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VISUALIZATION OF THREE-DIMENSIONAL REACHABLE SET FOR THE DUBINS CAR

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ABSTRACT

A mathematical model of controlled motion (called the Dubins car in the literature on optimal control) is considered. This model is widely used to describe various motions: an airplane in a horizontal plane, a car, etc. The paper deals with the problem of visualization of the three-dimensional reachable sets for the Dubins car. The developed visualization tool can be useful for research purposes, as well, for teaching a course of the optimal control.

KEYWORDS

Dubins Car, Three-Dimensional Reachable Set, Specialized Visualization System

1.   INTRODUCTION AND STATEMENT OF THE PROBLEM

The mathematical "Dubins car" (the origin of this name is the article by Dubins (1957)) is a model of controlled motion, in which two phase variables x, y are the coordinates of the point geometric position of the object on the plane, the third variable φ is the angle formed by the velocity vector w.r.t. the positive direction of the x axis. The value of the linear velocity is considered constant and equal to 1. The scalar control has the meaning of the angular rotation velocity (or, equivalently, instantaneous radius of rotation) and is restricted by the constraint $u \in [u_1, u_2]$. Such a model, often called "unicycle", is widely used when considering the motions of an aircraft in a horizontal plane, as well as, when simplifying the description of the motion of ground-controlled wheeled "bogies", see Laumond (1998).

When studying various control problems associated with the Dubins car, it is useful to describe the boundary of the three-dimensional reachable sets "*at the instant*". The reachable set at the instant t_f is the set of all phase states each of them can be achieve by the system with the help of some admissible control exactly at the instant t_f . The initial instant and the initial phase state are assumed to be zero. As the admissible controls, we take piecewise constant time functions with values from the interval $[u_1, u_2]$. The boundary of the reachable set $G(t_f)$ is generated by the controls satisfying the Pontryagin Maximum Principle (PMP). Such a necessary condition allows us to perform a detailed analysis of the boundary of the three-dimensional reachable set, see Patsko *et al* (2003), Patsko, Fedotov (2018).

The paper is devoted to visualization of the reachable set $G(t_f)$ under the assumption $-1 \le u_1 < 0 < u_2 = 1$. The relations $u_2 = 1$, $u_1 \ge -1$ are not essential: the geometric variables and time can always be normalized in such a way that the value of the linear velocity is equal to 1 and the conditions $u_2 = 1$, $u_1 \ge -1$ are satisfied. The inequalities $u_1 < 0 < u_2$ mean that the controlled object can rotate in both sides. In the case of $u_1 = -1$, $u_2 = 1$, there is a symmetry of the "right" and "left" turns. If the relations $-1 < u_1 < 0 < u_2 = 1$ are satisfied, then the rotation is possible in both sides, but the symmetry is broken.

The normalized mathematical description of the Dubins car dynamics on the plane x, y has the form

$$\dot{x} = \cos \varphi, \qquad \dot{y} = \sin \varphi, \qquad \dot{\varphi} = u; \qquad u \in [u_1, u_2].$$

We assume that the angle φ of the velocity vector direction is counted counterclockwise from the positive direction of the x-axis and takes values in the interval $(-\infty, \infty)$. That is, we do not identify the angle φ by modulo 2π. Under identification, the structure of the reachable set becomes significantly more complex that makes it difficult to understand its changes in time.

The resulting three-dimensional reachable sets can have a very nontrivial structure. The set $G(t_f)$ for $t_f > 0$ is not convex and can be non-simply connected. Its boundary is formed by pieces of smooth surfaces that do not always fit together smoothly. In the works of Patsko *et al* (2003), Patsko, Fedotov (2018), a parametric description of such smooth pieces was obtained.

The purpose of this work is to create a convenient specialized visualization system that allows us to view the development in time of three-dimensional sets that can have different mathematical and physical interpretations. There are a number of basic requirements that need to be satisfied:

1. It should be possible to get images from different angles of view and to make various cut-offs. This is necessary, for example, to study the time changes the structure of the surfaces forming the boundary of the reachable set.

2. In particular, the visual representation is required for the surfaces that form an internal cavity (not belonging to the reachable set). Note that such a cavity can occur on a very short time interval. In this interval, the reachable set is not simply connected, and its boundary is divided into two parts: "external" and "internal" ones.

3. Besides, it is also necessary to additionally display the extreme motions of the system (with the possibility of animation in time) that satisfy the PMP and go along the boundary of the time-varying three-dimensional reachable set. Such motions can split and, thus, generate trajectory bundle (going on the boundary). There are bundles that can again gather into one motion after a while. These splitting and merging occur at certain parts of the boundary. It is desirable to have a visual adequate reproduction of such processes.

For convenience, in this paper we denote our specialized visualization system as a visualization tool. The initial stage of creating the visualization tool is described in Vasev, Fedotov (2020). The possibilities of the developed visualization tool are shown by the example of the Dubins car reachable set.

2. VISUALUZATION TECHNOLOGY

2.1 Overview of the Visualization Method

We are in the field of optimal control, and it relies on phase space phenomena. The history of visualization of phase spaces and, in particular, phase trajectories is interesting and contains a set of efficient visualization methods. We can note, for example, approaches that use Poincare sections, return maps, and common methods like feature detection and feature outlining, various multidimensional projections onto 3d space, creation of synthetic helper spaces for visualization purposes, critical subsets outlining, interconnected separate views, symbolic representation of meaningful places (Figure 1a), filtering (Figure 1b), high-level entity reconstruction (Figure 1c).

Figure 1. Examples of phase space visualizations by ETH Zurich (https://cgl.ethz.ch/research/visualization/)
Left to right: (a) inertial particle dynamics, (b) visibility optimization, (c) vortex extraction

However, as we are in optimal control field, we have a special entity as a first-class object of interest – the reachable set at the instant, described in introduction section. We paint the reachable set almost as is, as we may effectively calculate this set for Dubins car. We paint it with an animation evolving over time. We apply various cross-sections, filtering, and linear transformations to the reachable set's geometry. And then we accompany it with specific phase trajectories of extreme motions. It is discussed in section 3.

2.2 Extension to the CinemaScience Format to Display the 3D Graphics

We have developed the visualization tool for the purpose of visualizing the reachable sets according to requirements defined in introduction section. Our tool is based on the CinemaScience data format, see Rogers *et al* (2018). We have created a universal viewer for this format with the 3d graphics support and then adapted this viewer for the reachable set visualization.

In the CinemaScience format, information is stored in a *data.csv* text file. It contains a table with columns for parameters and columns for data artifacts (artifacts can be images, grids, csv files; *i.e.*, any type of data that can be written to disk). Thus this table defines a combination of parameters and artifacts. The interpretation of these combinations is free and determined by the scope of an application.

Our extension suggests that artifacts may be 3d entities, and all such entities should be rendered into common 3d space. Each 3d entity type of artifact is defined by artifact name, currently by second underscores-surrounded word. For example, *FILE_points_alfa* artifact name defines "points" entity type. The list of supported types is extensible, and the following types are currently implemented: points, lines (segments), triangles, spheres, vrml (for VRML graphic files), obj (for OBJ files).

For each parameter column, a graphical control is generated: slider for numeric parameter and combobox for string parameter. User manipulates these controls and thus specifies combination of parameters values. This determines the current combination of artifacts 3d data files to be rendered.

Specifically for the task of visualization of reachable sets, the *data.csv* file looks like this:

```
T,      FILE_vrml_rset,  FILE_treki_all, FILE_treki_currentT
1.00,  Symm-20.vrml,   treki.csv,  pts_1.00.treki.csv
1.05,  Symm-21.vrml,   treki.csv,  pts_1.05.treki.csv
1.10,  Symm-22.vrml,   treki.csv,  pts_1.10.treki.csv
    …
4.00,  Symm-80.vrml,   treki.csv,  pts_4.00.treki.csv
```

Here, one parameter (*T*) and three artifacts are defined (*FILE_vrml_rset, FILE_treki_all, FILE_treki_currentT*). For the first artifact, *FILE_vrml_rset*, the vrml type is specified (by its name); that means loading and visualizing data in the VRML format. We use such format to store the surface of the reachable set. For the second and third artifacts, the *treki* type is specified for visualizing point trajectories.

The specially designed *treki* view type requires a csv file with the columns *N, X, Y, Fi.* Each line of this file defines a point in a 3d scene, and a set of such points form a trajectory. In the column *N*, there is a trajectory number. Then, in the graphical interface, a researcher can filter the view by the value of *N* and thus see only necessary trajectories.

The researcher selects the value *T* in the automatically created graphical user interface and the program calculates the corresponding values of artifact files and renders them. As a result, the overall view gets the necessary form: the surface for the selected instant, the points of the trajectories, and the points on the trajectories corresponding to this instant.

2.3 Technical Aspects

Procedures for visualization in this work consist of two parts: calculation and visualization. The first one performs the calculation and generates scene in the CinemaScience format. The visualization tool loads and displays scene interactively with the ability to apply visual filters.

In fact, the calculating and drawing parts are participants of a single visualization pipeline with calculation parameters at the input and interactive images on the screen at the output.

The described approach with the CinemaScience format allows one to program various 3d scenes, not just those shown in the paper. The list of artifacts is the list of 3d entities in scene. The applied semantics (selection of parameter values and rendering corresponding files of 3d entities into common 3d space) makes it possible to solve a wide range of visualization tasks. Other advantages of the used approach include CinemaScience's natural ability to create animated films using parameters specified in input files.

In addition, we added visual filters to our visualization tool. For the task of visualizing the reachable set (especially for the convenience of perceiving the structure of the boundary of the set), the following filters were found to be useful: half-space cross-section, cross-section series, sections edge tint. Different filters may be applied simultaneously, for example, half-space section on the X axis and section series on the Y axis.

The visualization tool is implemented using the WebGL and works in a web browser. We also made it compatible with the WebXR virtual reality standard. We observed our reachable sets in virtual reality with positive effects on perception. We have come to the conclusion that working in the visualization system requires using additional methods that could increase both the convenience of navigation in the graphic space and the convenience of managing the graphic filters.

Web technologies can be used together with virtual reality and with the addition of multi-user environment support and online connection with simulations. It brings us to the perspective of virtual test stands as described in the innovative work by Averbukh *et al* (2020).

The developed visualization tool for the reachable sets is available online at https://viewlang.ru/dubins/ as well as its source codes at https://github.com/pavelvasev/38parrots.

3. VISUALIZATION OF EXTREME MOTIONS

The description of extreme motions that go along the boundary of the time-varying reachable set has a theoretical and applied significance. For specialists in the field of optimal control, the answers to the following questions are of interest:
1. Extreme motions that satisfy the PMP go along smooth parts of the boundary, or may at some time intervals go along "edges of non-smoothness"?
2. Can extreme motions split and then merge back together into one motion?
3. Can extreme motion go into the interior of the reachable set?
4. If so, where on the boundary can this happen?

The developed tool provides ability to explore such issues. In particularly, it allows to see the location of the current phase point (x, y, φ) of the trajectory at the boundary of the reachable set $G(t)$ corresponding to a given instant. It is possible to trace at what instants the motion is on a smooth part of the boundary or on the non-smooth edge, and so on. Some trajectories are very interesting. For example, after the certain instant t^*, they cannot be continued by motions that going on the boundary of the reachable set with *any permissible control* for $t > t^*$. Such a motion goes to the interior of the reachable set. Also, there are situations when the extreme motion leading at some instant into some point on the boundary of the set $G(t_f)$ is not unique, and the splitting of the motions does not necessary occur at the initial instant. Thus, we are dealing with a bundle of extreme motions that exit at the instant t_0 from the same phase point and arrive also to the same phase point at the instant t_f.

At the address http://sector3.imm.uran.ru/confers/CGS2020/index_eng.html there are animation materials showing a three-dimensional reachable set, as well as some extreme trajectories.

In Figure 2, a frame is shown from a motion video of a trajectory bundle going on the boundary of the reachable set, which changes in time in the three-dimensional phase space x, y, φ. The bundle comes out from the origin at the initial instant and consists of motions that coincide at some initial time interval. The splitting effect is demonstrated by the six motions that are going on the boundary of the set $G(t_f)$ and connect at the end of a given time interval. The red dots show the phase trajectories (they coincide in the initial time interval) in the three-dimensional space. The yellow balls correspond to the current position of the motions at the time of the freeze frame. For the corresponding instant, the three-dimensional reachable set is also shown. The boundary of the reachable set is divided into color-highlighted parts generated by different types (in this case, 6 types) of extreme PMP controls leading to the boundary, see Patsko *et al* (2003).
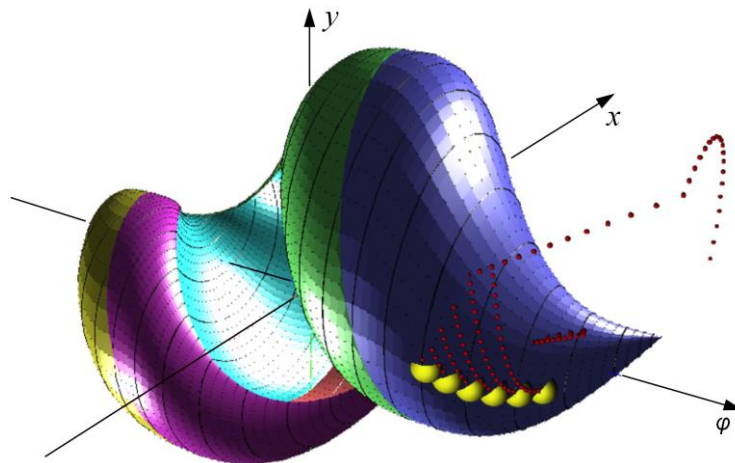
Figure 2. A frame from a video with the motion of a trajectory bundle going on the boundary of the time-evolving reachable set

4. CONCLUSION

The paper is devoted to visualization of the three-dimensional reachable set "at the instant" for the Dubins car model of the controlled object. The created visualization tool makes it possible to explore the features of the structure of the reachable set and the extreme motions that go on its boundary. The tool provides various visual filters for a better perception of the complex structure of the reachable set. The user's interface allows one to create animation movies on the basis of some scripting language. The suggested approach of scientific visualization is suitable for a big variety of visualization tasks besides the presented one.

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