Vol. 83, No. 2 November 1994

JOTABN 83(2) 237–446 (1994)

ISSN 0022-3239

JOURNAL OF OPTIMIZATION THEORY AND APPLICATIONS

PLENUM PUBLISHING CORPORATION
NEW YORK–LONDON

JOURNAL OF OPTIMIZATION THEORY AND APPLICATIONS: Vol. 83, No. 2, pp. 237-267, NOVEMBER 1994

Control of an Aircraft Landing in Windshear

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Communicated by N. V. Banichuk

Abstract. The problem of the feedback control of an aircraft landing in the presence of windshear is considered. The landing process is investigated up to the time when the runway threshold is reached. It is assumed that the bounds on the wind velocity deviations from some nominal values are known, while information about the windshear location and wind velocity distribution in the windshear zone is absent. The methods of differential game theory are employed for the control synthesis.

The complete system of aircraft dynamic equations is linearized with respect to the nominal motion. The resulting linear system is decomposed into subsystems describing the vertical (longitudinal) motion and lateral motion. For each subsystem, an auxiliary antagonistic differential game with fixed terminal time and convex payoff function depending on two components of the state vector is formulated. For the longitudinal motion, these components are the vertical deviation of the aircraft from the glide path and its time derivative; for the lateral motion, these components are the lateral deviation and its time derivative. The first player (pilot) chooses the control variables so as to minimize the payoff function; the interest of the second player (nature) in choosing the wind disturbance is just opposite.

The linear differential games are solved on a digital computer with the help of corresponding numerical methods. In particular, the optimal (minimax) strategy is obtained for the first player. The optimal control is specified by means of switch surfaces having a simple structure. The minimax control designed via the auxiliary differential

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0022-3239/94/1100-0237$07.00/0

game problems is employed in connection with the complete nonlinear system of dynamical equations.

The aircraft flight through the wind downburst zone is simulated, and three different downburst models are used. The aircraft trajectories obtained via the minimax control are essentially better than those obtained by traditional autopilot methods.

Key Words. Flight mechanics, landing, feedback control, windshear problems, differential games, longitudinal motion, lateral motion, numerical methods, linear differential games, switch surfaces.

Notations

b = mean aerodynamic chord, m;
c_x, c_y, c_z = aerodynamic force coefficients, body-axes system;
g = acceleration of gravity, m $\sec^{-2}$;
I_x, I_y, I_z, I_{xy} = inertia moments, kg m^2;
l = wing span, m;
m = aircraft mass, kg;
m_x, m_y, m_z = aerodynamic moment coefficients, body-axes system;
M_x, M_y, M_z = aerodynamic moments, N m;
P = thrust force, N;
q = dynamic pressure, kg m^{-1} $\sec^{-2}$;
S = reference surface, m^2;
V = absolute velocity, m $\sec^{-1}$;
$\hat{V}$ = relative velocity, m $\sec^{-1}$;
W = wind velocity, m $\sec^{-1}$;
V_{xg}, V_{yg}, V_{zg} = absolute velocity components, m $\sec^{-1}$;
$\hat{V}_{xg}, \hat{V}_{yg}, \hat{V}_{zg}$ = relative velocity components, m $\sec^{-1}$;
W_{xg}, W_{yg}, W_{zg} = wind velocity components, m $\sec^{-1}$;
x_g, y_g, z_g = coordinates of the aircraft center of mass, m, ground-fixed system;
α = angle of attack, deg;
β = sideslip angle, deg;
γ = bank angle, deg;
δ_a = aileron deflection, deg;
δ_e = elevator deflection, deg;
δ_r = rudder deflection, deg;
δ_{as} = aileron setting (control), deg;
δ_{es} = elevator setting (control), deg;
δ_{ps} = engine control lever setting, deg;

δ_{rs} = rudder setting (control), deg;
ϑ = pitch angle, deg;
ψ = yaw angle, deg;
ρ = air density, kg m^{-3};
σ = thrust inclination, deg;
ω_x, ω_y, ω_z = angular velocity components, body-axes system, deg sec^{-1}.

Acronyms

DG = differential game;
LM = lateral motion;
VM = vertical (longitudinal) motion;
RW = runway.

1. Introduction

The first papers studying the control of aircraft in landing and take-off under severe wind conditions appeared in the late 1970s/early 1980s. Both the stochastic and deterministic approaches for the description of wind disturbances were considered. One of the versions of the deterministic formulation assumes that the a priori information about the wind disturbances consists in the knowledge of the possible bounds on the wind velocity deviations from the nominal values. For such formulation, it is natural to apply the methods of differential game (DG) theory. The present paper is devoted to the application of numerical methods, based on the DG theory of Krasovskii and his school (Refs. 1–2), to the feedback control of an aircraft in landing. The first efforts in using the methods of DG theory for the aircraft landing problem were made in Refs. 3–6.

The investigations by Miele and his coworkers about aircraft control in take-off, abort landing, and penetration landing (see, e.g., Refs. 7–9) were followed by numerous papers by Chen, Leitmann, Bulirsch, and their coworkers (see Refs. 10–13 and references therein). In these papers, the wind disturbances associated with the aircraft passage through a downburst zone were considered. A downburst is a descending air column which spreads horizontally in the neighborhood of the ground.

In the majority of the papers, the downburst structure was supposed to be known a priori. The aircraft motion was considered in a vertical plane, and the angle of attack and the engine power setting were employed as control variables. As a rule, the methods of optimal control were used, so the results obtained characterize the potential control possibilities.

In Refs. 10–11, the take-off dynamics model of Ref. 7 was used, but global information about the wind field was supposed to be unknown *a priori*. The Lyapunov function was designed by means of robust control theory, and the feedback control was found by aiming along the antigradient of this function. Concerning numerical simulations, the proposed control algorithm was tested on different downburst models, and the results obtained were compared with those of Ref. 7.

In this paper, the landing problem for midsize transport aircraft is considered. As to the wind, we suppose that the bounds on the deviations of the wind velocity components from some average values are known. The aircraft dynamics is described via a sufficiently complete nonlinear system. The time lag of servomechanisms is taken into account. We consider the landing process from the altitude of 400 m up to the time when the runway (RW) threshold is reached. Numerical DG algorithms, developed at the Institute of Mathematics and Mechanics, Ural Branch, Russian Academy of Sciences, Ekaterinburg, are employed for the feedback control synthesis. The complete system is linearized with respect to the nominal motion along the descending glide path. Then, the resulting linear system is decomposed into a vertical motion (VM) subsystem and a lateral motion (LM) subsystem. For both subsystems, DGs with a terminal performance index are formulated. The numerical solution of these games gives the minimax feedback controls for the linear models. Then, the control laws are applied to the original nonlinear system. We test the control laws simulating the motion of the nonlinear system under various wind disturbances. Three models of wind downburst are employed (Refs. 7, 14–15). For these models, we compare the minimax laws with traditional autopilot methods.

2. Mathematical Model of Controlled Motion

2.1. Equations of Motion. For an aircraft on the approach trajectory, the motion is described by the following 12th order system of differential equations (see, e.g., Refs. 16–17):

$$\dot{x}_g = V_{xg}, \tag{1}$$

$$\begin{aligned}\dot{V}_{xg} = [&(P \cos \sigma - qSc_x) \cos \psi \cos \vartheta + (P \sin \sigma + qSc_y) \\ &\times (\sin \psi \sin \gamma - \cos \gamma \cos \psi \sin \vartheta) \\ &+ qSc_z(\sin \psi \cos \gamma + \cos \psi \sin \vartheta \sin \gamma)]/m,\end{aligned} \tag{2}$$

$$\dot{y}_g = V_{yg}, \tag{3}$$

$$\dot{V}_{yg} = [(P \cos \sigma - qSc_x) \sin \vartheta + (P \sin \sigma + qSc_y) \times \cos \vartheta \cos \gamma - qSc_z \cos \vartheta \sin \gamma]/m - g, \quad (4)$$

$$\dot{z}_g = V_{zg}, \quad (5)$$

$$\dot{V}_{zg} = [(P \cos \sigma - qSc_x)(-\sin \psi \cos \vartheta) + (P \sin \sigma + qSc_y) \times (\cos \psi \sin \gamma + \sin \psi \sin \vartheta \cos \gamma) + qSc_z(\cos \psi \cos \gamma - \sin \psi \sin \vartheta \sin \gamma)]/m, \quad (6)$$

$$\dot{\vartheta} = \omega_z \cos \gamma + \omega_y \sin \gamma, \quad (7)$$

$$\dot{\omega}_z = [I_{xy}(\omega_x^2 - \omega_y^2) - (I_y - I_x)\omega_x\omega_y]/I_z + M_z/I_z, \quad (8)$$

$$\dot{\psi} = (\omega_y \cos \gamma - \omega_z \sin \gamma)/\cos \vartheta, \quad (9)$$

$$\dot{\omega}_y = [(I_y - I_z)I_{xy}\omega_y + (I_z - I_x)I_x\omega_x]\omega_z/J + (I_xM_y + I_{xy}M_x)/J + \omega_zI_{xy}(I_x\omega_y - I_{xy}\omega_x)/J, \quad (10)$$

$$\dot{\gamma} = \omega_x - (\omega_y \cos \gamma - \omega_z \sin \gamma) \tan \vartheta, \quad (11)$$

$$\dot{\omega}_x = [(I_y - I_z)I_y\omega_y + (I_z - I_x)I_{xy}\omega_x]\omega_z/J + (I_yM_x + I_{xy}M_y)/J + I_{xy}\omega_z(I_{xy}\omega_y - I_y\omega_x)/J. \quad (12)$$

The state variables are: the coordinates x_g, y_g, z_g of the center of mass in the ground-fixed system (Fig. 1); the absolute velocity components V_{xg}, V_{yg}, V_{zg}; the angles of pitch, yaw, and bank ϑ, ψ, γ; the angular velocity components in the body-axes system $\omega_x, \omega_y, \omega_z$. The dynamic pressure q is computed with the formula

$$q = \rho \hat{V}^2/2.$$

The aerodynamic moments are

$$M_x = qSlm_x, \qquad M_y = qSlm_y, \qquad M_z = qSbm_z.$$

The quantity J is determined via the moments of inertia I_x, I_y, I_{xy} as

$$J = I_xI_y - I_{xy}^2.$$

The other variables and constants are explained in the notations.

The aircraft is controlled by means of the thrust force P, the elevator deflection δ_e, the rudder deflection δ_r, and the aileron deflection δ_a. Note that the aerodynamic force coefficients c_x, c_y, c_z and the aerodynamic moment coefficients m_x, m_y, m_z depend on $\delta_e, \delta_r, \delta_a$. The aerodynamic

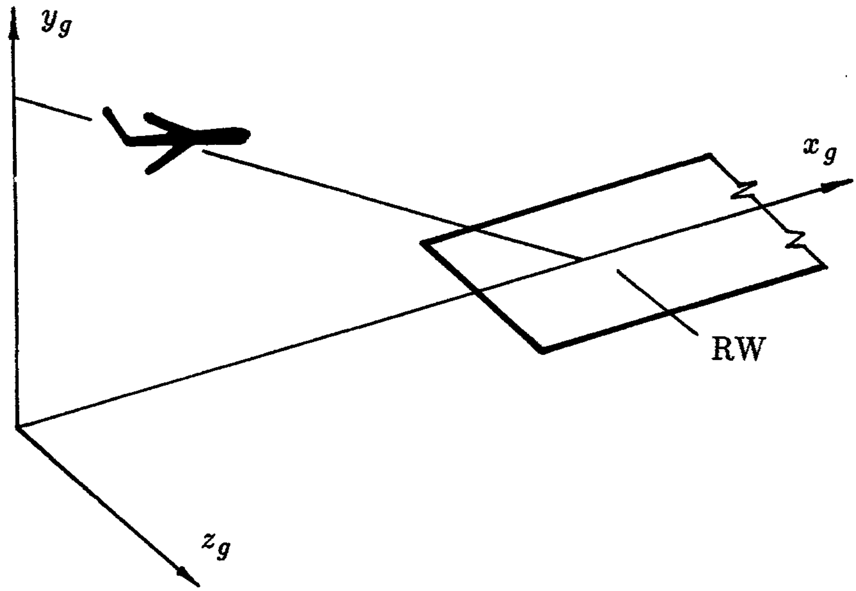

Fig. 1. Ground-fixed coordinate system.

coefficients depend also on the angle of attack α and the sideslip angle β, which can be computed as follows (Refs. 16–17):

$$\alpha = \arcsin\{[-\hat{V}_{xg}(\sin\psi\sin\gamma - \cos\psi\sin\vartheta\cos\gamma) - \hat{V}_{yg}\cos\vartheta\cos\gamma - \hat{V}_{zg}(\cos\psi\sin\gamma + \sin\psi\sin\vartheta\cos\gamma)]/(\hat{V}\cos\beta)\}, \tag{13}$$

$$\beta = \arcsin\{[\hat{V}_{xg}(\sin\psi\cos\gamma + \cos\psi\sin\vartheta\sin\gamma) - \hat{V}_{yg}\cos\vartheta\sin\gamma + \hat{V}_{zg}(\cos\psi\cos\gamma - \sin\psi\sin\vartheta\sin\gamma)]/\hat{V}\}. \tag{14}$$

The wind velocity components W_{xg}, W_{yg}, W_{zg} affect the relative velocity components $\hat{V}_{xg}$, $\hat{V}_{yg}$, $\hat{V}_{zg}$ via the relations

$$\hat{V}_{xg} = V_{xg} - W_{xg}, \quad \hat{V}_{yg} = V_{yg} - W_{yg}, \quad \hat{V}_{zg} = V_{zg} - W_{zg}, \tag{15}$$

where V_{xg}, V_{yg}, V_{zg} denote the absolute velocity components.

2.2. Numerical Data. In the numerical computations, we use the following data pertaining to a Tupolev TU-154 aircraft:

$m = 75 \times 10^3$ kg, $g = 9.81$ m sec^{-2}, $\rho = 1.207$ kg m^{-3},

$S = 201$ m^2, $l = 37.55$ m, $b = 5.285$ m,

$I_x = 2.5 \times 10^6$ kg m^2, $I_y = 7.5 \times 10^6$ kg m^2, $I_z = 6.5 \times 10^6$ kg m^2,

$I_{xy} = 0.5 \times 10^6$ kg m^2, $\sigma = 1.72$ deg.

2.3. Aerodynamic Coefficients. The aerodynamic force coefficients c_x, c_y, c_z in the system (1)–(12) refer to the body-axes system. They are related to the corresponding coefficients $\tilde{c}_x, \tilde{c}_y, \tilde{c}_z$ in the wind-body-axes system via the relations

$$c_x = \tilde{c}_x \cos\alpha - \tilde{c}_y \sin\alpha, \qquad c_y = \tilde{c}_y \cos\alpha + \tilde{c}_x \sin\alpha, \qquad c_z = \tilde{c}_z, \tag{16a}$$

where

$$\tilde{c}_x = 0.21 + 0.004\alpha + 0.47 \times 10^{-3}\alpha^2, \tag{16b}$$

$$\tilde{c}_y = 0.65 + 0.09\alpha + 0.003\delta_e, \tag{16c}$$

$$\tilde{c}_z = -0.0115\beta - (0.0034 - 6 \times 10^{-5}\alpha)\delta_r. \tag{16d}$$

Here and below, angular values are taken in degrees.

The aerodynamic moment coefficients m_x, m_y, m_z are specified by the following relations (Refs. 17–18), which pertain respectively to the rolling moment:

$$m_x = m_x^\beta \beta + m_x^r \delta_r + m_x^a \delta_a + (l/2\hat{V})(\pi/180)(m_x^x \omega_x + m_x^y \omega_y), \tag{17a}$$

$$m_x^\beta = -(0.0035 + 0.0001\alpha)\ \text{deg}^{-1}, \tag{17b}$$

$$m_x^r = -(0.0005 - 0.00003\alpha)\ \text{deg}^{-1}, \tag{17c}$$

$$m_x^a = -0.0004\ \text{deg}^{-1}, \tag{17d}$$

$$m_x^x = -0.61 + 0.004\alpha, \tag{17e}$$

$$m_x^y = -0.3 - 0.012\alpha, \tag{17f}$$

the yawing moment:

$$m_y = m_y^\beta \beta + m_y^r \delta_r + m_y^a \delta_a + (l/2\hat{V})(\pi/180)(m_y^x \omega_x + m_y^y \omega_y), \tag{18a}$$

$$m_y^\beta = -(0.004 + 0.00005\alpha)\ \text{deg}^{-1}, \tag{18b}$$

$$m_y^r = -(0.00135 - 0.000015\alpha)\ \text{deg}^{-1}, \tag{18c}$$

$$m_y^a = 0, \tag{18d}$$

$$m_y^x = 0.015\alpha, \tag{18e}$$

$$m_y^y = -0.21 - 0.005\alpha, \tag{18f}$$

and the pitching moment:

$$m_z = 0.033 - 0.017\alpha - 0.013\delta_e + 0.047\delta_{st} - 1.29\omega_z/\hat{V}. \tag{19}$$

Here, the constant δ_{st} is equal to 1.26 deg. The expressions (13)–(19) supplement Eqs. (1)–(12).

2.4. Time Lag of Servomechanisms. Let us assume that the change of the thrust force is subject to the relations

$$\dot{P} = -k_p P + \bar{k}_p(\delta_{ps} + \bar{\delta}_p), \tag{20a}$$

$$k_p = 1\ \mathrm{sec}^{-1}, \qquad \bar{k}_p = 3538\ \mathrm{N\,sec}^{-1}\,\mathrm{deg}^{-1}, \qquad \bar{\delta}_p = -41.3\ \mathrm{deg}, \tag{20b}$$

$$47 \leq \delta_{ps} \leq 112\ \mathrm{deg}. \tag{20c}$$

Here, δ_{ps} is the engine control lever setting. Substituting the extreme values $\delta_{ps} = 47$ deg and $\delta_{ps} = 112$ deg into the right side of Eq. (20a), we obtain for $\dot{P} = 0$ the corresponding stationary values $P \approx 2 \times 10^4$ N and $P \approx 25 \times 10^4$ N. If the initial value of P belongs to the segment $[2 \times 10^4, 25 \times 10^4]$, then it stays there later on.

The servomechanism dynamics for the control surfaces is specified in the simplest form via the following equations, which pertain respectively to the elevator:

$$\dot{\delta}_e = k_e(\delta_{es} - \delta_e), \tag{21a}$$

$$k_e = 4\ \mathrm{sec}^{-1}, \qquad |\delta_{es}| \leq 10\ \mathrm{deg}, \tag{21b}$$

the rudder:

$$\dot{\delta}_r = k_r(\delta_{rs} - \delta_r), \tag{22a}$$

$$k_r = 4\ \mathrm{sec}^{-1}, \qquad |\delta_{rs}| \leq 10\ \mathrm{deg}, \tag{22b}$$

and the ailerons:

$$\dot{\delta}_a = k_a(\delta_{as} - \delta_a), \tag{23a}$$

$$k_a = 4\ \mathrm{sec}^{-1}, \qquad |\delta_{as}| \leq 10\ \mathrm{deg}. \tag{23b}$$

The values $\delta_{es}, \delta_{rs}, \delta_{as}$ are the elevator setting, rudder setting, and aileron setting, respectively.

2.5. Complete Nonlinear System. Upon adding Eqs. (20)–(23) to the main system (1)–(12), we obtain a differential system in the vector form

$$\dot{\xi} = f(\xi, \delta_s, W), \tag{24}$$

where

$$\delta_s = (\delta_{ps}, \delta_{es}, \delta_{rs}, \delta_{as})^T, \qquad W = (W_{xg}, W_{yg}, W_{zg})^T$$

are the vectors of control and disturbance. Each of the control variables (scheduled deviations) $\delta_{ps}, \delta_{es}, \delta_{rs}, \delta_{as}$ has upper and lower bounds [see (20c), (21b), (22b), (23b)].

3. Minimax Control Law

The nominal aircraft motion during landing up to the time when the RW threshold is reached is a uniform motion (without rotation) along the descending glide path.

The aim of the control is to make the real motion close to the nominal motion. It is necessary to get the actual control without accurate a priori information on the location of the windshear zone and the wind velocity field in that zone. We assume the prior knowledge of approximate bounds for the deviations $\Delta W_{xg}, \Delta W_{yg}, \Delta W_{zg}$ of the wind velocity components W_{xg}, W_{yg}, W_{zg} from the nominal values $W_{xg0}, W_{yg0}, W_{zg0}$, which are assumed to be known as well. Some inertia characteristics of the wind velocity deviations can also be given. A minimax formulation of the control problem is natural, and the feedback control algorithm has to provide satisfactory landing trajectories for any disturbance realization chosen from the given class.

To solve the problem, we apply the minimax approach using the methods of DG theory (Refs. 1, 2, 19–22). Effective computer programs have been created (Refs. 5, 23–25) for finding the optimal control laws (strategies) in linear DGs with fixed terminal time and convex payoff function depending on two components of the state vector. The system (24) is nonlinear. However, we can linearize it, solve the auxiliary DGs for the longitudinal and lateral motions, and use the results for the original nonlinear system. So, having the nominal values $W_{xg0}, W_{yg0}, W_{zg0}$, the glide path inclination, and the nominal relative velocity, we can compute the values of the state variables corresponding to the nominal motion of the system (24). Linearizing (24) about the nominal motion, we obtain a linear controllable system decomposed into the VM and LM subsystems.

The state vector of the VM subsystem consists of the deviations Δx_g, Δy_g and the variables determining these deviations. The state vector of the LM subsystem consists of the deviation Δz_g and the quantities which determine it.

To take into account the inertia of the wind velocity variation along the trajectory, we introduce additional linear differential equations, for example,

$$\Delta \dot{W}_{xg} = k_1(\Delta F_{xg} - \Delta W_{xg}), \tag{25a}$$

$$\Delta \dot{F}_{xg} = k_2(w_{xg} - \Delta F_{xg}). \tag{25b}$$

Here, w_{xg} is a new independent variable and the constants k_1, k_2 determine the inertia of ΔW_{xg}. Similar equations are considered for $\Delta W_{yg}, \Delta W_{zg}$. The variables w_{xg}, w_{yg}, w_{zg} are interpreted as new disturbance factors. We add the equations for ΔW_{xg}, ΔW_{yg} to the VM subsystem and the equations for ΔW_{zg} to the LM subsystem.

For each of the subsystems, we consider an auxiliary DG with fixed terminal time t_f, geometric restrictions on the control variables and wind disturbances, and convex payoff function depending on two components of the state vector at the time t_f. In the VM subsystem, such components are Δy, $\Delta \dot{y}$; in the LM subsystem, such components are Δz, $\Delta \dot{z}$. The first player (pilot) chooses the control variables so as to minimize the payoff function. The second player (nature) chooses the wind disturbances so as to maximize the payoff function. In the auxiliary DG problems, it is not necessary to give the any physical meaning to the time t_f.

Upon solving the auxiliary DG problems on a digital computer, we find the optimal laws realized by means of a collection of switch lines. Each collection corresponds to a certain control variable and is defined on the grid of reverse time instants counted from the final time t_f. The switch line collections K_{es}, K_{as} for $\Delta\delta_{es}, \Delta\delta_{as}$ define the control components δ_{es}, δ_{as} in the system (24). Namely, we put $\delta_{es} = \Delta\delta_{es}$, $\delta_{as} = \Delta\delta_{as}$ because the nominal values $\delta_{es0}, \delta_{as0}$ are equal to zero. To use these controls, we continuously forecast the time remaining up to the time when the RW threshold is reached. Depending on it, certain switch lines from K_{es}, K_{as} are applied to choose the values δ_{es}, δ_{as}. With the help of δ_{es} [δ_{as}], we want to make sure that the deviations of the state coordinates y_g, V_{yg} [z_g, V_{zg}] from the nominal values along the trajectory are not too large; indeed, we are interested in making these deviations as small as possible at the time when the RW threshold is reached.

Each of the controls δ_{ps}, δ_{rs} has a special purpose during landing (stabilization of relative velocity and near-nullification of the sideslip angle); hence, it is not natural to make their choice in the system (24), similarly to δ_{es} and δ_{as}, on the base of the auxiliary DG problems mentioned above. Instead, we choose the controls δ_{ps}, δ_{rs} by the use of traditional (accepted nowadays) autopilot algorithms.

To sum up, concerning the minimax control, we find the controls δ_{es}, δ_{as} from the auxiliary linear DGs; on the other hand, we construct the controls δ_{ps}, δ_{rs} by traditional autopilot methods.

4. Auxiliary Linear Differential Games

4.1. Vertical (Longitudinal) Motion. The linear VM subsystem is given by

$$\dot{x} = A_* x + B_* u + C_* v, \qquad x \in R^{12}, \tag{26a}$$

$$A_* = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.0501 & 0 & -0.0973 & -2.6422 & 0 & 0.0628 & 0.9971 & 0.0501 & 0 & 0.0973 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2409 & 0 & -0.6387 & 45.2782 & 0 & 1.4479 & 0.0813 & -0.2409 & 0 & 0.6387 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0003 & 0 & 0.0069 & -0.5008 & -0.5263 & -0.3830 & 0 & -0.0003 & 0 & -0.0069 & 0 \\ & & & & & & -4 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & 0 & -1 & 0 & 0 & 0 & 0 \\ & & & & & & 0 & 0 & -0.5 & 0.5 & 0 & 0 \\ & & & & & & 0 & 0 & 0 & -3 & 0 & 0 \\ & & & & & & 0 & 0 & 0 & 0 & -0.5 & 0.5 \\ & & & & & & 0 & 0 & 0 & 0 & 0 & -3 \end{bmatrix}, \tag{26b}$$

$$B_* = \begin{bmatrix} 0, & 0, & 0, & 0, & 0, & 0, & 0 & 2.7, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 4, & 0, & 0, & 0, & 0, & 0 \end{bmatrix}^T, \tag{26c}$$

$$C_* = \begin{bmatrix} 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 3, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 3 \end{bmatrix}^T, \tag{26d}$$

$$x = [x_1, x_2, \dots, x_{12}]^T, \qquad u = [\Delta\delta_{ps}, \Delta\delta_{es}]^T, \qquad v = [w_{xg}, w_{yg}]^T. \tag{26e}$$

In this system, $x_1 = \Delta x_g$ and $x_3 = \Delta y_g$ are the deviations of x_g and y_g from the nominal motion, respectively; $x_5 = \Delta\vartheta$ is the deviation of the pitch angle; $x_7 = \Delta\delta_e$ is the deviation of the elevator from its trim position; $x_8 = \Delta P/m$ is the deviation of the thrust force-to-mass ratio. We describe the deviation ΔW_{xg} by means of the variables x_9 and x_{10}; the corresponding equations coincide with (25), with $x_9 = \Delta W_{xg}$. Similarly, the deviation ΔW_{yg} is described by the variables x_{11} and x_{12}, with $x_{11} = \Delta W_{yg}$. The control variables of the first player (pilot) are the deviations $\Delta\delta_{ps}$ of the engine control lever setting and $\Delta\delta_{es}$ of the elevator setting. The control variables w_{xg} and w_{yg} belong to the second player (nature) and are used to obtain the wind disturbances.

The bounds are the following:

$$|\Delta\delta_{ps}| \leq 27\pi/180 \text{ rad} = 27 \text{ deg}, \qquad |\Delta\delta_{es}| \leq 10\pi/180 \text{ rad} = 10 \text{ deg}, \tag{26f}$$

$$|w_{xg}| \leq 10 \text{ m sec}^{-1}, \qquad |w_{yg}| \leq 5 \text{ m sec}^{-1}. \tag{26g}$$

We introduce a function ϕ_* which depends on the coordinates $x_3 = \Delta y_g$ and $x_4 = \Delta\dot{y}_g$. We let M_* be a convex hexagon on the (x_3, x_4)-

plane, with apexes $(-3, 0)$, $(-3, 1)$, $(0, 1)$, $(3, 0)$, $(3, -1)$, $(0, -1)$. We suppose that

$$\phi_*(x_3, x_4) = \min\{c \geq 0: (x_3, x_4)^T \in cM_*\}. \tag{27}$$

We consider an antagonistic DG with dynamics (26), fixed terminal time t_f, and payoff function ϕ_* given by (27). The first player (pilot) tries to minimize the value of the function ϕ_* at the time t_f. The aim of the second player (nature) is the opposite. The set M_* can be considered as a tolerance for the deviations Δy_g and $\Delta \dot{y}_g$ at the time t_f; the function ϕ_* indicates a deviation from the tolerance. The optimal strategy of the first player in the game (26)–(27) will be used to define the elevator setting δ_{es} in the system (24).

4.2. Lateral Motion. The linear LM subsystem is given by

$$\dot{x} = A^*x + B^*u + C^*v, \qquad x \in R^{10}, \tag{28a}$$

$$A^* = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.0769 & -5.5553 & 0 & 9.2719 & 0 & -1.4853 & 0 & 0.0769 & 0 \\ 0 & 0 & 0 & 1.0013 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.0129 & -0.9339 & -0.2588 & -0.0883 & -0.0303 & -0.2456 & -0.0460 & 0.0129 & 0 \\ 0 & 0 & 0 & -0.0514 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -0.0331 & -2.3865 & -0.9534 & -0.2256 & -1.4592 & -0.2327 & -0.6894 & 0.0331 & 0 \\ & & & & & & -4 & 0 & 0 & 0 \\ & & & & & & 0 & -4 & 0 & 0 \\ & & & & & & 0 & 0 & -0.5 & 0.5 \\ & & & & & & 0 & 0 & 0 & -3 \end{bmatrix}, \tag{28b}$$

$$B^* = \begin{bmatrix} 0, & 0, & 0, & 0, & 0, & 0, & 4, & 0, & 0, & 0, \\ 0, & 0, & 0, & 0, & 0, & 0, & 0, & 4, & 0, & 0 \end{bmatrix}^T, \tag{28c}$$

$$C^* = [0, \ 0, \ 0, \ 0, \ 0, \ 0, \ 0, \ 0, \ 0, \ 3]^T, \tag{28d}$$

$$x = [x_1, x_2, \ldots, x_{10}]^T, \qquad u = [\Delta\delta_{rs}, \Delta\delta_{as}]^T, \qquad v = w_{zg}. \tag{28e}$$

In this system, $x_1 = \Delta z_g$ is the deviation of z_g from the nominal motion; $x_3 = \Delta\psi$ and $x_5 = \Delta\gamma$ are the deviations of the yaw and bank angles; $x_7 = \Delta\delta_r$ and $x_8 = \Delta\delta_a$ are the deviations of the rudder and the ailerons. We describe the deviation ΔW_{zg} by means of the variables x_9 and x_{10}, with $x_9 = \Delta W_{zg}$. The first player (pilot) control variables are the deviations $\Delta\delta_{rs}$

of the rudder and $\Delta\delta_{as}$ of the ailerons. The variable w_{zg} is used to obtain a wind disturbance and belongs to the second player (nature).

The bounds are the following:

$$|\Delta\delta_{rs}| \leq 10\pi/180 \text{ rad} = 10 \text{ deg}, \qquad |\Delta\delta_{as}| \leq 10\pi/180 \text{ rad} = 10 \text{ deg}, \tag{28f}$$

$$|w_z| \leq 10 \text{ m sec}^{-1}. \tag{28g}$$

We introduce a function ϕ^*, which depends on the coordinates $x_1 = \Delta z_g$ and $x_2 = \Delta\dot{z}_g$. We let M^* be a convex hexagon on the (x_1, x_2)-plane, with apexes $(-6, 0)$, $(-6, 1.5)$, $(0, 1.5)$, $(6, 0)$, $(6, -1.5)$, $(0, -1.5)$. We suppose that

$$\phi^*(x_1, x_2) = \min\{c \geq 0: (x_1, x_2)^T \in cM^*\}. \tag{29}$$

We consider an antagonistic DG with dynamics (28), fixed terminal time t_f, and payoff function ϕ^* given by (29). The first player (pilot) tries to minimize the value of the function ϕ^* at the time t_f. The aim of the second palyer (nature) is the opposite. The set M^* can be considered as a tolerance for the deviations Δz_g and $\Delta\dot{z}_g$ at the time t_f; the function ϕ^* indicates a deviation from the tolerance. The optimal strategy of the first player in the game (28)–(29) will be used to define the aileron setting δ_{as} in the system (24).

4.3. Remarks. In the systems (26) and (28), the geometric variables are in meters, the angles are in radians, the time is in seconds.

The coefficients of the systems (26) and (28) were obtained by the linearization of the system (24) with respect to the nominal motion. To compute the nominal motion, we use the following data: glide path inclination $\theta = -2.66$ deg; nominal relative velocity $\hat{V}_0 = 72.2 \text{ m sec}^{-1}$; nominal wind components $W_{xg0} = -5 \text{ m sec}^{-1}$, $W_{yg0} = W_{zg0} = 0$. For these data, the nominal values of the angle of attack and pitch angle are 5.42 deg and 2.94 deg, respectively; the nominal value P_0/m equals 1.66 N kg^{-1}; the nominal value δ_{ps0} is 74.43 deg. The values γ_0, ψ_0, β_0, ω_{x0}, ω_{y0}, ω_{z0}, δ_{e0}, δ_{r0}, δ_{a0}, δ_{es0}, δ_{rs0}, δ_{as0} are equal to zero.

The bounds on the control variables $\Delta\delta_{es}$, $\Delta\delta_{rs}$, $\Delta\delta_{as}$ in the auxiliary DGs are in concordance with those of (21b), (22b), (23b). The value $\delta_{ps0} = 74.43$ is not in the middle of the segment [47, 112]. We determine the bound on $\Delta\delta_{ps}$ by the quantity $\delta_{ps0} - 47 \approx 27$ deg.

The wind disturbance bounds in (26) and (28) are chosen according to reasonable consideration of the wind abilities. The constants k_1 and k_2 in the equations describing the inertia behavior of ΔW_{xg}, ΔW_{yg}, ΔW_{zg} [see (25)] are equal to 0.5 and 3, respectively.

5. Optimal First-Player Strategy in the Linear Differential Games

The main features of the linear DGs (26)–(27) and (28)–(29) are the following. Each game has a fixed stopping time and a convex payoff function depending only on two coordinates of the state vector. Such features simplify the solution of these games (Ref. 2). First, we transform the DG (26)–(27) [(28)–(29)] into an equivalent second-order game (Ref. 2). The relation between the state vector x of the system (26) [(28)] and the state vector $y = [y_1, y_2]^T$ of the equivalent game is described by the formula

$$y(t) = X_*(t_f - t)x(t) \qquad [y(t) = X^*(t_f - t)x(t)],$$

where $X_*(t_f - t)$ $[X^*(t_f - t)]$ is a matrix composed of the third and fourth [the first and second] rows of the fundamental Cauchy matrix for the homogeneous part of the system (26) [(28)]. The optimal control variables $\Delta\delta_{ps}$ and $\Delta\delta_{es}$ [$\Delta\delta_{rs}$ and $\Delta\delta_{as}$] in the DG (26)–(27) [(28)–(29)] are determined by means of switch surfaces in the space t, y_1, y_2 of the equivalent game (Refs. 5, 22–24, 26). On one side of the switch surface, the optimal control takes an extremal value with a particular sign; on the other side, the optimal control is opposite in sign. The algorithmic details can be found in Refs. 5, 23–25.

Every switch surface is realized on the computer via a set of sections on the given collection of reverse time instants $\tau = t_f - t$. These sections are called switch lines. The $\Delta\delta_{es}$ switch lines $\Pi_e(\tau)$, with $\tau = 7, 11, 14$, are shown in Fig. 2.

Denote by B_e the second column of the matrix B_*. This column corresponds to the control variable $\Delta\delta_{es}$ in the system (26). Let $x(t_i)$ be the state of the system (26) at the time t_i. If the point

$$y(t_i) = X_*(t_f - t_i)x(t_i)$$

lies in the direction of the vector $X_*(t_f - t_i)B_e$ with respect to the switch line $\Pi_e(t_f - t_i)$, then we use $\Delta\delta_{es} = -10$ on the next step of the discrete control scheme until the time t_{i+1}. We use $\Delta\delta_{es} = 10$ in the opposite situation at the point $\mathbf{y}(t_i)$. Similarly, the choice of the optimal control variable $\Delta\delta_{as}$ in the DG (28)–(29) at the time t_i is made with the help of the switch line $\Pi_a(t_f - t_i)$ and the vector $X^*(t_f - t_i)B_a$. Here, B_a is the second column of the matrix B^*. This column corresponds to the control variable $\Delta\delta_{as}$. As mentioned above, we use the optimal control variables $\Delta\delta_{es}$ and $\Delta\delta_{as}$ from the linear DGs (26)–(27) and (28)–(29) for choosing δ_{es} and δ_{as} in the original nonlinear system (24). The control variables δ_{ps} and δ_{rs} in the system (24) are chosen by traditional autopilot methods.

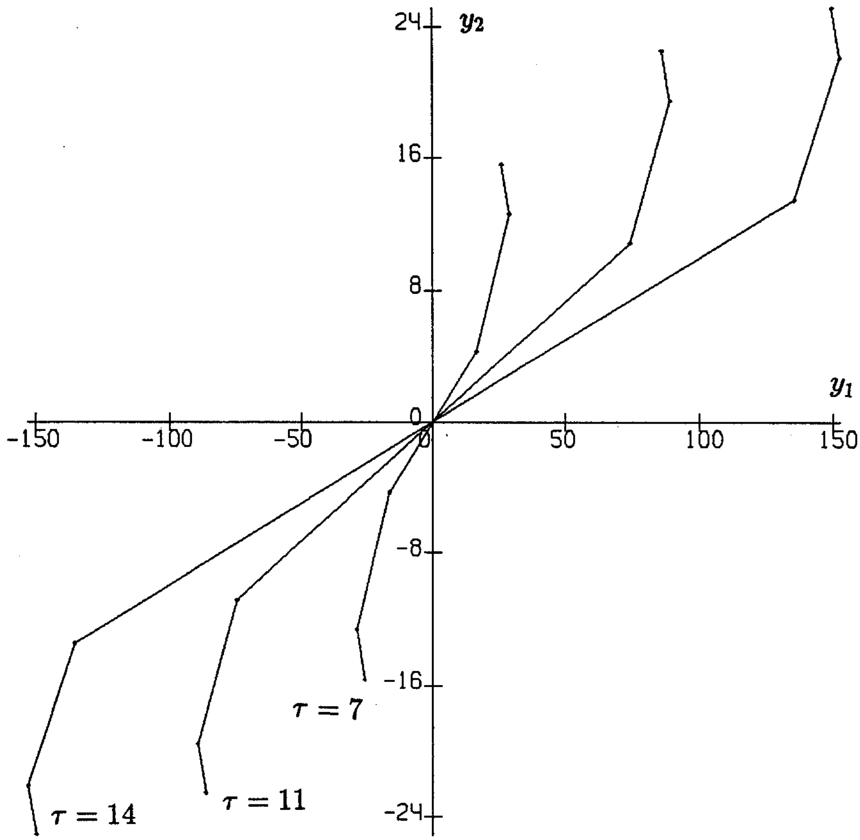

Fig. 2. Switch lines.

6. Downburst Models

To simulate the motion of the system (24), we suppose that the wind disturbance is associated with the aircraft flight through the downburst zone. Numerous papers devoted to downbursts have appeared recently. The downburst models used have been taken from Refs. 7 and 14–15. Below, we give a short outline of the models.

Model M1. See Ref. 14. The downburst is idealized as a three-dimensional axisymmetric vortex field. In this field, we distinguish the toroidal core region; in this region, the wind velocity is zero in the center and increases linearly along the radius to the frontier of the core. Outside the core, the vortex field is determined by the stream function. Differentiation of this function gives the radial and vertical components of the wind velocity. The radial component is resolved into two more components, the first parallel and the second orthogonal to the RW axis. The downburst is determined by three parameters: the modulus V_0 of the wind velocity vector in the central downburst point; the altitude H_0 of the central point; and the radius R_0 of the vortex. The core radius is equal to $0.8H_0$. The location of the downburst with respect to the glide path is determined by the coordinates of its center in the horizontal plane.

Model M2. See Ref. 15. The main distinction of this model from the former consists in the absence of explicit ring vortex streams.

Model M3. See Ref. 7. This model is given analytically. We modify this model by assuming that the wind velocity variations $\Delta W_{xg}, \Delta W_{zg}$ decrease to zero when the distance from the downburst center is large enough.

The downburst model M1 has the following parameters: $V_0 = 10\ \text{m sec}^{-1}$, $H_0 = 300\ \text{m}$, $R_0 = 500\ \text{m}$. The parameters of the downburst model M2 are chosen so as to have approximately the same downburst zone size as for the downburst model M1 and the same velocities at two particular points. A similar rule was used for the choice of the parameters of the downburst model M3. The structure of wind velocity field in the central vertical section for the downburst models M1–M3 is shown in Fig. 3.

7. Simulation Results

For the aircraft governed by the system (24), let the initial position x_{g*} at the time t_* be 8000 m from the RW threshold; let the values of all the state variables correspond to the nominal motion along the glide path.

We consider two methods of control for the system (24). Method I_1 uses the traditional autopilot algorithms (Refs. 18 and 27) for δ_{ps}, δ_{es}, δ_{rs}, δ_{as}. Method I_2 is the minimax law; in this method, the control variables δ_{es}, δ_{as} are constructed via the switch lines obtained from the auxiliary DGs (26)–(27) and (28)–(29). The control parameters δ_{ps}, δ_{rs} are constructed with the help of traditional autopilot algorithms. Let $t_f = 15$ sec for both auxiliary DGs.

The peculiarity of the minimax control is the possibility of frequent switches from one extremal control value to another. To diminish the number of switches, we introduce a buffer zone near the switch lines. Because of it, the control variables have a gradual change.

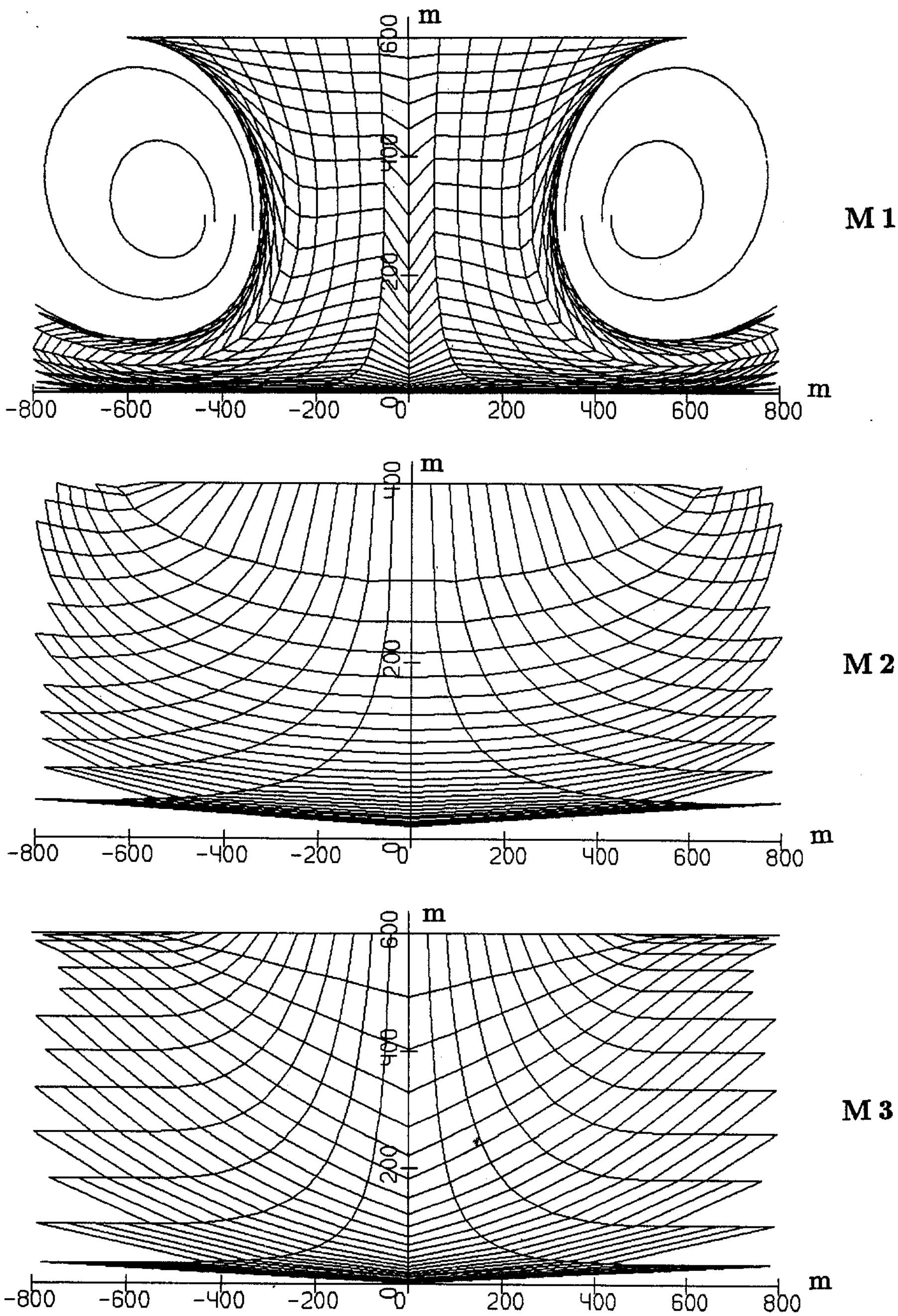

Fig. 3. Downburst models; wind velocity structure in a vertical plane.

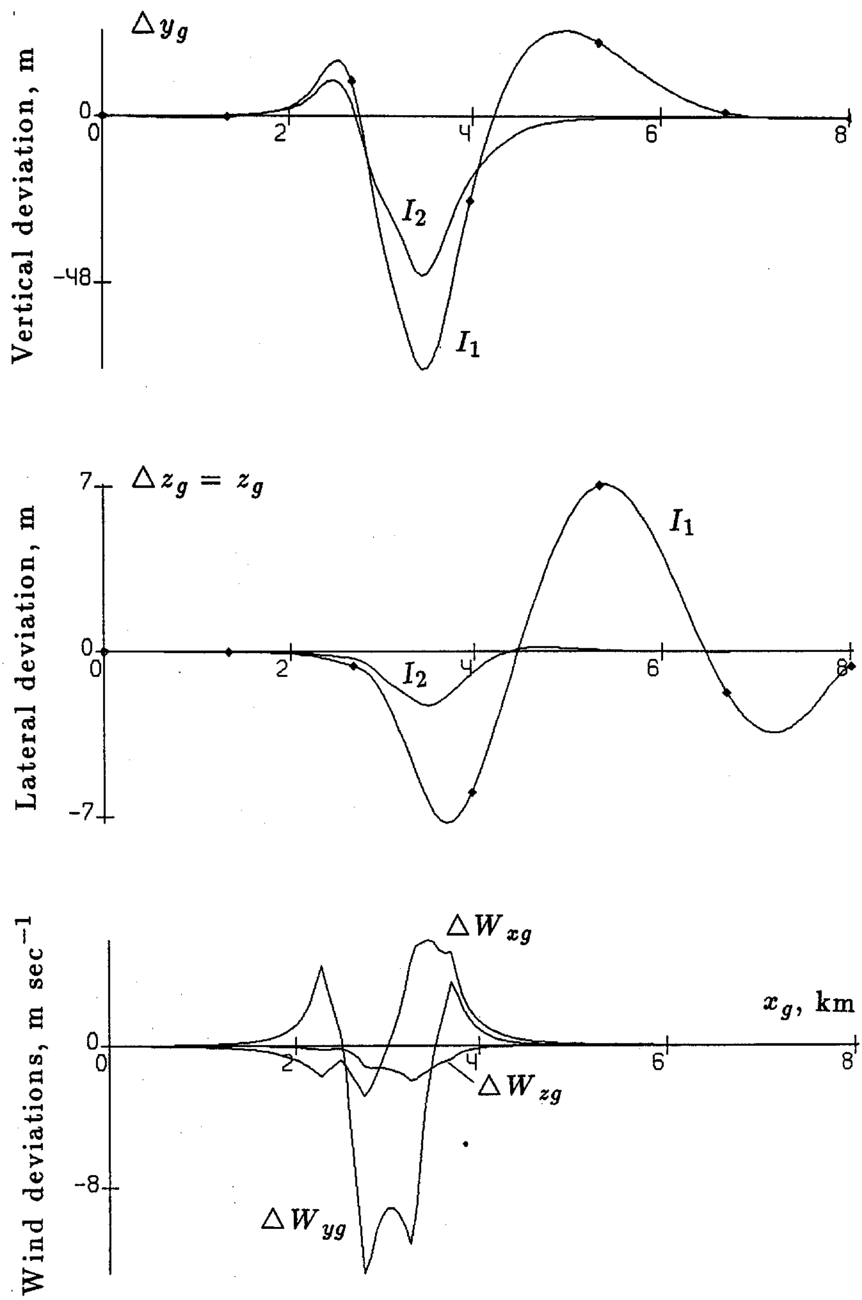

Fig. 4a. Landing simulation results for Model M1; downburst center $DX = 3000$ m, $DZ = 100$ m.

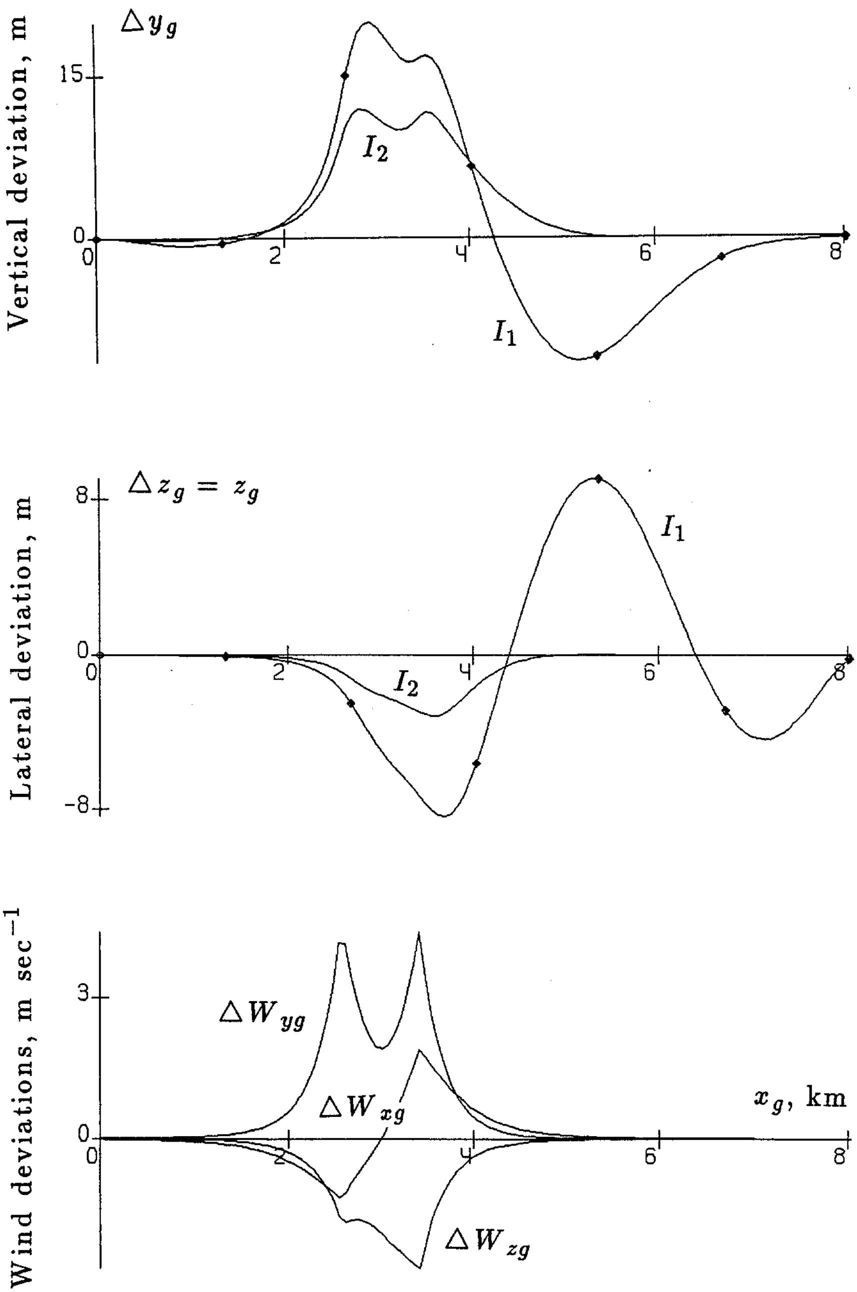

Fig. 4b. Landing simulation results for Model M1; downburst center $DX = 3000$ m, $DZ = 600$ m.

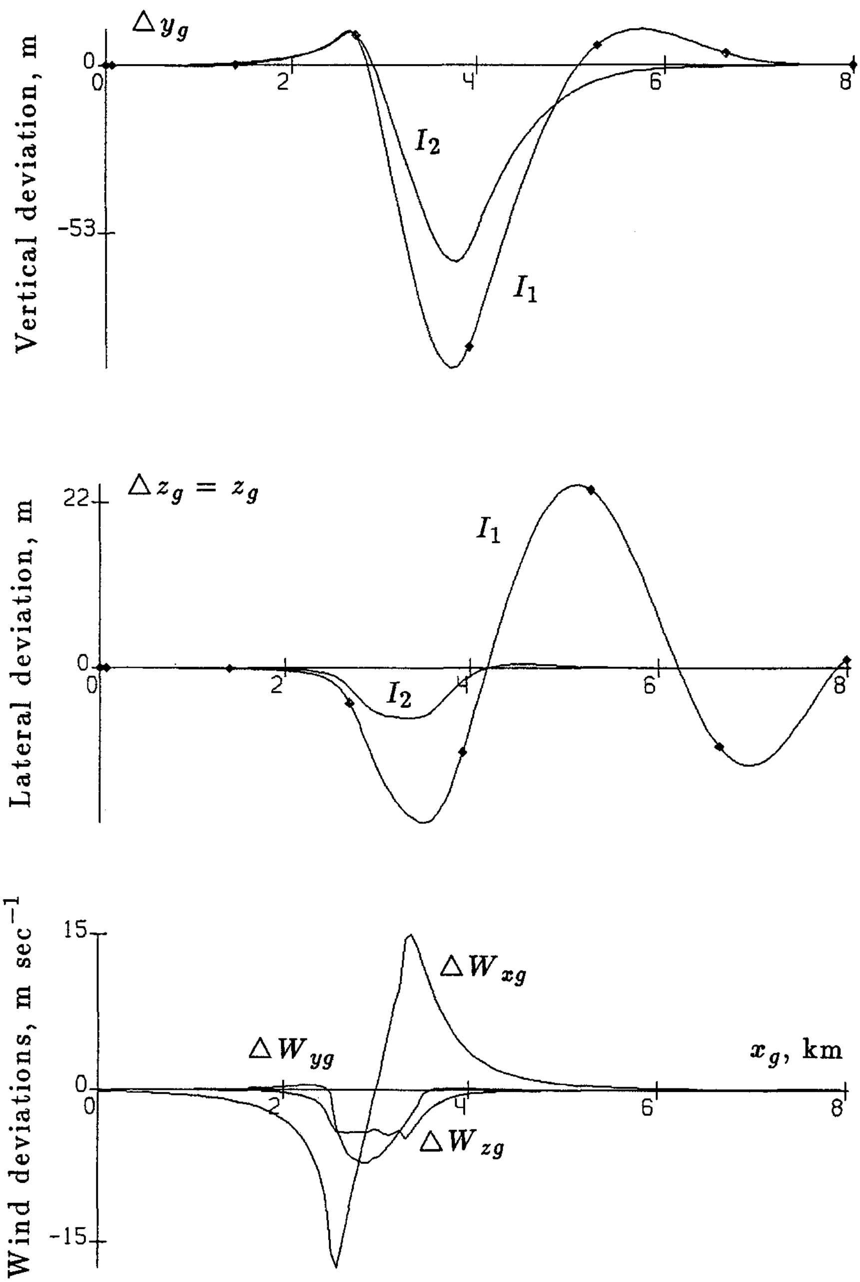

Fig. 5a. Landing simulation results for Model M2; downburst center $DX = 3000$ m, $DZ = 100$ m.

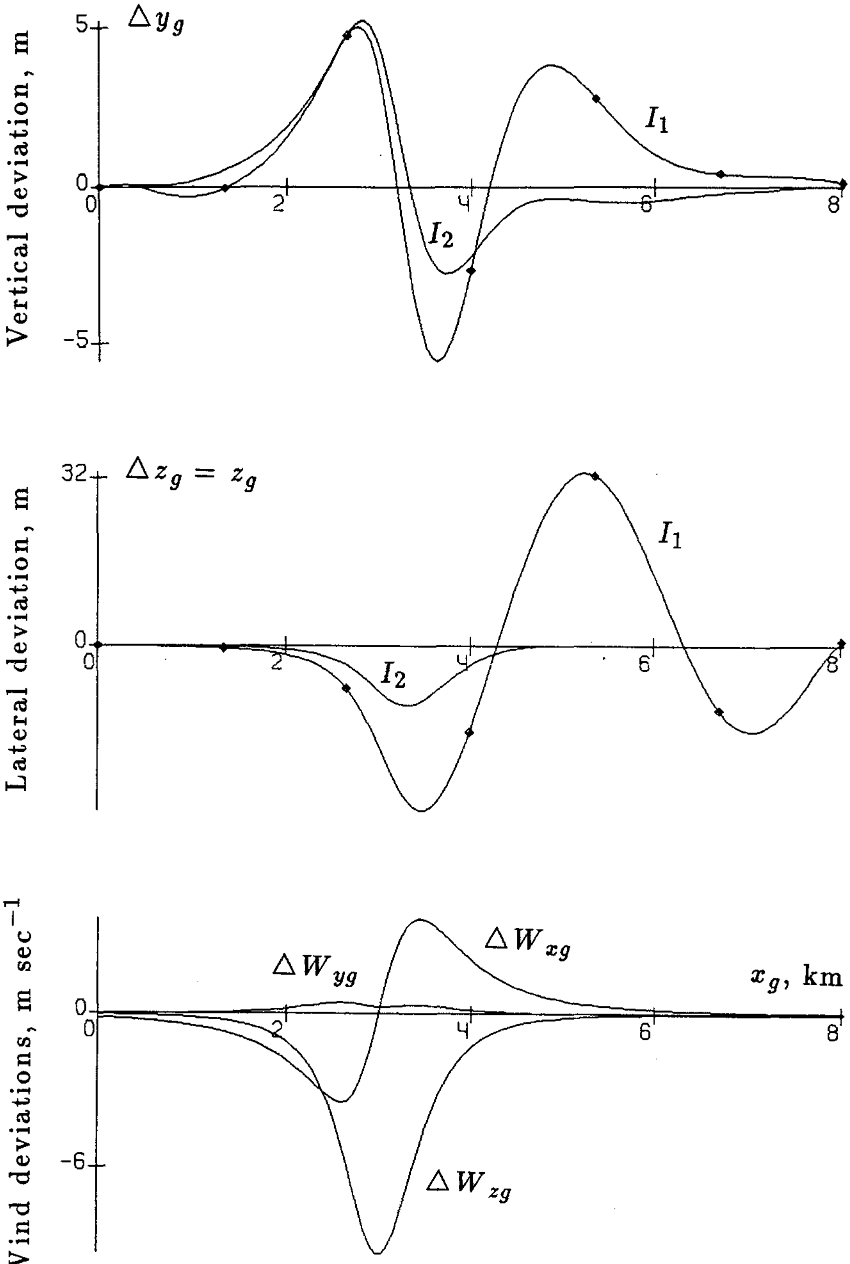

Fig. 5b. Landing simulation results for Model M2; downburst center $DX = 3000$ m, $DZ = 600$ m.

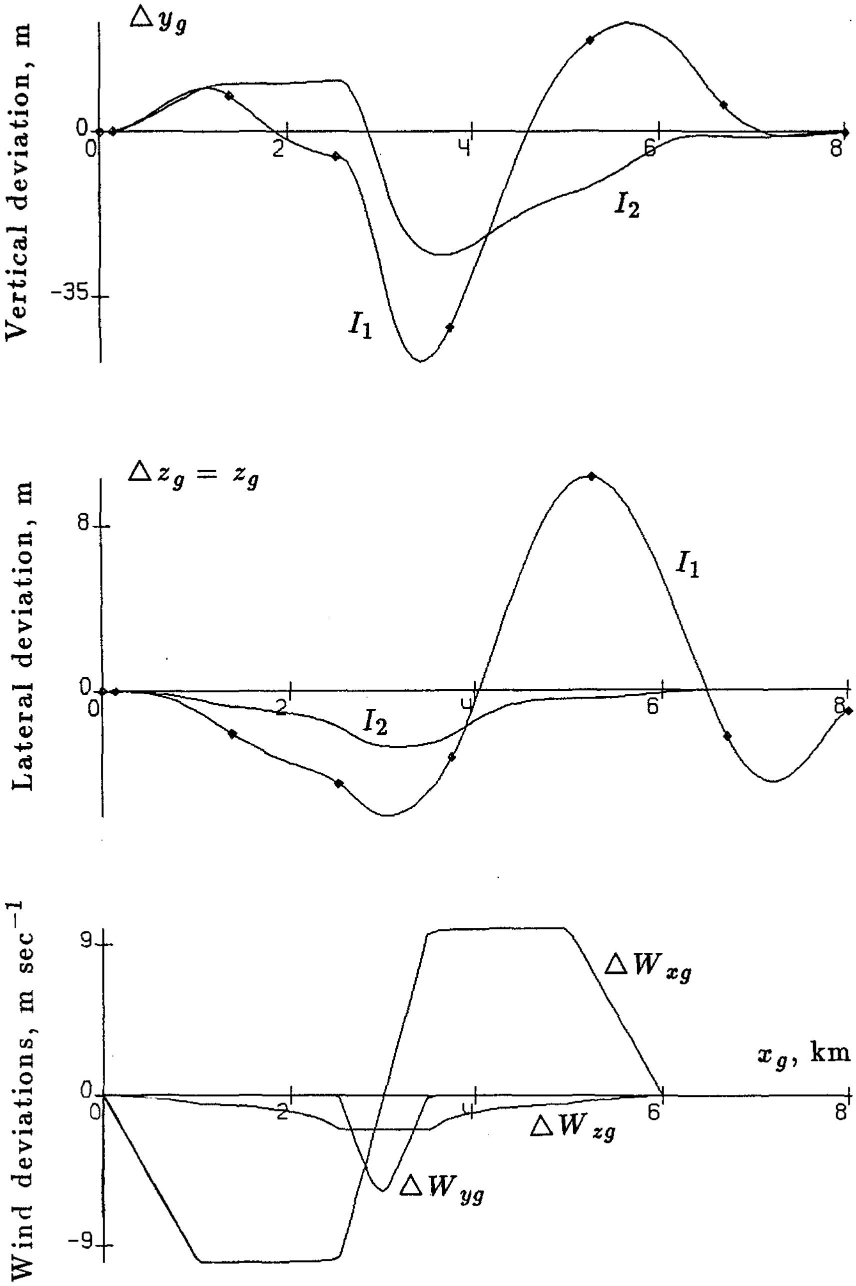

Fig. 6a. Landing simulation results for Model M3; downburst center $DX = 3000$ m, $DZ = 100$ m.

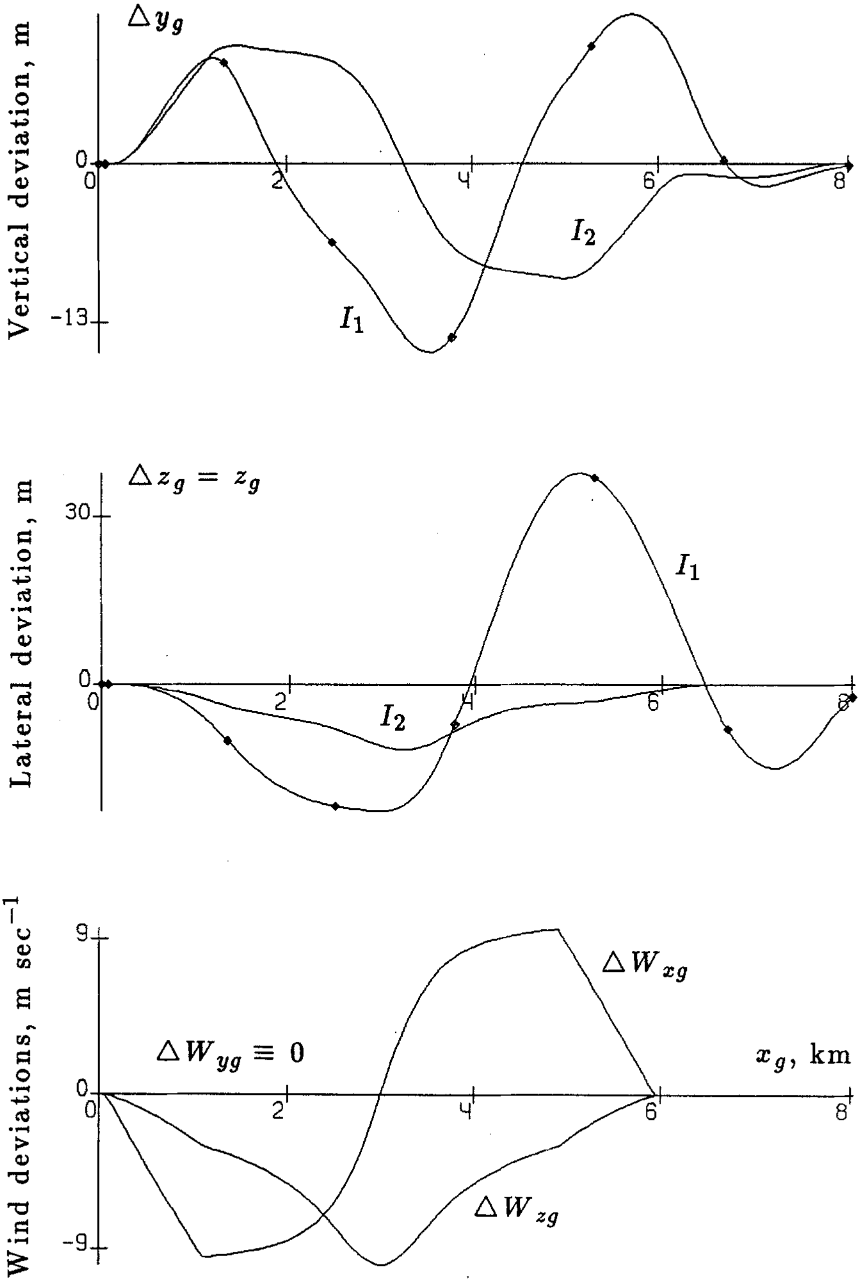

Fig. 6b. Landing simulation results for Model M3; downburst center $DX = 3000$ m, $DZ = 600$ m.

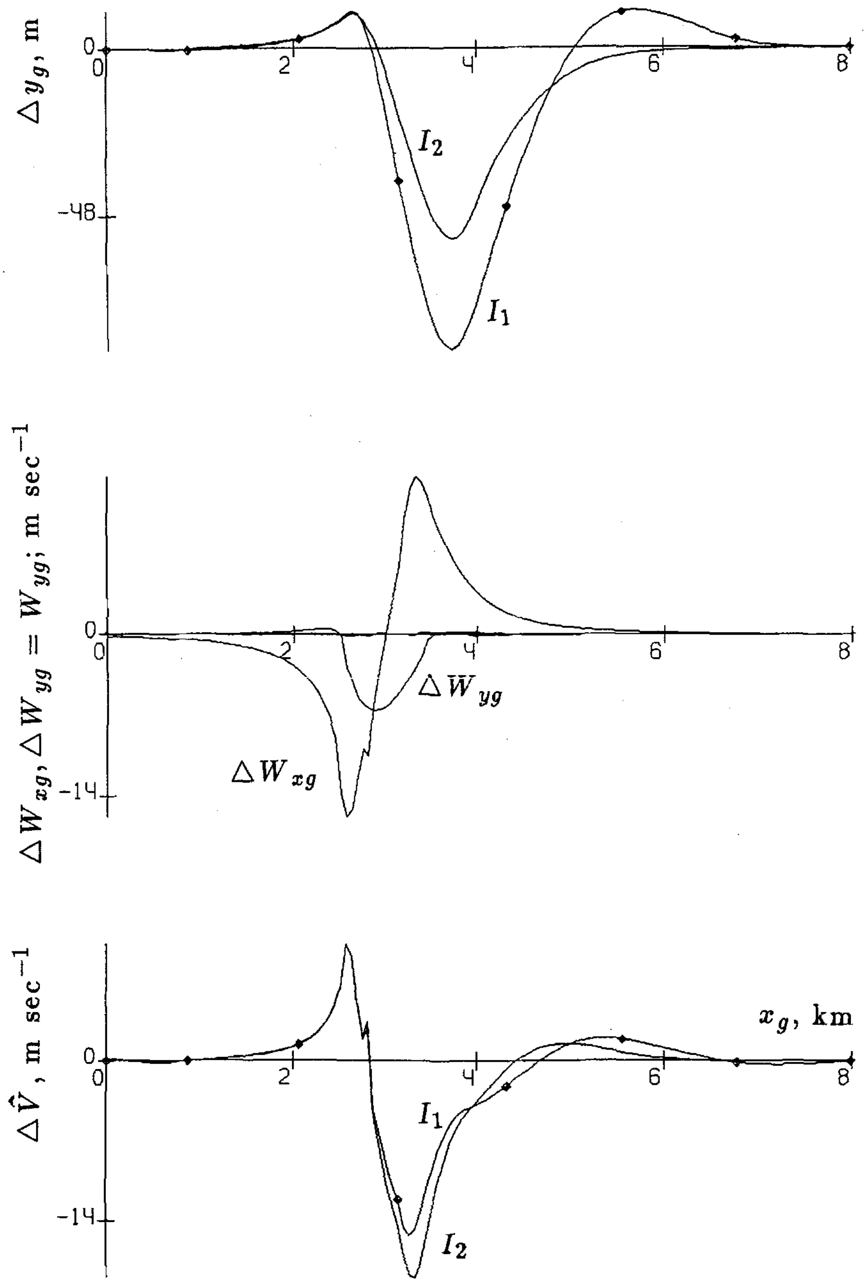

Fig. 7a. Deviation histories: altitude y_g, wind velocity components W_{xg} and W_{yg}, relative velocity $\hat{V}$; Model M2, downburst center $DX = 3000$ m, $DZ = 200$ m.

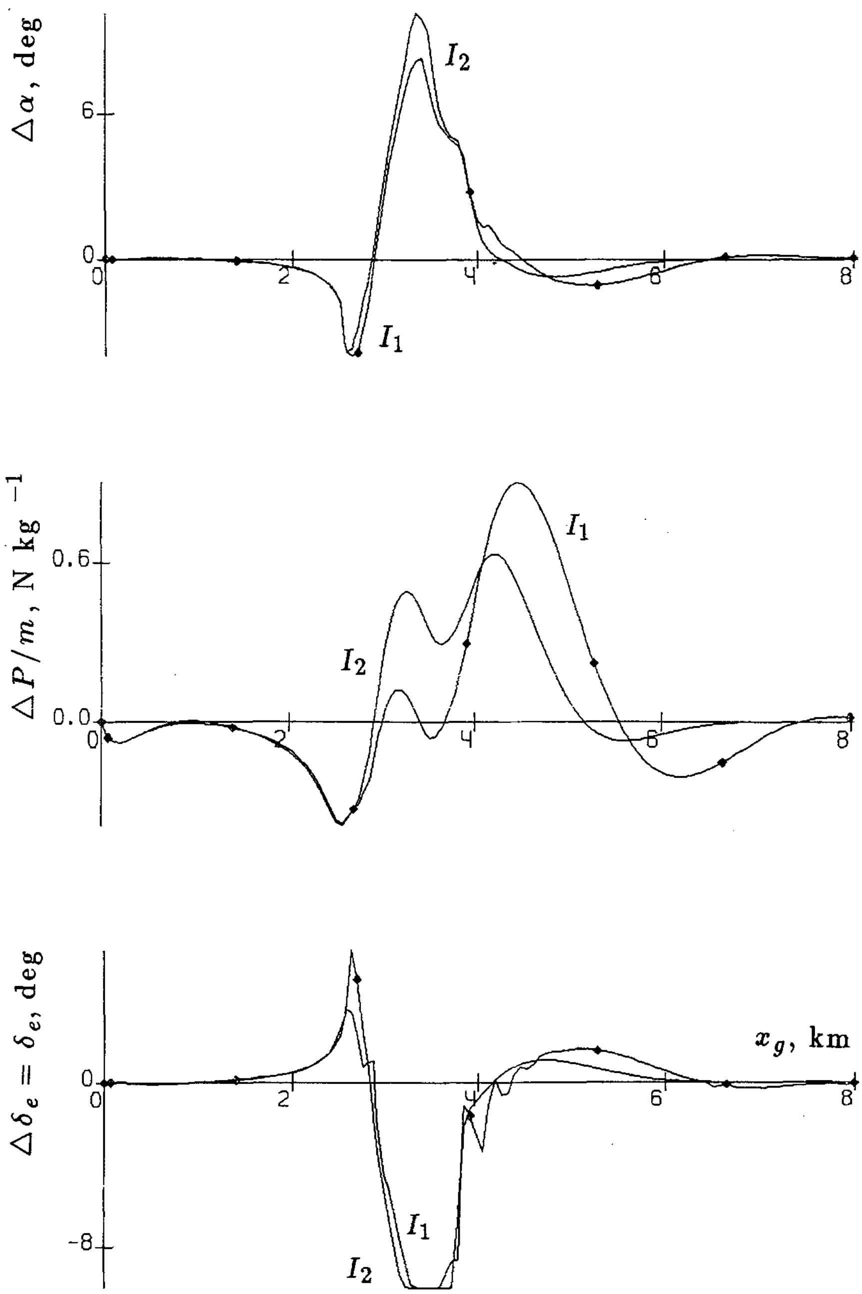

Fig. 7b. Deviation histories: angle attack α, ratio P/m, elevator deflection δ_e; Model M2, downburst center $DX = 3000$ m, $DZ = 200$ m.

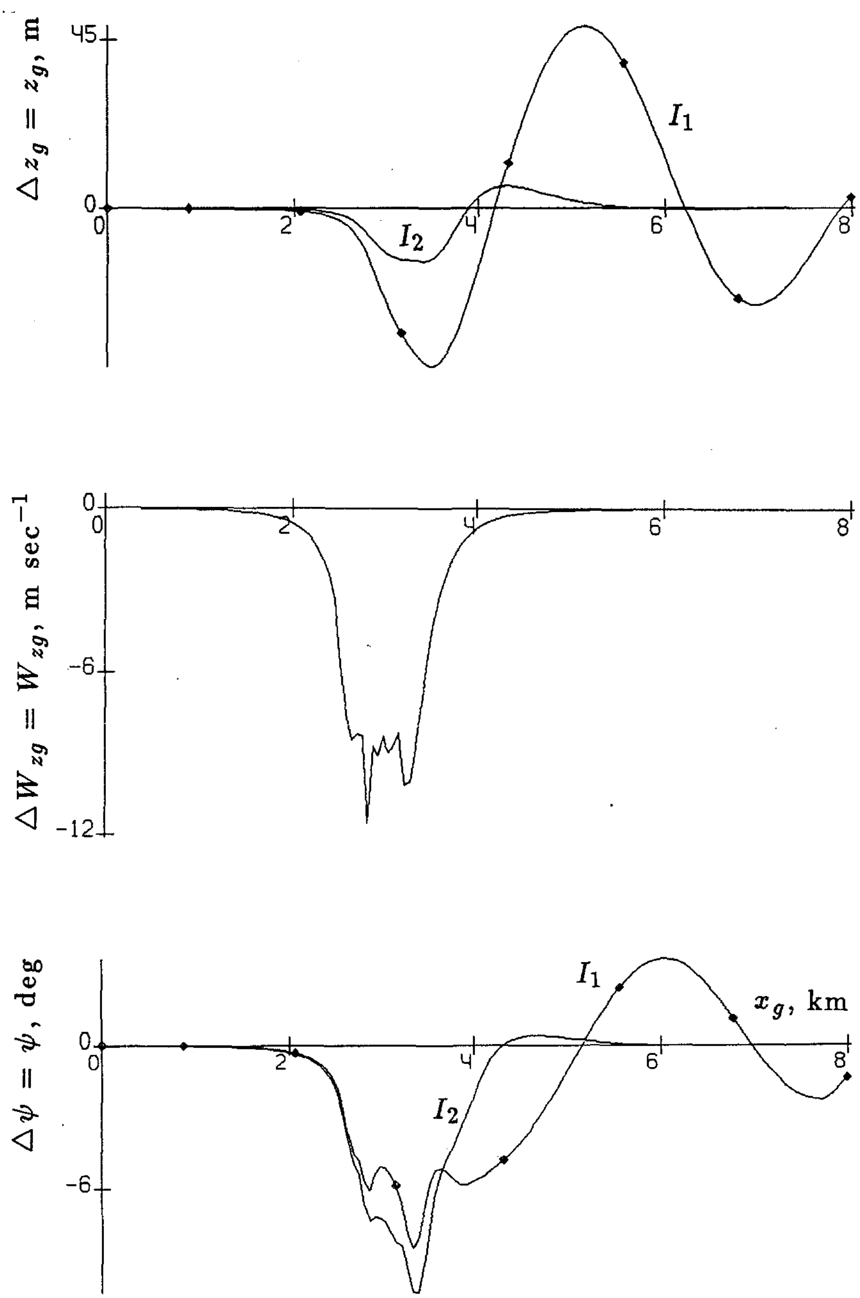

Fig. 7c. Deviation histories: lateral position z_g, wind velocity component W_{zg}, yaw angle ψ; Model M2, downburst center $DX = 3000$ m, $DZ = 200$ m.

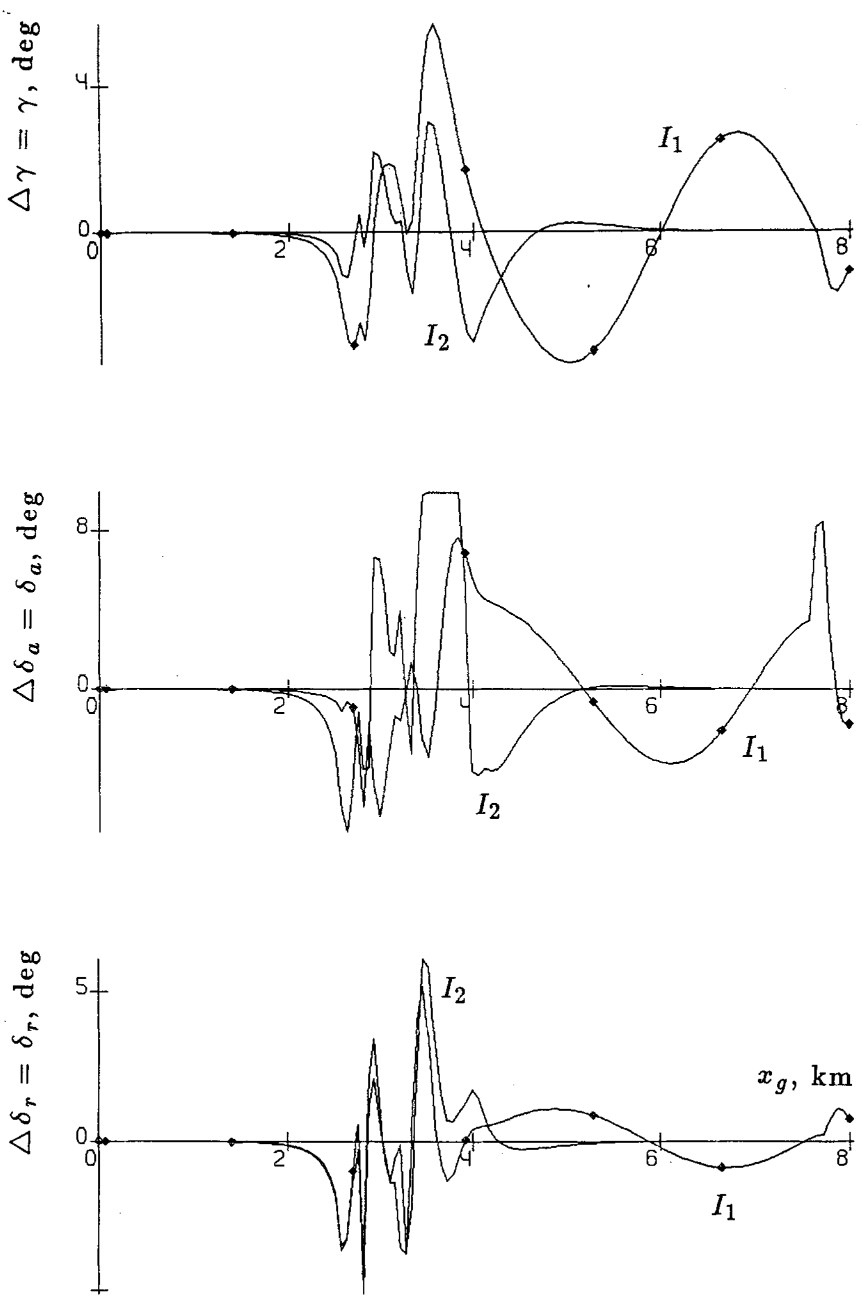

Fig. 7d. Deviation histories: bank angle γ, aileron deflection δ_a, rudder deflection δ_r; Model M2, downburst center $DX = 3000$ m, $DZ = 200$ m.

Denote by E a collection of reverse time instants τ_i on the interval $[0, t_f] = [0, 15]$. We suppose that the switch lines $\Pi_e(\tau_i)$, $\Pi_a(\tau_i)$ have been built for every $\tau_i \in E$. In Method I_2, the switch lines are used in the following way. Let $d(t)$ be the distance in x_g up to the RW threshold at the time $t \geq t_*$, and let V_{xg0} be the nominal velocity in x_g. Then, $s(t) = d(t)/V_{xg0}$ is the forecasted time remaining until the aircraft reaches the RW threshold. To obtain δ_{es} (δ_{as}) for $s(t) \geq t_f = 15$ sec, we use the same switch line corresponding to $\tau = t_f$. If $s(t) < t_f$, we use the line corresponding to the time $\tau_i \in E$ nearest to $s(t)$. So, our control is comparatively rough, while $d(t) \geq V_{xg0} t_f \approx 1000$ m. If $d(t) \leq V_{xg0} t_f$, the control is more qualitative.

The wind velocity components W_{xg}, W_{yg}, W_{zg} in the system (24) are calculated by the formulas

$$W_{xg} = \Delta W_{xg} + W_{xg0}, \qquad W_{yg} = \Delta W_{yg} + W_{yg0}, \qquad W_{zg} = \Delta W_{zg} + W_{zg0};$$

here, ΔW_{zg}, ΔW_{yg}, ΔW_{zg} are taken from the downburst model; also,

$$W_{xg0} = -5 \text{ m sec}^{-1}, \qquad W_{yg0} = W_{zg0} = 0.$$

The simulation results for Methods I_1 and I_2 are shown in Figs. 4–7. Traditional autopilot control curves are marked by diamonds, while minimax control curves are unmarked. The time step for control and wind disturbance computation was equal to 0.2 sec. In Method I_2, we had at our disposal the switch lines calculated with the time step 0.05 sec. For all the graphs, the horizontal axis yields the distance x_g from the initial point.

Figure 4 corresponds to Model M1; Fig. 5 was calculated for Model M2, and Fig. 6 was calculated for Model M3. Each of Figs. 4–6 contains two parts. Part a corresponds to the following downburst center location in the horizontal plane: longitudinal displacement from the initial aircraft position is $DX = 3000$ m; lateral displacement is $DZ = 100$ m. For Part b, these displacements are $DX = 3000$ m and $DZ = 600$ m. We give graphs of the vertical and lateral deviations Δy_g, Δz_g from the nominal motion, and also the wind deviations ΔW_{xg}, ΔW_{yg}, ΔW_{zg} for Method I_2; the wind deviations ΔW_{xg}, ΔW_{yg}, ΔW_{zg} for Method I_1 are practically the same. Note that the deviations of the wind velocity components during modelling do not necessarily satisfy (see Figs. 4a and 5a) the bounds of the auxiliary linear DGs.

It can be seen that the results for the minimax Method I_2 are better than those for the traditional autopilot Method I_1.

For Model M2 with center location $DX = 3000$ m and $DZ = 200$ m, Fig. 7 shows the deviation histories of the following quantities: altitude, wind velocity components W_{xg}, W_{yg}, and relative velocity (Fig. 7a); angle of attack, ratio P/m, and elevator deflection (Fig. 7b); lateral position, wind velocity component W_{zg}, yaw angle (Fig. 7c); bank angle, aileron deflection, rudder deflection (Fig. 7d).

8. Conclusions

This paper is devoted to aircraft guidance in landing under the action of severe wind disturbances. To find an appropriate feedback control, we apply numerical methods based on DG theory. We consider the motion of the aircraft in landing up to the time when the RW threshold is crossed. A priori information about the wind is supposed to be minimal: only the average values and deviation scales are assumed to be known.

Our approach is the following. We linearize the complete nonlinear system of the aircraft dynamics equations with respect to the nominal motion. The resulting linear system is decomposed into subsystems describing the vertical (longitudinal) and lateral motions. For each subsystem, an auxiliary DG with fixed terminal time t_f, bounds on control variables and wind disturbances, and convex payoff function depending on two components of the state vector at the time t_f is formulated. The first player (pilot), who governs the control variables, minimizes the payoff function. The aim of the second player (nature) is the opposite. In the auxiliary DG problems, it is not necessary to give any physical meaning to the time t_f.

Reducing the auxiliary DGs to two-dimensional DGs by certain transformations, we make them solvable with the help of specialized and efficient computer programs. As a result, optimal minimax controls are designed via switch surfaces. Every switch surface is realized on the computer via a set of sections (switch lines) on the given time grid.

The control obtained is applied to the original nonlinear problem. In particular, to use the switch lines properly, we correct continuously the time of process termination. In simulating the aircraft motion, we employ different variants of the wind disturbances. In this paper, we suppose that the wind disturbance is associated with the aircraft flight through the downburst zone. The simulation results are essentially better for the proposed control than for traditional autopilot algorithms.

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