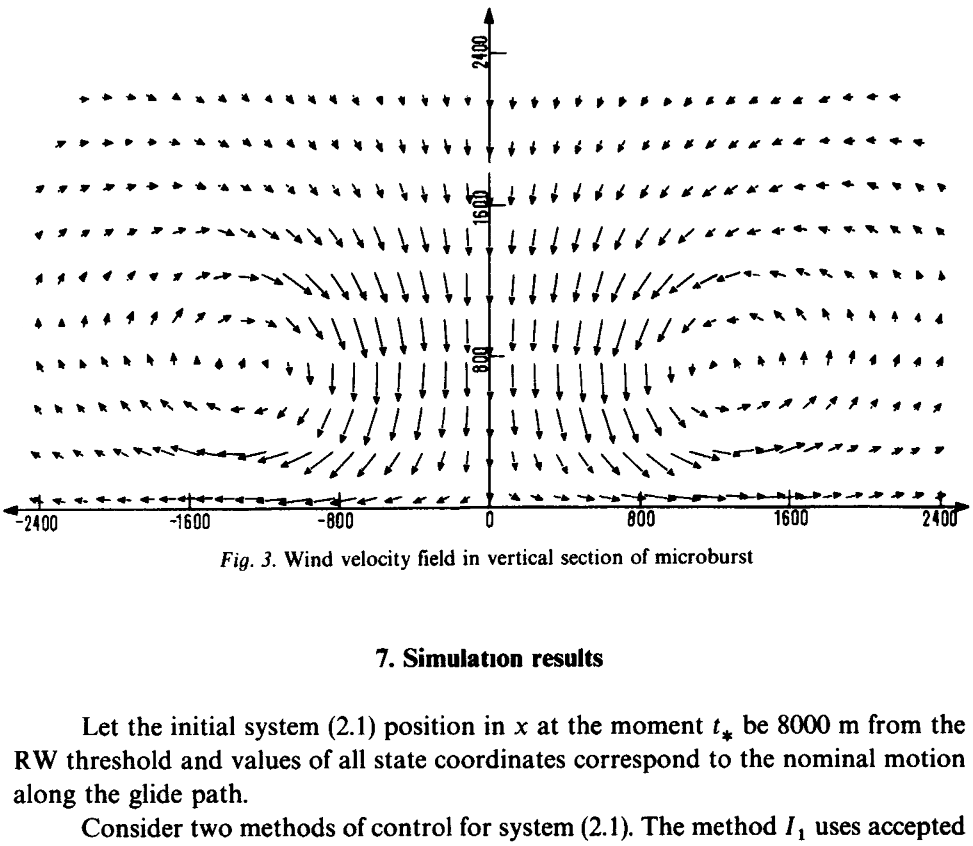

Fig. 3. Wind velocity field in vertical section of microburst

7. Simulation results

Let the initial system (2.1) position in x at the moment t_* be 8000 m from the RW threshold and values of all state coordinates correspond to the nominal motion along the glide path.

Consider two methods of control for system (2.1). The method I_1 uses accepted nowadays autopilot algorithms for constructing δ_{ps}, δ_{es}, δ_{rs} and δ_{as}. The second method I_2 is the minimax law. In this method factors δ_{es}, δ_{as} are constructed by means of switch lines obtained from auxiliary differential games (4.1), (4.2). Factors δ_{ps}, δ_{rs} are constructed with the help of accepted algorithms. Let $T = 15$ sec for problems (4.1), (4.2).

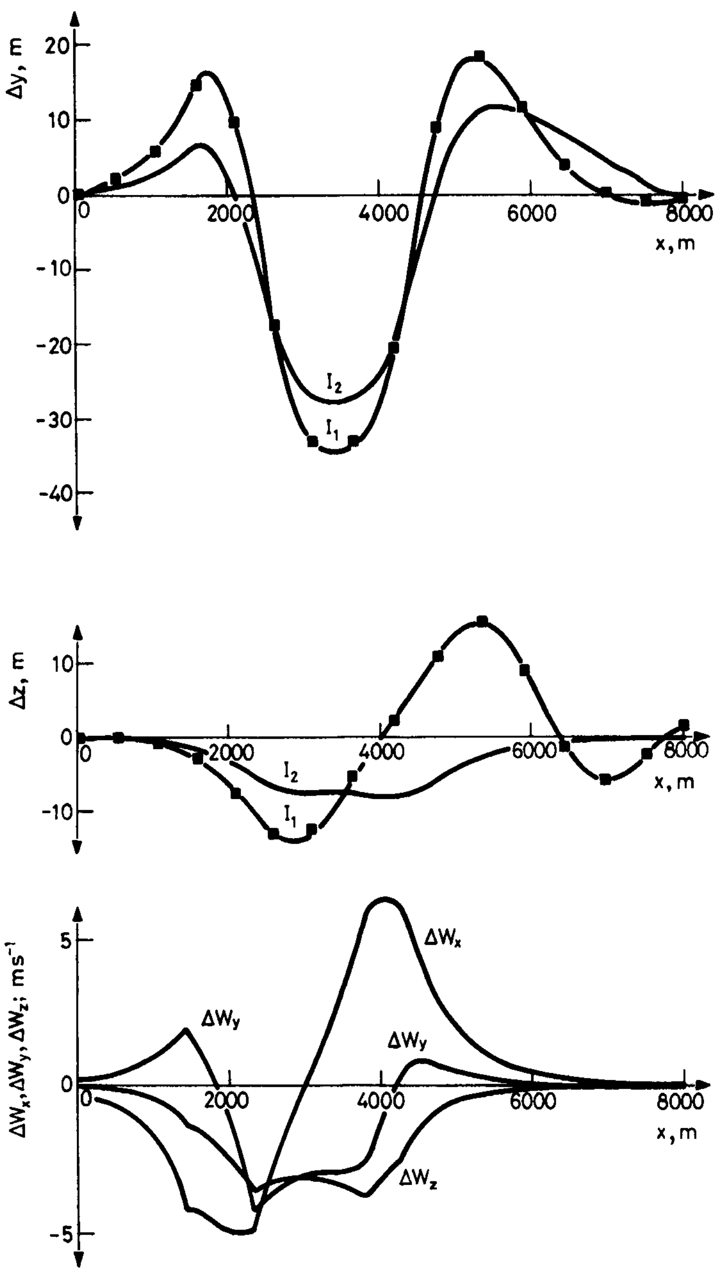

Fig. 4. Landing simulation results. Microburst center coordinates: $DX = 3000$ m, $DZ = 500$ m

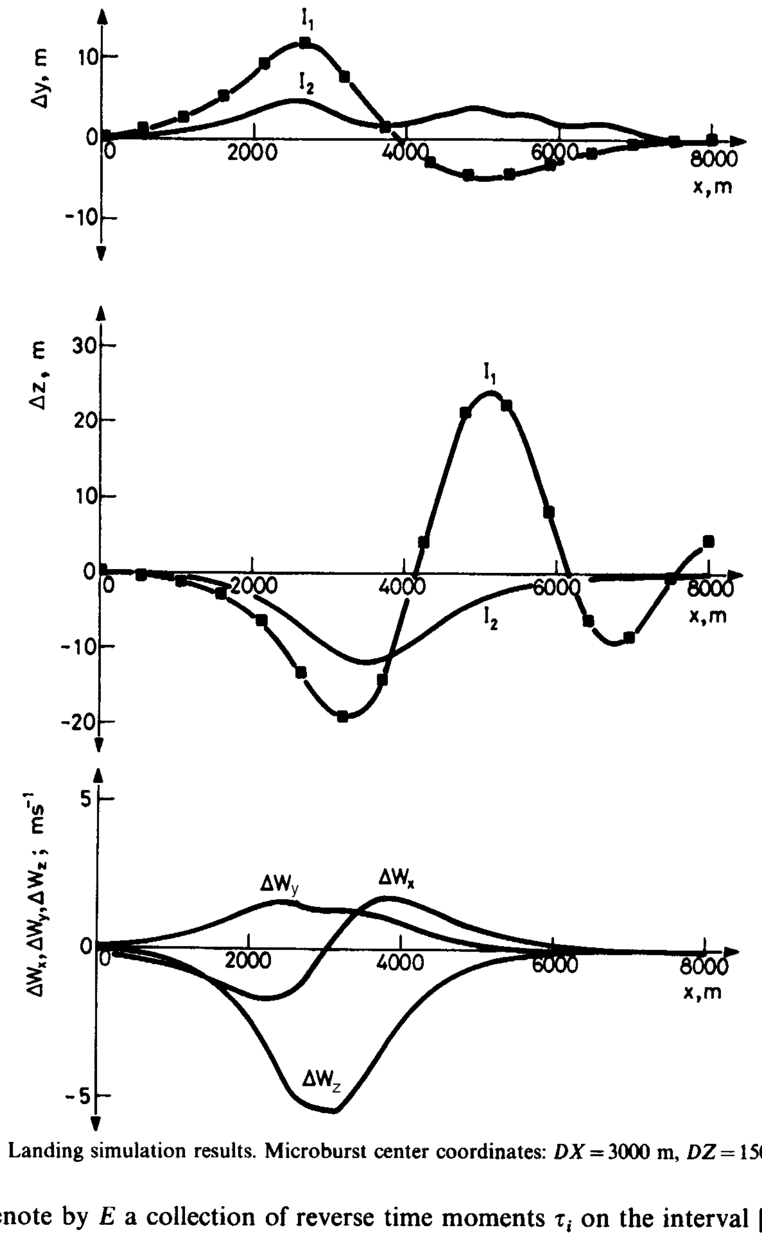

Fig. 5. Landing simulation results. Microburst center coordinates: $DX = 3000$ m, $DZ = 1500$ m

Denote by E a collection of reverse time moments τ_i on the interval $[0, T] = [0, 15]$. We suppose the switch lines have been built for every $\tau_i \in E$. In the method I_2 the switch lines are used in the following way. Let $d(t)$ be the distance in x up to the RW threshold at a moment $t \geqq t_*$ and V_{x0} be the nominal motion velocity in x. Then $s(t) = d(t)/V_{x0}$ is the prognose time remained till passing the RW threshold. For obtaining $\delta_{es}(\delta_{as})$, if $s(t) \geqq T = 15$ sec, we use the same switch line corresponding to $\tau = T$. If $s(t) < T$, we use the line corresponding to the moment $\tau_i \in E$ nearest to $s(t)$. So, our control is comparatively rough while $d(t) \geqq V_{x0} T \approx 1000$ m. If $d(t) < V_{x0} T$, the control is more qualitative.

In system (2.1) wind velocity components W_x, W_y, W_z are calculated by formulas $W_x = \Delta W_x + W_{x0}$, $W_y = \Delta W_y + W_{y0}$, $W_z = \Delta W_z + W_{z0}$ where ΔW_x, ΔW_y, ΔW_z are taken from the microburst model; $W_{x0} = -5$ m sec^{-1}, $W_{y0} = W_{z0} = 0$. Consider two variants of the microburst center disposition in the horizontal plane: 1) the displacement DX in x from the initial aircraft position is 3000 m (or 5000 m from the RW threshold), the displacement DZ in z orthogonally to the RW axis is 500 m, 2) $DX = 3000$ m, $DZ = 1500$ m.

Simulation results for the control methods I_1, I_2 are shown in Figs 4 and 5. We give graphs of vertical Δy and lateral Δz deviations from the nominal motion and also realizations of deviations ΔW_x, ΔW_y, ΔW_z. The last curves correspond to the control method I_2. Realizations ΔW_x, ΔW_y, ΔW_z for the method I_1 are practically the same. For all the graphs the horizontal axis is the distance passed in x. Figure 4 (5) corresponds to the first (second) variant of microburst center disposition. The time discrete for computing of control factors and wind disturbances was equal to 0.1 sec.

It can be seen that the results for the minimax method I_2 are better than for the traditional method I_1.

8. Concluding remark

In conclusion we emphasize once more that the computation of the minimax control method demands neither accurate information about the disposition of the extremal wind disturbance zone nor any information about the wind velocity field in that zone. It is enough to describe an approximate amplitude of the wind velocity variation. This is the principal difference of the approach, based on the differential game theory, from the methods, given in [4-6], where such information is essential.

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Управление самолетом на посадке при сдвиге ветра

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Рассматривается задача управления средним транспортным самолетом на посадке в условиях резкого изменения скорости ветра (сдвиг ветра). Процесс посадки исследуется до момента пролета торца взлетно-посадочной полосы. Предполагается, что относительно ветра известны ориентировочно лишь пределы возможных отклонений его скорости от некоторого номинального постоянного значения и само это значение. Какая-либо информация о пространственном расположении зоны сдвига ветра, как и сведения о поле скорости ветра в ней, считаются отсутствующими.

Для решения задачи об управлении привлекаются методы теории позиционных дифференциальных игр [7, 8]. Исходная нелинейная система дифференциальных уравнений линеаризуется относительно номинального движения по прямолинейной глиссаде снижения. Полученнная в результате линейная система распадается на подсистему движения в вертикальной плоскости (продольное движение) и подсистему бокового движения. Для каждой из подсистем ставится вспомогательная линейная дифференциальная игра с фиксированным моментом окончания и выпуклой терминальной функцией платы, зависящей от двух координат фазового вектора. За такие координаты в подсистеме продольного движения берутся отклонение по высоте и его скорость, в подсистеме бокового движения — боковое отклонение и его скорость. Первый игрок выбором

управления минимизирует значения функции платы, интересы второго, распоряжающегося ветровой помехой, противоположны.

Указанные линейные дифференциальные игры поддаются решению на ЭВМ при помощи разработанных в настоящее время численных методов [9, 13, 18–20]. Элементом решения, в частности, является оптимальная (минимаксная) стратегия первого игрока. Оптимальное управление при этом определяется набором линий переключения, имеющих несложную структуру.

Минимаксный способ управления, полученный в рамках вспомогателных задач, используется затем в полной нелинейной системе. Приводимые в статье результаты моделирования процесса посадки относятся к случаю, когда ветровое возмущение обусловлено прохождением самолета через зону микровзрыва. Микровзрыв представляет собой нисходящий поток воздуха, ударяющийся о поверхность земли и растекающийся затем с образованием вихря. Математическая модель микровзрыва заимствована из работы [14].

При моделировании минимаксный способ управления сравнивается с традиционным, принятым в настоящее время. В целом результаты моделирования для минимаксного способа существенно лучше.

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