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ROBUST CONTROL IN A PROBLEM OF AIRCRAFT LANDING UNDER WIND DISTURBANCES [1]

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Abstract: Currently, methods for solving antagonistic differential games with geometric constraints for players' controls are well developed. In the paper, these methods are used to construct a feedback robust control in situations when a geometric constraint is given *a priori* for the minimizing player's control (the useful control) only and there is no any constraint for the disturbance. The suggested method of robust control is applied to a problem of aircraft landing under wind disturbances.

Keywords: differential games, robust control, aircraft landing, wind disturbance

1. INTRODUCTION

During the last 20 years, there were a lot of works concerning application of modern methods of the control theory and differential game theory to problems of aircraft landing and take-off under wind disturbances (see, for example, (Miele *et al.*, 1988; Leitmann and Pandey, 1991) and references therein).

In some possible formulation of aircraft landing problem, the result is computed at the instant of passing the runway threshold (Kein, 1985; Patsko *et al.*, 1994). At the stage of descent until crossing the runway threshold, the aircraft moves along a rectilinear glide path. Deviation from it is not too large, so for mathematical investigations a linearization of the dynamics is reasonable. Control laws obtained in the framework of linearized dynamics can be tested in the original nonlinear system.

The time interval of descent from the altitude 60 m until passing the runway threshold at the altitude 15 m is very important. Duration of this stage for a midsize transport aircraft is 15 sec. Any errors in control at this stage are unacceptable. So, it is natural to use approaches of the differential game theory when creating methods for automatic control. In the framework of these approaches, the optimal control of the minimizing player (the autopilot) is of guaranteeing type. At the instant of passing the runway threshold, a tolerance on deviation of the most important phase variables of the aircraft from the nominal values can be defined. After that we can consider a differential game where the objective of the useful control is to lead the system to a terminal set constructed on the basis of the tolerance. Since the aircraft velocity along the glide path is quite large in comparison with its deviation from the nominal

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value due to wind disturbances, it is reasonable to compute the result of entering to the tolerance not at the true instant of passing the threshold, but at the fixed instant $\theta = 15\,\mathrm{sec}$.

The scheme mentioned above was realized in (Patsko *et al.*, 1994). After linearization of the original nonlinear system, the resulting dynamics is disjoined into two independent subsystems of lateral and vertical motions. For both of them, an auxiliary linear differential game with fixed terminal time was considered. The terminal set for lateral (vertical) channel was built as a convex set defined in the plane of two coordinates: lateral deviation and its velocity (vertical deviation and its velocity). Some constraints for the useful controls were taken. The controls for the lateral channel are the scheduled aileron and rudder angles, and for the vertical channel they are the thrust deviation from the nominal value and the scheduled elevator angle. On the basis of terminal sets, payoff functions were constructed as Minkowskii functions. To apply methods for solving differential games worked out earlier, it is necessary also to define some geometric constraint for instantaneous value of wind disturbance. The obtained control laws were tested by modelling under different types of wind disturbance including the case of various types of windshear.

A vulnerable point of the described scheme of differential game theory approach is in the strict demand for definition of a constraint for wind disturbance. In engineering practice, defining such a constraint is quite problematic. At the same time, the obtained control depends essentially on the taken constraint. Another defect is that the realization of useful control involves extremal values even if wind disturbance is weak. This is specifics of antagonistic differential games: the optimal control law is generated taking into account the worst disturbance.

In this work, an attempt is made to overcome the mentioned drawbacks. A method for constructing a feedback control for game problems with linear dynamics and fixed terminal time is suggested. The first player (the minimizer) tries to lead the system to a given terminal set M at the terminal instant θ. The components u_i, $i = \overline{1,p}$, of the first player's control u are bounded by independent constraints $|u_i| \leqslant \mu_i$. No constraint for the second player's control (the disturbance) is given *a priori*. For initial states of the system close to the origin the feedback control to be constructed (we call it *robust*) has the following properties:

- if the second player applies a "low level" control, the first player leads the system to the terminal set closely to its center at the terminal instant. Moreover, the realization of the first player's control should be also of a "low level";
- if the second player's control is "stronger", the first player should still lead the system to the terminal set, maybe, by a "stronger" or even maximal control;
- in the case, when the second player involves "very strong" control and the first player (acting within the framework of his constraint) cannot guarantee reaching the terminal set, he may allow some terminal miss, but tries to minimize it.

The basis for construction of the robust control is in introduction of a family of inserted *stable bridges* (stable tubes) in the space *time* $\times$ *phase variable*. Each stable bridge corresponds to some constraint for the second player's control. The property of stability means (Krasovskii and Subbotin, 1988) that the first player guarantees keeping the system inside a tube if the second player applies a control from the constraint, which corresponds to this tube. The family of constraining sets (and, therefore, the family of the tubes) is parametrized by nonnegative real numbers.

In the work (Ganebny *et al.*, 2005), an approach for constructing such a system of stable tubes has been suggested. One of the main questions is whether we should keep the whole family of tubes (in some time grid from the game interval) to generate the feedback control in real time. In (Ganebny *et al.*, 2005), the case was considered when the first player's control u is scalar and bounded by the inequality $|u| \leqslant \mu$. In such a situation, one can store only one bridge (called the main bridge) and some switching surface changing in time. On the one side of this surface, the control has the plus sign, on the other side, the control has the minus sign. The absolute value of the control is computed on the basis of location of the current state of the system with respect to the corresponding time section of the main bridge.

In this paper, we expand this method for the case of vector control u, which components are independently bounded by the inequalities $|u_i| \leqslant \mu_i$.

The paper has the following structure. Section 2 describes the method for constructing a robust control for a linear game problem. In Section 3, the original nonlinear system of landing aircraft motion is discussed shortly. Also, two linearized systems of motions in the lateral and vertical channels are given. They are used in construction of the robust control. The results of simulation of the original nonlinear system with the robust control taken from the auxiliary problems are shown in Section 4. During simulation, the wind disturbance was generated on the basis of a windshear model (Ivan, 1985). The paper is completed by a short Conclusion.

2. ROBUST FEEDBACK CONTROL

Consider a linear differential game with fixed terminal time:

$$\begin{aligned}&\dot{\mathbf{x}} = \mathbf{A}(t)\mathbf{x} + \mathbf{B}(t)u + \mathbf{C}(t)v,\\ &\mathbf{x} \in R^m,\ t \in T,\ v \in R^q,\\ &u \in P = \{u \in R^p \colon |u_i| \leqslant \mu_i,\ i = \overline{1,p}\}.\end{aligned} \tag{1}$$

Here, P is the constraint for the first player's control u, $T = [\theta_0, \theta]$ is the time interval of the game. The matrix-functions $\mathbf{A}$ and $\mathbf{C}$ are continuous. The matrix-function $\mathbf{B}$ is Lipschitzian in the time interval T. Unlike the standard formulation (Krasovskii and Subbotin, 1988) of a differential game, system (1) does not include any constraint for the second player's control v.

The first player tries to lead system (1) to a set M at the terminal instant θ. The control v is regarded as the disturbance. The set M is supposed to be a convex compactum in a subspace $R^n \subset R^m$ of some n chosen components of the vector $\mathbf{x}$. Let us assume that the set M includes a neighborhood of the origin of this subspace.

By means of the standard change of variables (Krasovskii and Subbotin, 1988, pp. 89–91) let us pass to a system, which right-hand side does not include the phase vector:

$$\begin{aligned}&\dot{x} = B(t)u + C(t)v,\\ &x \in R^n, \quad t \in T, \quad u \in P, \quad v \in R^q.\end{aligned} \tag{2}$$

The passage is provided by the following relations:

$$x(t) = X_{n,m}(\theta,t)\mathbf{x}(t),$$
$$B(t) = X_{n,m}(\theta,t)\mathbf{B}(t), \quad C(t) = X_{n,m}(\theta,t)\mathbf{C}(t),$$

where $X_{n,m}(\theta,t)$ is the matrix combined of n (corresponding to the subspace R^n which contains the terminal set M) rows of the Cauchy matrix of the system $\dot{\mathbf{x}} = \mathbf{A}(t)\mathbf{x}$. Here again, the first player tries to lead system (2) to the set M at the terminal instant θ, and the second one hinders this. The set M is a convex compact set in R^n including a neighborhood of the origin.

Below, a description of construction of a robust control for systems (2) and (1) is given.

1) Let us choose a set $Q_{\max} \subset R^q$ that will represent the "maximal" constraint for control of the second player, which the first player "agrees" to consider reasonable for the problem of guiding system (2) to the set M. The set $Q_{\max}$ should include the origin of its space. Denote by W the maximal stable bridge with dynamics (2) corresponding to the constraint P for the first player's control, the constraint $Q_{\max}$ for the second player's control, and the terminal set M. Assume the set $Q_{\max}$ is such that $0 \in \operatorname{int} W(t)$ for any $t \in T$. Here, $W(t) = \{x \in R^n : (t,x) \in W\}$ is the time section of the set W at the instant t.

2) Additionally, a tube $\widehat{W} \subset T \times R^n$ is constructed as the reachable set of the second player for system (2) starting at the instant θ_0 from a ball in the space R^n with the center at the origin. We assume here that $u \equiv 0$ and the constraint for the control v of the second player is $Q_{\max}$.

3) Consider a family of sets W_k, which time sections $W_k(t)$ are defined as

$$W_k(t) = \begin{cases} kW(t), & 0 \leqslant k \leqslant 1,\\ W(t) + (k-1)\widehat{W}(t), & k > 1.\end{cases}$$

In this formula, multiplication by a scalar and addition are usual algebraic operations. For any $0 < k_1 < k_2 < 1 < k_3 < k_4$, the inclusions $W_{k_1} \subset W_{k_2} \subset W \subset W_{k_3} \subset W_{k_4}$ are true.

It is proved (Ganebny *et al.*, 2005) that the sets W_k, when $0 \leqslant k \leqslant 1$, are maximal stable bridges corresponding to the constraint kP for the first player's control, the constraint $kQ_{\max}$ for control of the second player, and the terminal set kM. The sets W_k, when $k > 1$, are stable bridges corresponding to the constraints P, $kQ_{\max}$, and the terminal set $M + (k-1)\widehat{W}(\theta)$. So, with increasing the coefficient k, one obtains a growing collection of stable bridges where any larger bridge corresponds to a larger constraint for the second player's control.

On the basis on the family of stable sets W_k by means of the method of extremal aiming (Krasovskii and Subbotin, 1988), one can define a feedback control of the first player, which possesses the following properties of guarantee. Let $k^* \geqslant 0$. If $v(t) \in k^* Q_{\max}$ and $(t_0, x_0) \in W_{k^*}$, then system (2) can be kept in some small neighborhood of the bridge W_{k^*} under a discrete scheme of control with a sufficiently small time step. With that, the realization of the first player's control belongs to the set $\min(k^*, 1)P$. In particular, if $k^* < 1$, then the system can be led to the terminal set by means of the control, which has a level less than the extremal one. So, we obtain a control, which is robust in the sense of our definition. To construct such a control by the method of extremal aiming, one needs to keep in the computer memory a very large amount of information describing a quite dense subfamily of the family W_k, $k \geqslant 0$. But if to use the specifics of the set P constraining the first player's control, one can suggest more constructive variant.

4) Define a function $V : T \times R^n \to R$ as $V(t,x) = \min\{k \geqslant 0 : (t,x) \in W_k\}$. The level sets (Lebesgue sets) of this function coincide with the stable bridges W_k.

By $B_i(t)$, $i = \overline{1,p}$, we denote the ith column of the matrix $B(t)$.

Denote by $\mathcal{A}(i,t,x)$ a line in the space R^n parallel to the vector $B(t)$ and passing through the point x. Let $\mathcal{V}(i,t,x) = \min\{V(t,z) : z \in \mathcal{A}(i,t,x)\}$. The minimum is reached since the function $x \mapsto V(t,x)$ is continuous and tends to infinity as $|x| \to \infty$. Since the function is quasiconvex (i.e., all its Lebesgue sets are convex), the set of points, where the minimum is reached, is either a point or a segment. If $B_i(t) = 0$, it is assumed $\mathcal{V}(i,t,x) \equiv V(t,x)$.

5) For any $t \in T$, let

$$\Pi(i,t) = \{x \in R^n \colon V(t,x) = \mathcal{V}(i,t,x)\},$$
$$\Pi_-(i,t) = \{x \in R^n \colon x + \alpha B_i(t) \notin \Pi(i,t),\ \forall \alpha \geqslant 0\},$$
$$\Pi_+(i,t) = \{x \in R^n \colon x + \alpha B_i(t) \notin \Pi(i,t),\ \forall \alpha \leqslant 0\}.$$

The set $\Pi(i,t)$ is closed, the sets $\Pi_-(i,t)$ and $\Pi_+(i,t)$ are on different sides of $\Pi(t)$.

6) Define the function $\overline{V}(t,x) = \min\{V(t,x), 1\}$. For each $i = \overline{1,p}$ consider a multifunction

$$\mathbf{U}_i^0(t,x) = \begin{cases} -\overline{V}(t,x)\mu_i, & x \in \Pi_-(i,t), \\ \overline{V}(t,x)\mu_i, & x \in \Pi_+(i,t), \\ \left[-\overline{V}(t,x)\mu_i, \overline{V}(t,x)\mu\right], & x \in \Pi(i,t). \end{cases}$$

As the feedback control U of the first player, any one-valued selection from the multifunction $\mathbf{U}^0 = (\mathbf{U}_1^0, \mathbf{U}_2^0, \ldots, \mathbf{U}_p^0)^{\mathrm{T}}$ can be taken. Thus, the component $U_i(t,x)$ "switches" on the set $\Pi(i,t)$. For simplicity, the set $\Pi(i,t)$ is called the *switching surface* corresponding to the instant t for the ith component of control.

7) Returning to system (1), introduce the function $\widetilde{U}(t,\mathbf{x}) = U\big(t, X_{n,m}(\theta,t)\mathbf{x}\big)$. The function $\widetilde{U}$ can be taken as the robust control for system (1).

Under some additional assumptions concerning possible intersections of the surfaces $\Pi(i,t)$, one can prove a theorem about guarantee, which is provided by the control U in system (2) and the control $\widetilde{U}$ in system (1). Properly speaking, this guarantee for system (2) coincides with one, which is described at the end of Subsection **3)**.

Below, the case $n = 2$ is discussed. In this case, the sets $W_k(t)$ and $\Pi(i,t)$ are in the plane. So, the set $\Pi(i,t)$ can be called the *switching line* for the ith component of control. In the case $n = 2$, the assumption mentioned above becomes very simple: the lines $\Pi(i,t)$, $i = \overline{1,p}$, intersect at the origin only.

To construct the robust control numerically in the case $n = 2$, one should keep time sections $W(t)$ of the bridge W and the switching lines $\Pi(i,t)$ in some time grid $\{t_j\}$. At the instant t, having the position $\mathbf{x}(t)$ of system (1), one can transfer it to the coordinates of system (2) by the formula $x(t) = X_{n,m}(\theta,t)\mathbf{x}(t)$. The sign of the control $\widetilde{U}_i\big(t,\mathbf{x}(t)\big) = U_i\big(t,x(t)\big)$ is defined by the location of the point $x(t)$ with respect to the switching surface $\Pi(i,t)$. Analyzing the location of the point $x(t)$ with respect to the boundary of the time section $W(t)$, one can compute the absolute value $\big|\widetilde{U}\big(t,\mathbf{x}(t)\big)\big|$ of the control. Here, the homothety of the sets $W_k(t)$ for $k \leqslant 1$ is used.

3. LANDING AIRCRAFT DYNAMICS

The aircraft motion during landing is described by a differential equation system of the 12th order. The state vector includes three coordinates x, y, z (longitudinal, vertical, and lateral) of the center of mass in the coordinate system connected with runway surface, angles of pitch ϑ, yaw ψ, bank γ, and corresponding linear and angular velocities. This system is specified in (Patsko *et al.*, 1994).

The control factors are deviations of the elevator, the rudder, the ailerons, and change of the thrust force. Equations of servo and engine dynamics are supplemented to the main system of aircraft motion. So, the noted factors are included to the extended state vector and new control parameters are the scheduled (command) deviations δ_{es}, δ_{rs}, δ_{as}, δ_{ps}. Every parameter has upper and lower bounds. As a result, we get a complete system of differential equations, which we write down in the vector form as follows:

$$\dot{\xi} = f(\xi, u, w). \tag{3}$$

Here, $u = (\delta_{ps}, \delta_{es}, \delta_{rs}, \delta_{as})^{\mathrm{T}}$ is the control vector, $w = (w_x, w_y, w_z)^{\mathrm{T}}$ is the disturbance vector consisting of three wind components along the x, y, z axes. Linear variables are measured in meters, angles are in radians, and time is in seconds.

The original nonlinear system after linearization near the nominal motion along the descent glide path is disjoined into two subsystems of lateral motion (LM) and vertical motion (VM).

The linear LM system is

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu + Cw,$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.077 & -5.555 & 0 & 9.272 & 0 & -1.485 & 0 \\ 0 & 0 & 0 & 1.001 & 0 & 0 & 0 & 0 \\ 0 & -0.013 & -0.934 & -0.259 & -0.088 & -0.030 & -0.246 & -0.046 \\ 0 & 0 & 0 & -0.051 & 0 & 1 & 0 & 0 \\ 0 & -0.033 & -2.387 & -0.953 & -0.226 & -1.459 & -0.233 & -0.689 \\ & & & & & & -4 & 0 \\ & & & & & & 0 & -4 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix}^{\mathrm{T}},$$

$$C = \left[\, 0,\ 0.0769,\ 0,\ 0.0129,\ 0,\ 0.0331,\ 0,\ 0 \,\right]^{\mathrm{T}}.$$

Here, $\mathbf{x}_1 = \Delta z$ is a deviation in z from the nominal motion; $\mathbf{x}_2 = \Delta\dot{z}$ is a deviation in $\dot{z}$ from the nominal motion; $\mathbf{x}_3 = \Delta\psi$, $\mathbf{x}_5 = \Delta\gamma$ are deviations of the yaw and bank angles. The coordinates $\mathbf{x}_7$, $\mathbf{x}_8$ are deviations of the rudder and ailerons. The controls of the first player are the scheduled deviations $u_1 = \Delta\delta_{rs}$ of the rudder and $u_2 = \Delta\delta_{as}$ of the ailerons. Constraints are

$$\Delta\delta_{rs} \leq 10^\circ, \qquad \Delta\delta_{as} \leq 10^\circ.$$

The variable $w = w_z$ denotes the lateral component of the wind velocity. In real life, the wind changes inertially. To describe this we use the following wind dynamics:

$$\dot{w}_z = 0.5(F_z - w_z), \quad \dot{F}_z = 3(v_z - F_z).$$

The terminal set in the plane $\mathbf{x}_1 = \Delta z$, $\mathbf{x}_2 = \Delta\dot{z}$ is taken as the hexagon with vertices $(-6, 0)$, $(-6, 1.5)$, $(0, 1.5)$, $(6, 0)$, $(6, -1.5)$, $(0, -1.5)$.

The linear VM system is

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu + Cw,$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.050 & 0 & -0.098 & -2.642 & 0 & 0.063 & 0.997 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0.241 & 0 & -0.639 & 45.278 & 0 & 1.448 & 0.081 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.007 & -0.501 & -0.526 & -0.383 & 0 \\ & & & & & & -4 & 0 \\ & & & & & & 0 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \end{bmatrix}^{\mathrm{T}},$$

$$C = \begin{bmatrix} 0 & 0.0501 & 0 & -0.2409 & 0 & -0.0003 & 0 & 0 \\ 0 & 0.0973 & 0 & 0.6387 & 0 & -0.0069 & 0 & 0 \end{bmatrix}^{\mathrm{T}}.$$

Here, $\mathbf{x}_1 = \Delta x$ and $\mathbf{x}_3 = \Delta y$ are deviations in x and y from the nominal motion; $\mathbf{x}_2 = \Delta\dot{x}$ and $\mathbf{x}_4 = \Delta\dot{y}$ are deviations in $\dot{x}$ and $\dot{y}$ from the nominal motion; $\mathbf{x}_5 = \Delta\vartheta$ is a deviation of the pitch angle. The coordinates $\mathbf{x}_7$, $\mathbf{x}_8$ are deviations of the elevator and thrust force. The controls of the first player are the scheduled deviations $u_1 = \Delta\delta_{ps}$ of the thrust force and $u_2 = \Delta\delta_{es}$ of the elevator. Constraints are

$$\Delta\delta_{ps} \le 27^\circ, \qquad \Delta\delta_{es} \le 10^\circ.$$

The variables $w_1 = w_x$ and $w_2 = w_y$ denote the longitudinal and vertical components of the wind velocity. Taking into account inertia of the wind, we write the law of change of these variables as

$$\dot{w}_{x,y} = 0.5(F_{x,y} - w_{x,y}), \quad \dot{F}_{x,y} = 3(v_{x,y} - F_{x,y}).$$

The terminal set in the plane $\mathbf{x}_3 = \Delta y$, $\mathbf{x}_4 = \Delta\dot{y}$ is taken as the hexagon with vertices $(-3, 0)$, $(-3, 1)$, $(0, 1)$, $(3, 0)$, $(3, -1)$, $(0, -1)$.

To construct the robust control for these linear systems, one has to define the constraint $Q_{\max}$ for the second player's control. For the lateral channel the set is taken as $|v_z| \le 10$ m/sec. For the vertical channel, the set $Q_{\max}$ is equal to $|v_x| \le 10$ m/sec, $|v_y| \le 5$ m/sec.

4. SIMULATION RESULTS

To simulate the landing process, the original nonlinear system was used. The controls δ_{rs}, δ_{as} in the lateral channel and δ_{ps}, δ_{es} in the vertical one were generated by means of robust control laws obtained for the linear systems.

Wind disturbance w was generated on the basis of the wind microburst model from (Ivan, 1985). In Fig. 1, the field of wind velocity is shown for this model. The center of the microburst is in 500 m from the runway threshold along the axis x and in -150 m aside along the axis z. Geometric parameters of the microburst are: the height of the central point is 100 m, the radius of the torus is 220 m, the radius of the torus ring 80 m, the wind velocity at the central point is 6 m/sec.

The initial deviations of the aircraft position from the nominal one are: the lateral deviation is 20 m, the vertical one is 10 m, all others are equal to zero.

In Fig. 2, the phase trajectories of the nonlinear system are shown in projection into the plane of the lateral deviation Δz and its velocity $\Delta\dot{z}$ (the upper picture) and into the plane of the vertical deviation Δy and its velocity $\Delta\dot{y}$ (the lower picture). In Fig. 3, one can see the graphs of the components δ_{rs}, δ_{as}, δ_{ps}, and δ_{es} of the useful control u. The extremal levels of these controls are marked by dashed lines. The graphs of the realizations of the wind velocity components w_z, w_x, w_y are given in Fig. 4.

The realizations of components of the control u have frequent switches. But since $u(t)$ is the

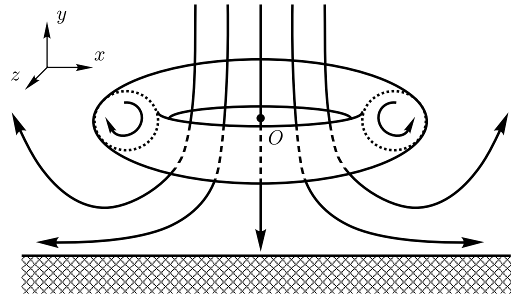

Fig. 1. Scheme of streamlines of the wind microburst

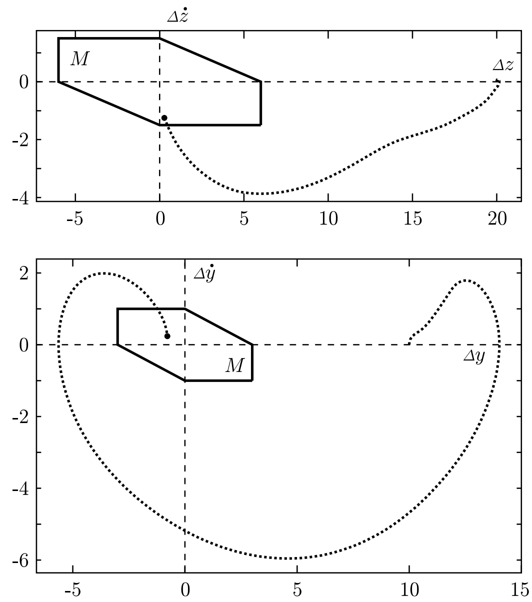

Fig. 2. Phase trajectories for the lateral (the upper picture) and vertical (the lower picture) channels

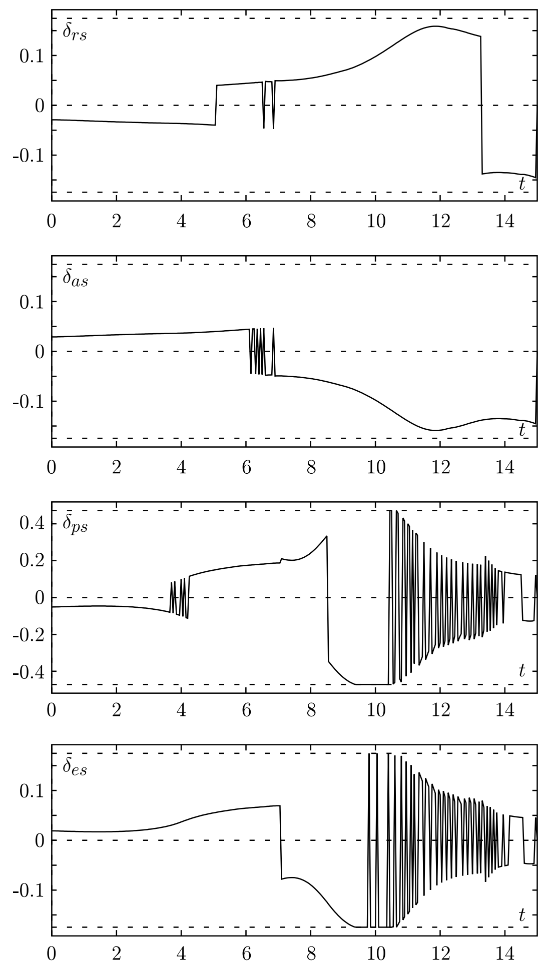

Fig. 3. Realizations of components of the useful control u

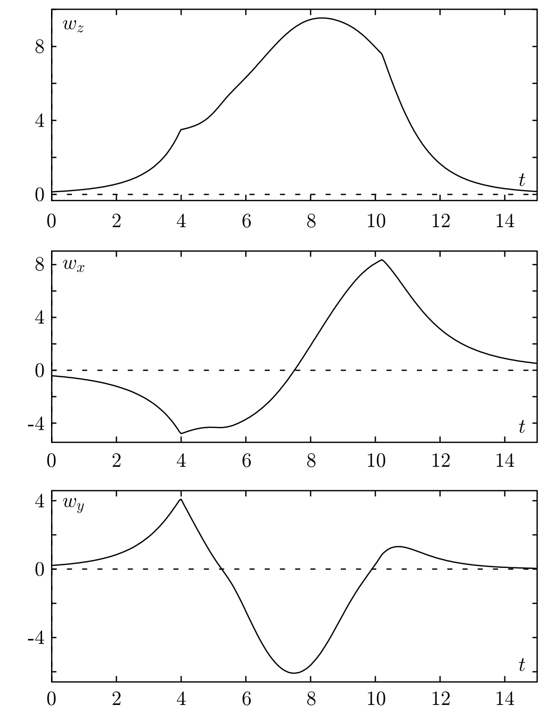

Fig. 4. Realizations of components of the wind velocity w

scheduled control vector, this chattering will be smoothed in servo-mechanisms, which dynamics is included in system (3).

One can see that the final point enters to the tolerance in both channels. The controls on the thrust force and the elevators reach the maximal level, but only in some small time interval near 10 sec. Let us note that the component w_y of the disturbance exceeds the level 5 m/sec, which corresponds to the chosen set $Q_{\max}$.

5. CONCLUSION

Traditional methods for control of an aircraft during landing work unsatisfactory when the wind velocity changes abruptly. A new method for constructing a control is suggested, which is suitable for wind disturbances from a wide range. It uses ideology of guaranteed control and can autotune to actual level of the disturbance. Simulation results of the landing process are given for the case when the disturbance is generated by a wind microburst. Modelling is made in the framework of the original nonlinear system. The results show that the suggested control method successfully works with hard wind disturbances.

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