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Informational Sets in a Model Problem of Homing[1]

S. I. KUMKOV[2] AND V. S. PATSKO[3]

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Abstract. A linear pursuit problem in the plane under incomplete pursuer information about the evader is investigated. At discrete time instants, the pursuer measures with errors the angle of vision to the evader, the angular velocity of the line of sight, and the relative distance. Other combinations of measurable parameters are possible (for example, angle of vision and relative distance or angle of vision only). The measurements errors obey certain geometric constraints. The initial uncertainties on the evader coordinates and velocities are given in advance. Having a resource of impulse control, the pursuer tries to minimize the miss distance. The evader control is bounded in modulus.

The problem is formulated as an auxiliary differential game. Here, the notion of informational set is central. The informational set is the totality of pointwise phase states consistent with the history of the observation-control process. The informational set depends on the current measurements; it changes in time and plays the role of a generalized state, which is used for constructing the pursuer control.

A control method designed for the linear pursuit problem is used in the planar problem of a vehicle homing toward a dangerous space object. The nonlinear dynamics is described by the Kepler equations. Nonlinear terms of the equations in relative coordinates are small and are replaced by an uncertain vector parameter, which is bounded in modulus and is regarded as an evader control. As a result, we obtain the mentioned control problem in the plane.

The final part of the paper is devoted to the simulation of a space vehicle homing toward a dangerous space object. In testing the control method developed, two variants are considered: random measurement errors and game method of constructing the measurements; the latter is also described in the paper.

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[2]Senior Research Scientist, Institute of Mathematics and Mechanics, Russian Academy of Sciences, Ekaterinburg, Russia.

[3]Senior Research Scientist, Institute of Mathematics and Mechanics, Russian Academy of Sciences, Ekaterinburg, Russia.

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Notations

B = geometric constraint on the evader initial relative position vector;
d = true relative distance, m;
d_m = measured value of the relative distance, m;
D = geometric constraint on the evader initial relative velocity vector;
e = modulus of the nominal initial relative velocity vector, m sec^{-1};
E_{nom} = nominal initial state of the evader in the relative coordinate system;
h = time step of the discrete control scheme, sec;
H_x, H_α, H_ω = uncertainty sets of measurements: longitudinal distance, angle of vision, angular velocity;
I_-, I, I_+ = informational sets: before a measurement, after it, after applying an impulse control;
$k(t_i)$ = impulse control at the time instant t_i, m sec^{-1};
K = gravitational constant of the Sun, km^3 sec^{-2};
r_V, r_D = vehicle and dangerous space object radial distances to the Sun, km;
u_x, u_z = components of the vehicle control vector, m sec^{-2};
U_s = strategy of maintaining the symmetry of the forecast miss distance;
v_x, v_z = evader acceleration components, m sec^{-2};
V_x, V_z = relative velocity components, m sec^{-1};
V_{nom} = nominal value of the initial relative velocity vector, m sec^{-1};
x, z = axes of the relative coordinate system;
x_D, z_D = longitudinal and lateral coordinates of the dangerous space object in an inertial heliocentric coordinate system, km;
x_V, z_V = longitudinal and lateral coordinates of the space vehicle in an inertial heliocentric coordinate system, km;
α = true angle of vision, rad;
α_m = measured angle of vision, rad;
β = measurement error of the relative distance, m;
ϵ = longitudinal distance threshold for interruption of control and construction of the informational sets, m;
ζ = measurement error of the angle of vision, rad;
η_*, η^* = left and right edges of the interval of the forecast miss distance, m;
μ = bound on the resource of impulse control, m sec^{-1};

μ_1, μ_2 = lower and upper bounds on the modulus of the impulse control at the current time instant, m sec^{-1};
ν = bound on the modulus of the evader acceleration, m sec^{-2};
ξ = measurement error of the line-of-sight angular velocity, rad sec^{-1};
Π = passive forecast miss distance, m;
ω = true line-of-sight angular velocity, rad sec^{-1};
ω_m = measured line-of-sight angular velocity, rad sec^{-1}.

Acronyms

DG = differential game;
DSO = dangerous space object;
IS = informational set;
SV = space vehicle.

1. Introduction

Quite often in control problems, the phase state vector cannot be measured in full. Instead, a certain vector depending on the phase state is measured with an induced error. In the latter case, two types of problem formulation are known. One formulation includes certain probabilistic assumptions about the measurement error. In the other formulation, one imposes certain geometric constraints on the measurement error. The latter approach is investigated in this paper.

Under geometric constraints, it is known only that the measurement error belongs to some given set. No more data on the error are assumed to be known. Using the incoming measurements, it is hypothetically possible to find the totality of all phase states that do not contradict the obtained information and therefore are admissible; this is called the informational set (IS). The IS changes in time, plays the role of a generalized state, and can be used for constructing the pursuer feedback control.

Since the IS depends on the current measurement, minimax formulations are natural. In such formulations, the first player governs the object control and tries to minimize a certain performance index, while the second player (nature) forms the measurements by maximizing the value of the performance index.

The minimax approach to control problems with incomplete information had been studied intensively in the middle of the 70's; see Refs. 1–5 and references therein. The main efforts were directed toward problem formulation and transfer of the basic definitions and statements of differential game (DG) theory to problems with incomplete information. We note that, in real problems, the construction of the IS is a difficult task, and this is more complicated when building optimal or near-optimal feedback controls. For some actual problems solved analytically, see Refs. 3 and 6–7.

This paper deals with the following model problem of pursuit. Two material points (pursuer and evader) move in the plane. The pursuer measures the direction to the evader (i.e., the angle of vision), the angular velocity of the line of sight, and the relative distance. Other variants are possible; for example, the measured quantities might include the angle of vision and the relative distance or the angle of vision only. The measurement errors obey certain geometric constraints. The aim of the control is the minimization of the miss distance (sometime shortened to miss). The pursuer control is applied impulsewise. When applying a control impulse, the pursuer velocity changes stepwise. The impulses operate orthogonally to some chosen direction that is kept constant in time: this is the direction leading from the initial pursuer position to the nominal initial position of the evader.

The following assumptions are made: by appropriate choice of the initial velocity, a pursuer can force the nominal relative velocity vector to be opposite to the mentioned direction; the resource of impulse control is given in advance; the evader control affects its acceleration and obeys certain geometric constraints. Also, we assume that the variation of the relative velocity vector, which occurs during the motion due to the pursuer and evader controls, is small (weak controllability assumption). Indeed, the pursuer impulse control is applied to correct a motion, which is governed entirely by the initial conditions.

A pursuit problem close to the above was investigated in Refs. 8–9, where the stochastic nature of the measurement error was assumed.

In this paper, the pursuer feedback control is constructed on the basis of the investigation of a special auxiliary DG, in which the state at the current time instant is represented by the IS. A functional of the miss distance is introduced on the IS motions. This functional is minimized by the first player who governs the choice of the impulse control; the second player (nature) maximizes the functional.

The simulation results of pursuit in the plane, when the pursuer uses the control method obtained from the analysis of the aforementioned DG, can be found in Refs. 10–14. In this paper, we apply such method to the construction of the control in the problem of a space vehicle (SV) homing toward an asteroid of a relatively small size or toward some other dangerous space object (DSO). The aim of homing is the DSO destruction or at least a change of the DSO trajectory. Similar problems, related to the Earth protection from DSO, are investigated nowadays; see e.g. Refs. 15–17.

We assume that the SV and DSO orbits are coplanar. The Kepler equations of motion are used. The original nonlinear problem is replaced by a linear problem of pursuit. The evader control is a fictitious one. It is regarded as a parameter that describes an error induced by a transformation of the nonlinear system into the linear one. The constraint on the evader control is given from the estimation of the linearization error. The problem of pursuit in the plane is an intermediate one between the original nonlinear problem and the auxiliary DG.

The results of computer simulation of the SV-to-DSO homing process are presented. To test the suggested method of SV control, two variants of the measurement construction are applied: one variant uses a random number generator; another variant is the game construction, which results from the auxiliary DG.

2. Pursuit Problem in the Plane

Consider now a problem of pursuit of one material point by another under incomplete information of the pursuer P about the evader E. Let us join the origin of the relative coordinate system with the pursuer. Assume that, at the initial time instant t_0, a nominal (precalculated) evader state E_{nom} and a nominal velocity vector V_{nom} are given in the relative coordinate system. We suppose that the vector V_{nom} is applied at the point E_{nom} and is directed to the point P, i.e., to the origin of the system. The direction of the x-axis of the relative coordinate system is opposite to the direction of the vector V_{nom}; the z-axis is orthogonal to the x-axis (Fig. 1). The directions of both axes are constant in the time t. The angle between the x-axis and the line of sight, joining the origin P of the relative coordinate system with the point E, is called the angle of vision α. The true evader position E at the initial time instant can differ from the nominal one, and the true initial relative velocity vector can differ from the nominal one. The modulus of the vector V_{nom} is denoted by e.

The pursuer control is impulsive. The pursuer can change his velocity, and correspondingly the relative velocity, in a stepwise manner. The impulses operate perpendicularly to the x-axis, i.e., along the z-axis positively or negatively. Such directions of the impulses provide economical expenditure of the control resource in the problem of the miss distance minimization. The resource μ of the control impulses is given in advance. The evader control vector v, with components v_z and v_x, has the dimension of an acceleration and is bounded by the condition

$$|v| \leq \nu.$$

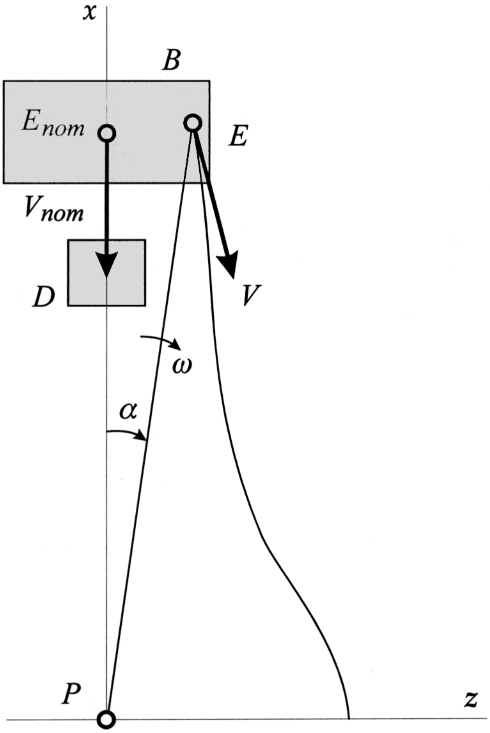

Fig. 1. Relative coordinate system and initial uncertainties.

It is assumed that the pursuer control is realized in a discrete scheme with a time step h. Let $k(t_i)$ be the impulse control at the time instant t_i. It could be equal to zero or differ from it. In the latter case, let us assume that

$$\mu_1 \leq |k(t_i)| \leq \mu_2,$$

where μ_1, μ_2 are given constants. The constraint on the total expenditure of the control is

$$\sum_i |k(t_i)| \leq \mu.$$

Let $n(t_i)$ be the control resource at the current time instant t_i (before applying the control at this instant). At the initial instant, $n(t_0) = \mu$.

The dynamic equations in the relative coordinates z, x are the following:

$$\ddot{z}(t) = v_z - \sum_i k(t_i)\delta(t - t_i), \tag{1a}$$

$$\ddot{x}(t) = v_x, \tag{1b}$$

$$k(t_i) \in \{0\} \cup \{k: \mu_1 \leq |k| \leq \mu_2\}, \qquad \sum_i |k(t_i)| \leq \mu, \qquad |v| \leq \nu. \tag{1c}$$

Here, δ is the delta function. The impulse control at the time instant t_i changes the velocity $\dot{z}$ impulsively by the value $\Delta\dot{z} = -k(t_i)$.

The angle of vision and independently the angular velocity of the line of sight are measured at some time instants t_i (with the given step in t) before building the pursuer control. A variant is possible when only one of these variables is measured. Additionally, measurements of the relative distance between P and E can be carried out. The models of the measurement errors are

$$\alpha_m(t_i) = \alpha(t_i) + \zeta(t_i), \qquad |\zeta(t_i)| \leq a, \qquad a \geq 0, \tag{2}$$

$$\omega_m(t_i) = \omega(t_i) + \xi(t_i), \qquad |\xi(t_i)| \leq c_1|\omega(t_i)| + c_2, \qquad 1 > c_1 \geq 0, \quad c_2 \geq 0, \tag{3}$$

$$d_m(t_i) = d(t_i) + \beta(t_i), \qquad |\beta(t_i)| \leq bd(t_i), \qquad 1 > b \geq 0. \tag{4}$$

Here, $\alpha_m(t_i)$, $\omega_m(t_i)$, $d_m(t_i)$ are the measured values of the angle of vision, angular velocity, and the relative distance; $\alpha(t_i)$, $\omega(t_i)$, $d(t_i)$ are the true values; $\zeta(t_i)$, $\xi(t_i)$, $\beta(t_i)$ are the measurement errors. The error of the angle of vision measurement is bounded by the constant a. If the constant c_1 is equal to zero, then the bound on the error of the angular velocity measurement does not depend on the value of the true angular velocity. If the constant c_2 is equal to zero, then the relative error of the angular velocity measurement is bounded. The relative error of the relative distance measurement is bounded geometrically by the constant b.

The initial uncertainties of the relative geometric state $[z(t_0), x(t_0)]^T$ and relative velocity $[V_z(t_0), V_x(t_0)]^T$ are written as follows:

$$[z(t_0), x(t_0)]^T \in B, \qquad [V_z(t_0), V_x(t_0)]^T \in D. \tag{5}$$

Here,

$$B = [z_0, z^0] \times [x_0, x^0], \qquad D = [V_{z0}, V_z^0] \times [V_{x0}, V_x^0]$$

are rectangles (Fig. 1), symmetric with respect to points E_{nom} and V_{nom}; the superscript T denotes transposition of vector.

Formulas (1)–(5) are known to the pursuer. The variant of obtaining the measurements is also given. Let us assume that the measurements of the angle of vision (if any) occur with the time step $\sigma_\alpha h$; the time step of the angular velocity measurements is $\sigma_\omega h$; for the relative distance measurements, the time step is $\sigma_d h$. Here, σ_α, σ_ω, σ_d are positive integers.

The aim of the pursuit is the miss distance minimization. The minimal value of the distance $d(t)$ through all time of the pursuit process is regarded as the miss distance. Our goal is to construct the pursuer feedback control that gives a satisfactory solution of the miss distance minimization problem under incomplete information.

Investigations of this problem are implemented under the following simplifying assumption. We shall suppose that the variations of the vector of the relative velocity caused by the pursuer and evader controls are relatively small during the pursuit process. It is also assumed that the size of the set D and the span of the set B along the z-axis are small. More precisely, the last assumption means that ratio z^0/x_0 is small.

3. Auxiliary Game with Incomplete Information

We formulate now an auxiliary DG of two persons in which the state at the current time instant t_i is a pair: the IS and the residual resource of the impulse control.

3.1. Equivalent Coordinates. Let us rewrite the system (1) in the coordinates α, ω, x, V_x. We consider $x>0$, $|\alpha|<\pi/2$. Differentiating the relation $z(t)=x(t)\tan\alpha(t)$ twice with respect to t, we obtain

$$\begin{aligned}\ddot{z}(t) &= \ddot{x}(t)\tan\alpha(t)\\ &\quad + [2\dot{x}(t)\dot{\alpha}(t)+2x(t)\tan\alpha(t)\dot{\alpha}^2(t)+x(t)\ddot{\alpha}(t)]/\cos^2\alpha(t),\end{aligned}$$

with the implication that

$$\begin{aligned}\ddot{\alpha}(t) &= [\ddot{z}(t)\cos^2\alpha(t)-\ddot{x}(t)\sin\alpha(t)\cos\alpha(t)-2\dot{x}(t)\dot{\alpha}(t)]/x(t)\\ &\quad -2\tan\alpha(t)\dot{\alpha}^2(t).\end{aligned}$$

In first-order form, we have

$$\dot{\alpha}(t)=\omega(t), \tag{6a}$$

$$\begin{aligned}\dot{\omega}(t) &= -2V_x(t)\omega(t)/x(t)-2\tan\alpha(t)\omega^2(t)\\ &\quad -\sin\alpha(t)\cos\alpha(t)v_x/x(t)+\cos^2\alpha(t)v_z/x(t)\\ &\quad -[\cos^2\alpha(t)/x(t)]\sum_i k(t_i)\delta(t-t_i),\end{aligned} \tag{6b}$$

$$\dot{x}(t)=V_x(t), \tag{6c}$$

$$\dot{V}_x(t)=v_x, \tag{6d}$$

$$k(t_i)\in\{0\}\cup\{k: \mu_1\le|k|\le\mu_2\}, \qquad \sum_i |k(t_i)|\le\mu, \qquad |v|\le\nu. \tag{6e}$$

3.2. Dynamic Equations and Measurement Formulas in the Auxiliary Game. Let us simplify the system (6). In the problem under discussion, the pursuer is interested in the miss distance minimization. The assumption regarding weak controllability leads to the fact that, taking into account the vector V_{nom} direction along the x-axis, the calculation of the miss distance along each actual trajectory of the system (1) can be approximated by the value of the coordinate z in modulus at the time instant when this trajectory crosses the z-axis. Additionally, the value of the miss distance depends more on the variation of the velocity along the z-axis, and less on the variation of the velocity along the x-axis. Therefore, simplifying the system (6), we can consider $V_x(t_0)$ to be known exactly and to be coincident with $V_{x\,\mathrm{nom}} = -e$, with the control v_x equal to zero. Then, the variation of the coordinate x is described by the relation

$$x(t) = x(t_0) - e(t - t_0).$$

Furthermore, the weak controllability assumption and the discussed conditions on the sets B and D allow one to regard the angle of vision α to be small on a long time interval beginning from the initial time instant. Therefore, in the description of the dynamics of the auxiliary problem, we can replace $\sin\alpha$ by zero and $\cos\alpha$ by one. Assuming also that the acceleration v_z is chosen from the segment $[-\nu, \nu]$, we obtain the simplified system

$$\dot{\alpha}(t) = \omega(t), \tag{7a}$$

$$\dot{\omega}(t) = 2e\omega(t)/x(t) + v_z/x(t) - (1/x(t)) \sum_i k(t_i)\delta(t - t_i), \tag{7b}$$

$$x(t) = x(t_0) - e(t - t_0), \tag{7c}$$

$$k(t_i) \in \{0\} \cup \{k: \mu_1 \le |k| \le \mu_2\}, \qquad \sum_i |k(t_i)| \le \mu, \qquad |v_z| \le \nu. \tag{7d}$$

Under the action of an impulse control at the time instant t_i, the coordinates α and x do not vary, but the coordinate ω changes stepwise by the value

$$\Delta\omega = -k(t_i)/x(t_i).$$

The formulas for the variations of the coordinates α, ω on the interval $[t_i, t_{i+1})$ when $v_z \equiv \pm\nu$ are the following:

$$\alpha(t) = \alpha(t_i) + [\omega(t_i)(t - t_i)x(t_i) \pm \nu(t - t_i)^2/2 - k(t_i)(t - t_i)]/x(t), \tag{8}$$

$$\omega(t) = [\omega(t_i)x^2(t_i) \pm \nu(t - t_i)(x(t_i) - e(t - t_i)/2) - k(t_i)x(t_i)]/x^2(t). \tag{9}$$

To construct the measurements in the auxiliary problem, let us replace the relative distance d by its projection onto the x-axis. The relations for the distance measurements are

$$x_m(t_i) = x(t_i) + \beta(t_i), \qquad |\beta(t_i)| \le bx(t_i), \qquad 1 > b \ge 0. \tag{10}$$

Thus, for the miss distance minimization problem, we have substituted the system (1) by the system (7) and the formulas (4) by the relations (10). The formulas (2)–(3) for the angle of vision and angular velocity measurements stay unchanged.

3.3. Informational Sets. For the system (7), the totality of all states consistent with the history of the observation-control process is called the informational set.

The angle of vision α and the angular velocity ω correspond to the quadruple z, x, V_z, V_x and are calculated as follows:

$$\alpha = \arctan(z/x), \qquad \omega = (V_z x - V_x z)/(x^2 + z^2). \tag{11}$$

Using these formulas, we can put the initial informational set $I\_(t_0)$ in the coordinates α, ω, x in correspondence to the set $B \times D$. Namely, for each $x \in [x_0, x^0]$, let us introduce a rectangle with its sides parallel to the axes α, ω and the coordinates of the vertex points

$$\alpha_0(x) = \arctan(z_0/x), \qquad \alpha^0(x) = \arctan(z^0/x),$$
$$\omega_0(x) = (V_{z0}x - V_{x0}z_0)/x^2, \qquad \omega^0(x) = (V_z^0 x - V_{x0}z^0)/x^2.$$

The set $I\_(t_0)$ is described by the totality of such rectangles. The set $I\_(t_0)$ gives the upper estimate for the image of the set $B \times D$ when the latter is mapped into the space α, ω, x via the formulas (11).

Let us define a recurrent procedure of the IS calculation in time. Taking into account that the system (7) has a singularity at $x = 0$, a threshold $\epsilon > 0$ is introduced. Assume that, at the current time instant t_i, we have the informational set $I\_(t_i)$, which was constructed in the space α, ω, x, with $x \geq \epsilon$.

If at the time instant t_i a measurement $\alpha_m(t_i)$ occurs, then denote by $H_\alpha(t_i)$ the corresponding uncertainty set, i.e., the totality of all points α, ω, x such that, for each of them, the measurement α_m is possible according to the formulas

$$\alpha_m(t_i) = \alpha + \zeta, \qquad |\zeta| \leq a.$$

The set $H_\alpha(t_i)$ is cylindrical in ω, x. Its projection $\mathscr{H}_\alpha(t_i)$ onto the α-axis is the segment $[\alpha_m(t_i) - a, \alpha_m(t_i) + a]$. If at the time instant t_i a measurement of the angle of vision does not occur, then the set $H_\alpha(t_i)$ is formally defined to be coincident with R^3.

Let us put a set of uncertainty $H_\omega(t_i)$ in correspondence to the measurement $\omega_m(t_i)$. It is the totality of all points α, ω, x such that, for each of them, the measurement ω_m is possible according to the formulas

$$\omega_m(t_i) = \omega + \xi, \qquad |\xi| \leq c_1|\omega| + c_2.$$

The set $H_\omega(t_i)$ is cylindrical in α, x. Its projection $\mathscr{H}_\omega(t_i)$ onto the ω-axis is described by the following relations:

$$\mathscr{H}_\omega = \begin{cases} [(\omega_m - c_2)/(1 + c_1), (\omega_m + c_2)/(1 - c_1)], & \text{if } \omega_m \geq c_2, \\ [(\omega_m - c_2)/(1 - c_1), (\omega_m + c_2)/(1 - c_1)], & \text{if } -c_2 < \omega_m < c_2, \\ [(\omega_m - c_2)/(1 - c_1), (\omega_m + c_2)/(1 + c_1)], & \text{if } \omega_m \leq -c_2. \end{cases}$$

The span of the interval $\mathscr{H}_\omega$ depends on ω_m (Fig. 2). If at the time instant t_i a measurement of the angular velocity does not occur, we suppose that $H_\omega(t_i) = R^3$.

Similarly, by the formulas (10), an uncertainty set $H_x(t_i)$, which corresponds to a measurement $x_m(t_i)$ of the relative distance, is cylindrical in α, ω. Its projection $\mathscr{H}_x(t_i)$ onto the x-axis is the segment $[x_m(t_i)/(1+b), x_m(t_i)/(1-b)]$. If at the time instant t_i, the relative distance measurement does not appear, then it is supposed that $H_x(t_i) = R^3$.

The informational set $I(t_i)$ is defined by the relation

$$I(t_i) = I_-(t_i) \cap H_\alpha(t_i) \cap H_\omega(t_i) \cap H_x(t_i).$$

If the measurements of the angular velocity and relative distance are absent, then evidently we have

$$I(t_i) = I_-(t_i) \cap H_\alpha(t_i).$$

If at the time instant t_i all measurements are absent, then

$$I(t_i) = I_-(t_i).$$

After the construction of the set $I(t_i)$, the control $k(t_i)$ is determined. While the control is applied, the set $I(t_i)$ is transformed into the set $I_+(t_i)$.

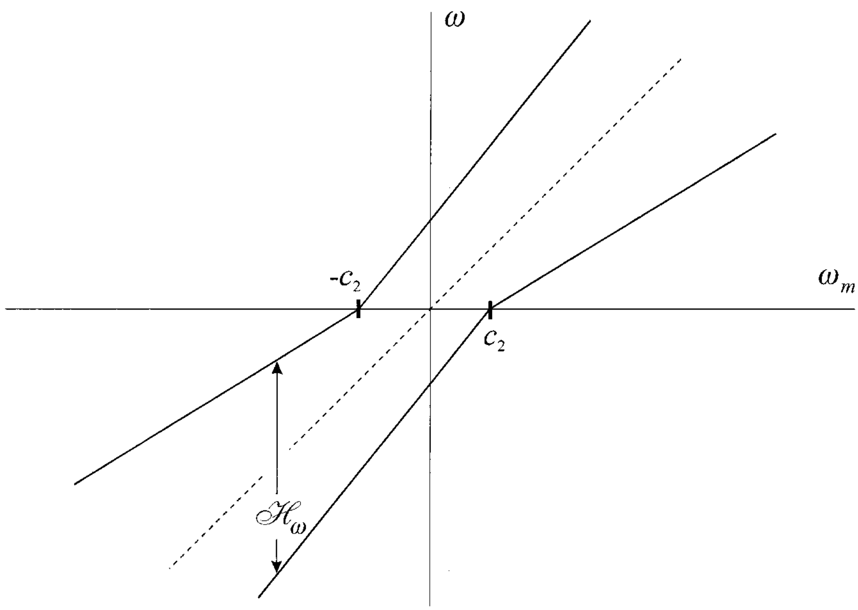

Fig. 2. Span of the measurement uncertainty interval.

The mapping $I(t_i)\rightarrow I_+(t_i)$ is a shift of each x-section of the set $I(t_i)$ by the value $-k(t_i)/x$ along the ω-axis. If $k(t_i)=0$, then

$$I_+(t_i)=I(t_i).$$

We denote by $J_\epsilon(t_i)$ the part of the set $I_+(t_i)$ such that $x<\epsilon+eh$. This part will go under the threshold ϵ on the x-axis at the instant $t_{i+1}=t_i+h$. We put

$$I_{+\epsilon}(t_i)=I_+(t_i)\backslash J_\epsilon(t_i).$$

Let $I_-(t_{i+1})$ be the phase state forecast of the system (7) at the time instant t_{i+1} corresponding to its state $I_{+\epsilon}(t_i)$ at the instant t_i, zero impulse control, and all admissible variations of the parameter v_z, $|v_z|\leq\nu$. The set $I_-(t_{i+1})$ is the result of the linear system (7) action onto the x-sections of the set $I_{+\epsilon}(t_i)$. Here, each convex set is transformed into a convex one. Therefore, a recurrent sequence of the IS is defined.

Each of the sets $I_-(t_i)$, $I(t_i)$, $I_+(t_i)$, $I_{+\epsilon}(t_i)$ is called the informational set (before a measurement, after it, after applying the impulse control, and after ϵ-cutoff, correspondingly). The set $I_-(t_i)$ is also called the forecast informational set.

3.4. Formalization of the Auxiliary Game. By motion, we mean the IS evolution in time. The first player governs the impulse control. The second player constructs the measurements. The influence of the parameter v_z is taken into account when constructing the sets $I_-(t_i)$.

The triplet (t_i, n, I) is called the game position for the first player. Here, t_i is a time instant, n describes the residual resource of the impulse control, I is the informational set after the measurement. As an admissible strategy U of the first player, a rule $(t_i, n, I)\rightarrow k$ maps each game position into the impulse control k, bounded by the following condition: if $k\neq 0$, then

$$\mu_1\leq|k|\leq\mu_2, \qquad |k|\leq n.$$

The triplet (t_i, n, I_-) is called the game position for the second player. Here, I_- is the informational set before a measurement. As an admissible strategy Ω for the second player, a rule $(t_i, n, I_-)\rightarrow(\alpha_m, \omega_m, x_m)$ maps each position into the measurements. If at the time instant t_i some of the parameters mentioned above are not measured, then the construction of the corresponding measurements is not implemented. We require that

$$I_-\cap H_\alpha\cap H_\omega\cap H_x\neq\varnothing.$$

Here, H_α, H_ω, H_x are the uncertainty sets constructed by the measurements α_m, ω_m, x_m. The nonemptiness of the intersection reflects the fact that, at the time instant t_i, some states of the system (7) must be true; therefore, they must belong to both the set I_- and each of the sets H_α, H_ω, H_x.

Having an initial position $(t_0, n(t_0), I_-(t_0))$, the concrete admissible strategies U, Ω, the step h, and the parameter ϵ, it is possible to speak about the IS motion in time.

Let us define a payoff functional. For an arbitrary pair ω, x, with $x > 0$, let

$$\Pi(\omega, x) = |\omega| x^2/e. \tag{12}$$

The value $\Pi(\omega, x)$ approximates a passive forecast miss distance from the state ω, x, i.e., the miss distance computed when the free motion of the system (1) crosses the z-axis. For the exact calculation of the passive forecast miss distance, it would be necessary to have not only the values ω, x, but also the angle of vision α, namely, $|\omega| x^2/e \cos^2 \alpha$. The meaning of this formula is clarified by the fact that $\omega x/\cos^2 \alpha$ is the horizontal component of the velocity vector of the system (1) in the representation that uses the line of sight, and x/e is the time left until the crossing of the z-axis. Neglecting the small angle α, we obtain the formula (12). Let

$$\hat{\Pi}(\omega, x) = \Pi(\omega, x) + \nu(x/e)^2/2.$$

The term $\nu(x/e)^2/2$ is the maximal possible increment of the miss distance due to the acceleration v_z, $|v_z| \leq \nu$. The number

$$\bar{\Pi}(M) = \sup_{(\omega,x) \in M} \hat{\Pi}(\omega, x)$$

is put in correspondence to each arbitrary set M in the space ω, x.

On every actual motion of the IS, the totality of time instants t_i such that $J_\epsilon(t_i) \neq \varnothing$ is denoted by the symbol T_ϵ. The number

$$\Phi(t_0, n(t_0), I_-(t_0), U, \Omega, \epsilon, h) = \max_{t_i \in T_\epsilon} \bar{\Pi}(J_\epsilon(t_i))$$

is called the miss distance corresponding to the initial position $(t_0, n(t_0), I_-(t_0))$, strategies U, Ω, step h, and parameter ϵ.

Let us clarify the meaning of the value Φ. For a given threshold ϵ, it is assumed that, after the time instant t_i, the impulse control ceases to operate for the part of the informational set $I_+(t_i)$ that, at the instant t_{i+1}, will be below the level ϵ. Namely, this is the set $J_\epsilon(t_i)$. The single-point motions arising from $J_\epsilon(t_i)$ terminate. But at the same time, the motions that originate from the part $I_{+\epsilon}(t_i) = I_+(t_i) \backslash J_\epsilon(t_i)$ continue. The payoff for $J_\epsilon(t_i)$ is naturally defined as $\bar{\Pi}(J_\epsilon(t_i))$. Furthermore, the maximum is chosen through all the time instants t_i for which $J_\epsilon(t_i)$ is not empty.

The best guarantee for the first player is defined by the relation

$$\Gamma^{(1)}(t_0, n(t_0), I_-(t_0)) = \inf_U \overline{\lim_{\epsilon\to 0}}\, \overline{\lim_{h\to 0}} \sup_\Omega \Phi(t_0, n(t_0), I_-(t_0), U, \Omega, \epsilon, h).$$

The strategy on which the outer extremum is achieved is called optimal. Constructively building the optimal strategy seems to be a very hard problem. Below, one reasonable variant of the first player strategy is suggested.

3.5. Strategy of Maintaining the Forecast Miss Distance Symmetry. Let us put the projection of the x-section of the set I in the plane ω, x in correspondence to each x. This projection is a line segment. We denote by $L(I)$ the union of such segments. Let $\partial^* L(I)$ be the totality of the right edges of the segments (each for its own x) composing the set $L(I)$. Similarly, $\partial_* L(I)$ is the totality of the left edges. Denote

$$\eta^*(I) = \max\{\omega x^2/e + \nu(x/e)^2/2 \colon (\omega, x) \in \partial^* L(I)\}, \tag{13}$$

$$\eta_*(I) = \min\{\omega x^2/e - \nu(x/e)^2/2 \colon (\omega, x) \in \partial_* L(I)\}. \tag{14}$$

Let (ω^*, x^*), (ω_*, x_*) be the corresponding points on which the maximum in (13) and minimum in (14) are achieved. The segment η with the edges $\eta_*(I)$, $\eta^*(I)$ is called the interval of the forecast miss distance that corresponds to the set I. The marginal (in modulus) edge of the interval η is interpreted as the maximal forecast miss distance.

Just when the impulse control is applied, the miss distance for the former maximizing point in (13) is changed instantly by the value

$$(\Delta\omega) x^{*2}/e = (-k/x^*) x^{*2}/e = -k x^*/e,$$

and for the minimizing point in (14) it is changed by

$$(\Delta\omega) x_*^2/e = -k x_*/e.$$

Here, $\Delta\omega$ is the impulsive jump of the angular velocity. Let us write the symmetry relation

$$\eta^* - k x^*/e = -(\eta_* - k x_*/e).$$

Solving it with respect to the unknown k, we obtain

$$k_s = (\eta^* + \eta_*) e/(x^* + x_*). \tag{15}$$

Let I be the current informational set, and let n be the residual resource of the impulse control. The feedback control U_s is defined as the function that puts a number k_s in correspondence to each pair I, n. The number k_s is calculated via the formula (15) if

$$\mu_1 \le |k_s| \le \mu_2, \qquad |k_s| \le n;$$

note that

$$\text{either } k_s = \min\{\mu_2, n\}\operatorname{sign} k_s, \qquad \text{if } |k_s| \geq \mu_1,\ |k_s| > \min\{\mu_2, n\};$$
$$\text{or } k_s = 0, \qquad \text{if } |k_s| < \mu_1 \quad \text{or} \quad n < \mu_1.$$

The control U_s tries to maintain the symmetry of the interval η with respect to zero. The maximal forecast miss distance does not increase in the time intervals between the control impulses. At the instants while the impulses are applied, the forecast miss distance decreases. The control ceases when the IS drops completely below the threshold ϵ along the x-axis.

3.6. Remark. Let us discuss now a special case when the only measurable parameter is the angular velocity, while the angle of vision and the relative distance are not measured.

Since the angle of vision α does not appear in the formulas of the payoff functional nor in the strategy of maintaining the forecast miss distance symmetry, and since the uncertainty set H_ω is cylindrical in α, we disregard the equation $\dot{\alpha}(t) = \omega(t)$ in the system (7), and we can carry out all the constructions described in the coordinates ω, x. In this case, the IS is the totality of the segments in the plane ω, x. Each segment in the IS is parallel to the ω-axis and corresponds to the actual value of the coordinate x. Here, the computational operations related to the IS constructions and transformations in time are simplified significantly. We have shown that, if the initial IS consists of a single segment, then the strategy U_s of maintaining the forecast miss symmetry is optimal; see Refs. 11, 18.

4. Nonlinear Problem of Homing

We now consider the problem of a SV homing toward a DSO in outer space. The motions of the SV and DSO are assumed to be coplanar and can be described in a heliocentric inertial coordinate system via the equations (see e.g. Ref. 19)

$$\ddot{z}_V = -Kz_V/r_V^3 + u_z, \tag{16a}$$

$$\ddot{x}_V = -Kx_V/r_V^3 + u_x, \tag{16b}$$

$$\ddot{z}_D = -Kz_D/r_D^3, \tag{17a}$$

$$\ddot{x}_D = -Kx_D/r_D^3. \tag{17b}$$

Here, z_V, x_V are the SV coordinates; z_D, x_D are the DSO coordinates; u_z, u_x are the components of the SV control vector; r_V, r_D are the SV and DSO radial distances to the Sun; K is the Sun gravitational constant.

The control $u = (u_z, u_x)^T$ is implemented in the form of relatively strong accelerations which act on short time intervals. The accelerations are applied orthogonally to the vehicle longitudinal axis. Idealizing the character of the control, we assume it to be impulsewise. The resource of the impulse control is bounded. Let h be the time step of the discrete scheme of control.

Using the relative coordinates

$$z = z_D - z_V, \qquad x = x_D - x_V,$$

we can define the relative motion of the DSO as follows:

$$\ddot{z} = -Kz/r_V^3 - Kz_D(r_V^3 - r_D^3)/r_V^3 r_D^3 - u_z, \tag{18a}$$

$$\ddot{x} = -Kx/r_V^3 - Kx_D(r_V^3 - r_D^3)/r_V^3 r_D^3 - u_x. \tag{18b}$$

The nonlinear components of the right-hand sides contain the values r_V, r_D, z_D, x_D that are represented using the coordinates of the original system (16)–(17).

When the SV self-homing process begins, the nominal DSO position at the initial time instant and the nominal velocity vector are known. Assume that the x-axis of the relative inertial system coincides with the longitudinal SV axis and is oriented from the SV center of mass toward the point of the nominal DSO position; the z-axis is orthogonal to this direction. With such orientation, the component u_x of the control vector is equal to zero. We also assume that the nominal relative velocity vector has the direction opposite to the x-axis. The initial uncertainties of the state and the velocity are given in the relative coordinates in the form of some rectangles B, D.

During the homing process, the angle of vision measurements are obtained with the time step $\sigma_\alpha h$, and the distance measurements between the SV and DSO are obtained with the time step $\sigma_d h$. The measurements errors are bounded by (2), (4). The angular velocity measurements are not implemented. If at the time instant t_i the impulse control $k(t_i) \neq 0$, then it satisfies the constraint

$$\mu_1 \leq |k(t_i)| \leq \mu_2.$$

The general constraint is

$$\sum_i |k(t_i)| \leq \mu.$$

Thus, the system (18) is written as follows:

$$\ddot{z} = -Kz/r_V^3 - Kz_D(r_V^3 - r_D^3)/r_V^3 r_D^3 - \sum_i k(t_i)\delta(t - t_i), \tag{19a}$$

$$\ddot{x} = -Kx/r_V^3 - Kx_D(r_V^3 - r_D^3)/r_V^3 r_D^3, \tag{19b}$$

$$k(t_i) \in \{0\} \cup \{k: \mu_1 \leq |k| \leq \mu_2\}, \qquad \sum_i |k(t_i)| \leq \mu. \tag{19c}$$

Let us consider the right-hand sides of the equations (19a), (19b). Each side contains two nonlinear terms. The first term depends on the corresponding true relative coordinate and, during the homing process, is small since the magnitude r_V in the denominator is approximately constant and exceeds the value of the relative coordinate by many orders. Furthermore, the coordinate x decreases monotonically during the homing process. For the same reason, the second term is small, too.

Let us replace the nonlinear terms in (19a) by the disturbance factor v_z, and let us replace the nonlinear terms in (19b) by the disturbance factor v_x. We assume $v = (v_z, v_x)^T$ to be the fictitious evader accelerations that can change within the constraint limit $|v| \leq \nu$ during the homing process. The value ν is the upper estimate (in modulus) of the vector whose components on the z-axis and x-axis are the nonlinear terms in (19a), (19b). As a result, the system (19) becomes the system (1), and we can use the control method U_s described in Section 3.

5. Numerical Constructions and Computation of Measurements

The control method U_s is used in the systems (16)–(17). We assume that the SV onboard control system carries out all the necessary calculations for the IS construction and operating the controls.

In the numerical realization, the IS is defined by a finite number of its convex x-sections. During the transition from the time instant t_i to the time instant t_{i+1}, each section is recalculated independently of all the others. The explicit formulas (8)–(9) are used for the integration of the system (7). The convex addenda appearing because of the parameter v_z is substituted by a segment. As a result, the x-section is approximated by a convex polygon. During the homing process, the number of x-sections in the IS can decrease. At some time instant, if the residual number of sections becomes smaller than the predefined value, then additional sections are introduced.

The initial informational set $I_-(t_0)$ is calculated on the basis of the sets B and D that define the initial uncertainties in position and velocity. The set $I_-(t_0)$ is stored as a collection of rectangles in the plane α, ω. Each rectangle corresponds to a certain value of the longitudinal distance x.

During simulation of an actual homing process, the true phase point (represented by the coordinates α, ω, x) can go out of the IS. It is governed by the fact that the system (7) used for the IS construction differs from the system (6). As a result, after the next performance of the intersection procedure, the informational set $I(t_i)$ may become empty. Thus, the process of further IS construction and the algorithm building the impulse control could be disrupted. To avoid the true point falling outside the IS, we increase slightly the initial informational set $I_{-}(t_0)$ along the coordinate x and also increase the bound ν of the parameter v_z. Here, we use the structure of Eq. (6b). Increasing properly the bound ν, we compensate (almost throughout the time of the homing process) the influence of the following factors on the variation of ω: the parameter v_x, variation of the velocity V_x, and the term $2\tan\alpha(t)\omega^2(t)$. These three factors are not taken into account in (7b).

According to (2), the measurement $\alpha_m(t_i)$ of the angle of vision at the time instant t_i must belong to the interval $[\alpha^{(1)}(t_i), \alpha^{(2)}(t_i)]$, where

$$\alpha^{(1)}(t_i) = \alpha(t_i) - a, \qquad \alpha^{(2)}(t_i) = \alpha(t_i) + a,$$

and $\alpha(t_i)$ is the true angle of vision. By virtue of (4), the distance measurement $d_m(t_i)$ must be in the interval $[d^{(1)}(t_i), d^{(2)}(t_i)]$, where

$$d^{(1)}(t_i) = d(t_i)(1-b), \qquad d^{(2)}(t_i) = d(t_i)(1+b),$$

and $d(t_i)$ is the true distance. When building the sets $H_x(t_i)$, we assume that

$$x_m(t_i) = d_m(t_i).$$

To test the control method U_s, two algorithms of measurement construction are applied: the random generation algorithm and the game construction algorithm. In the first algorithm, the values $\alpha_m(t_i)$, $d_m(t_i)$ are obtained by means of the random number generator using the uniform distribution laws in the intervals $[\alpha^{(1)}(t_i), \alpha^{(2)}(t_i)]$, $[d^{(1)}(t_i), d^{(2)}(t_i)]$. When computing the game measurements, the sets $I_{-}(t_i)$ are used. The game error is designed to keep the point of the maximal forecast miss distance of the set $I_{-}(t_i)$ in the set $I(t_i)$ during the transition from $I_{-}(t_i)$ to $I(t_i)$, if possible.

The game rule for distance measurement construction is described as follows. Having the set $I_{-}(t_i)$ at the instant t_i, we find the largest $\eta^*(I_{-})$ and the least $\eta_*(I_{-})$ forecast miss distances; to simplify the formulas, we omit the symbol t_i here and below. The calculations are carried out using (13)–(14), where the set I_{-} is substituted for the set I. Let $(\alpha^*, \omega^*, x^*)$ and $(\alpha_*, \omega_*, x_*)$ be the triplets on which the misses $\eta^*(I_{-})$ and $\eta_*(I_{-})$ are achieved.

Between the points x^* and x_*, we denote by x' the one that corresponds to the maximal (in modulus) forecast miss distance. Two cases are possible:

(C1) $x' = \max\{x^*, x_*\}$,

(C2) $x' = \min\{x^*, x_*\}$.

Consider Case C1. Let us assume that

$$d' = x'(1-b).$$

Let $\hat{d}_m$ be the nearest point to d' of the interval $[d^{(1)}, d^{(2)}]$. Let $\bar{x}$ be the least value of the projection of the set L_- onto the x-axis. Denote

$$\bar{d} = \bar{x}(1 + b).$$

If $\hat{d}_m \geq \bar{d}$ or if $\hat{d}_m < \bar{d}$ and $\hat{d}_m = d^{(2)}$, then assume that

$$d_m(t_i) = \hat{d}_m.$$

If $\hat{d}_m < \bar{d}$ and $\hat{d}_m \neq d^{(2)}$, then assume that

$$d_m(t_i) = \min\{\bar{d}, d^{(2)}\}.$$

Let us clarify the mentioned method of construction of the relative distance measurement. In the case considered, the worst of the points x^*, x_* is denoted by x' and is above the second extremum point. The preliminary appointed measurement $\hat{d}_m$ pulls the upper border of the interval

$$\mathscr{H}_x(\hat{d}_m) = [\hat{d}_m/(1-b), \hat{d}_m/(1-b)]$$

closer to the point x'. Apparently, if $\hat{d}_m < \bar{d}$ and $\hat{d}_m \neq d^{(2)}$, then both points x^* and x_* lie in the interval $\mathscr{H}_x(\hat{d}_m)$. In such case, the preliminary appointed measurement $\hat{d}_m$ is modified to keep both extremum points in the interval $\mathscr{H}_x$ and simultaneously to increase the x-span of the intersection set $L_- \cap H_x$. Thus, the transition to the measurement $d_m(t_i)$ is implemented.

Consider now Case C2. Let us assume that

$$d' = x'(1 + b).$$

Denote by $\hat{d}_m$ a point of the interval $[d^{(1)}, d^{(2)}]$ that is the nearest to d'. Let $\tilde{x}$ be the largest value of the x-projection of the set L_-. Denote

$$\tilde{d} = \tilde{x}(1 - b).$$

If $\hat{d}_m \leq \tilde{d}$ or if $\hat{d}_m > \tilde{d}$ and $\hat{d}_m = d^{(1)}$, then assume that

$$d_m(t_i) = \hat{d}_m.$$

If $\hat{d}_m > \tilde{d}$ and $\hat{d}_m \neq d^{(1)}$, then assume that

$$d_m(t_i) = \max\{\tilde{d}, d^{(1)}\}.$$

Similarly, we construct the game-defined angle measurements using the formulas for the interval $\mathscr{H}_\alpha$.

6. Simulation Results

The coplanar SV-to-DSO approach is simulated in a region with a mean distance to the Sun of 150 million km. The dynamics is described by the Kepler equations (16)–(17). The Sun gravitational constant is $K = 1.324948 \times 10^{11}\,\text{km}^3\,\text{sec}^{-2}$. The following numerical data are given.

The nominal initial distance between the SV and DSO in the relative system is 5801.3 km. The uncertainty set of the initial DSO positions in the relative coordinates z, x is

$$B = [-10\,\text{km}, 10\,\text{km}] \times [5301.3\,\text{km}, 6301.3\,\text{km}].$$

The nominal value of the relative velocity is $-58.0\,\text{km sec}^{-1}$. The uncertainty set of the initial DSO velocity is

$$D = [-0.1\,\text{km sec}^{-1}, 0.1\,\text{km sec}^{-1}] \times [-59.0\,\text{km sec}^{-1}, -57.0\,\text{km sec}^{-1}].$$

The sets, B, D correspond to the data obtained from the outer aiming informational system. More accurate information is unavailable to the SV onboard control system prior to the beginning of the homing process.

For the mentioned initial conditions, the constant ν, which bounds the fictitious control in the auxiliary system (7), is $0.005\,\text{m sec}^{-2}$. The time step of the discrete control scheme is $h = 0.5$ sec. The impulse control resource is $\mu = 1000\,\text{m sec}^{-1}$. The constant $\mu_1 = 0.25\,\text{m sec}^{-1}$, and the constant $\mu_2 = 7.5\,\text{m sec}^{-1}$.

The time step of the angle of vision measurements coincides with h. The maximal (in modulus) value of the error in the angle of vision measurement is $a = 0.0002$ rad (0.0115 deg). If the distance meter operates, the distance is measured throughout the homing process with the time step $4h = 2$ sec. The maximal value of the relative error in the distance measurement is $b = 0.05$, namely, 5 percent.

The number of x-sections in the initial IS is equal to 29. In the IS construction procedures, the threshold on the longitudinal distance is $\epsilon = 5$ km. The control ceases when the IS drops completely below the level ϵ along the coordinate x.

The true miss distance during the homing process is calculated as the minimal distance between the SV and DSO centers of mass.

Figure 3 shows the control processes. The initial relative state is $z = -8$ km, $x = 5801.3$ km; the initial relative velocity is $V_z = 0$, $V_x = -58.0\,\text{km sec}^{-1}$. In this run, the distance measurements are absent. Depending on the time t, the curves of the measured and true vision angles, current angular velocity, and current impulse control are represented. Figure 3a corresponds to the variant of random generation of the angle of vision measurements. Figure 3b represents the game method of the measurement elaboration. The relative distance is 2.8 m in the case of the random disturbance. For the game case, the miss distance is 6.7 m. In the first case, the impulse control expenditure was $214.7\,\text{m sec}^{-1}$, in the second case, it was $552.2\,\text{m sec}^{-1}$.

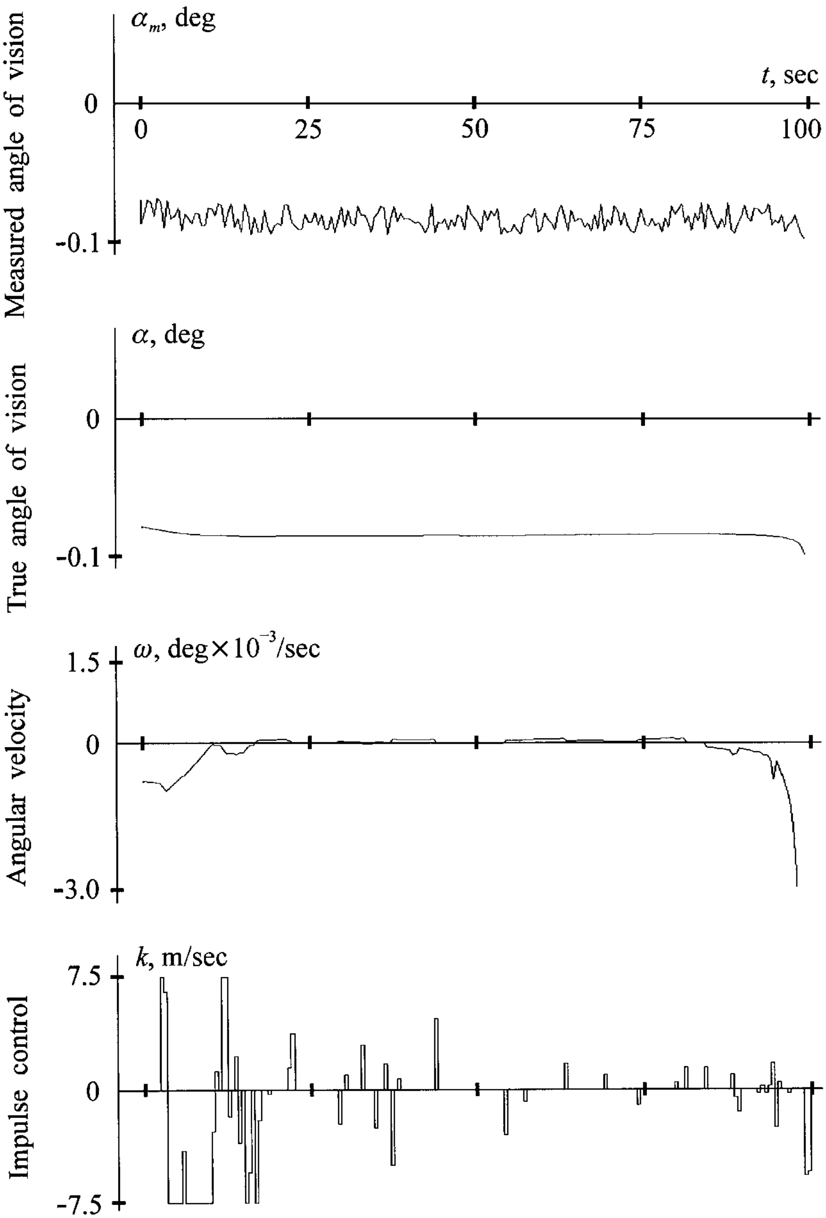

Fig. 3a. Simulation results; case of random disturbances.

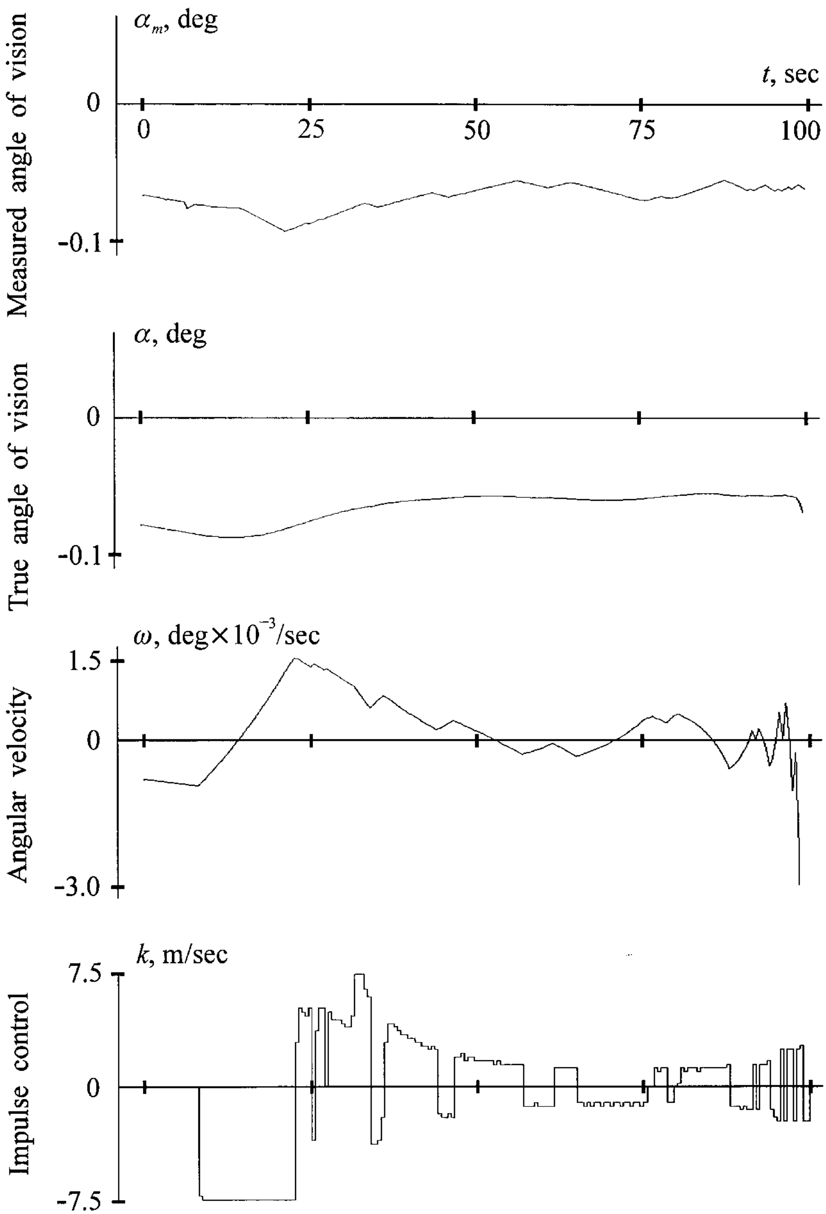

Fig. 3b. Simulation results; case of game disturbances.

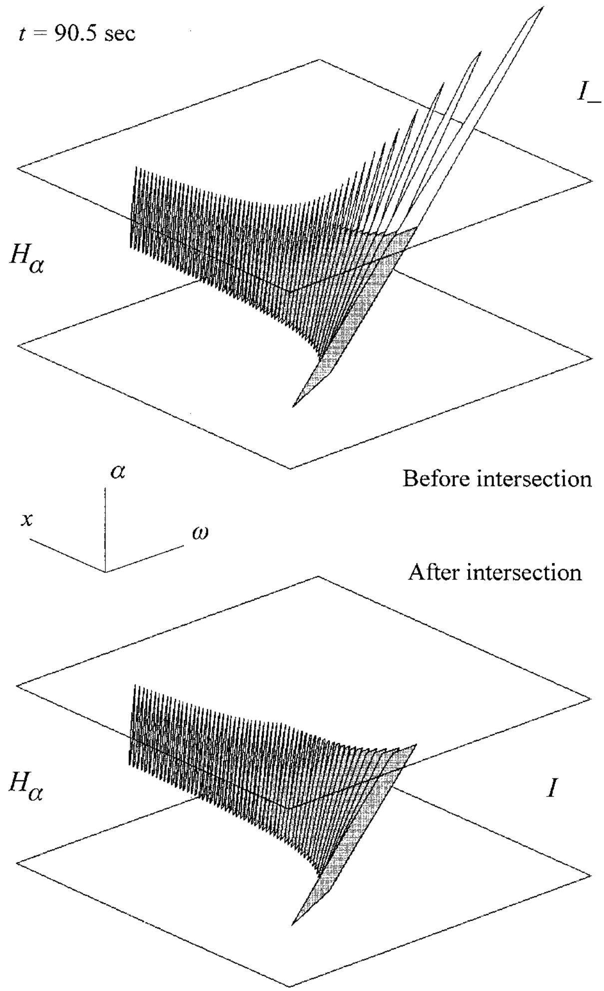

Fig. 4a. Informational sets in the final stage of homing; case of random disturbances.

Figure 4 demonstrates the dynamics of the IS in the final stage of the SV-to-DSO approach. Here, I_- denotes the IS obtained at the time instant $t = 90.5$ sec before the measurement occurs, H_α is the uncertainty set corresponding to the measurement $\alpha_m(90.5)$, I is the IS after taking into account the measurement. It is seen that the span of the set I decreases in both the coordinates α and ω with respect to the span of the set I_-. Note also that, for the random error variant (Fig. 4a), the total size of the IS is smaller than the size of the IS under the game error variant (Fig. 4b).

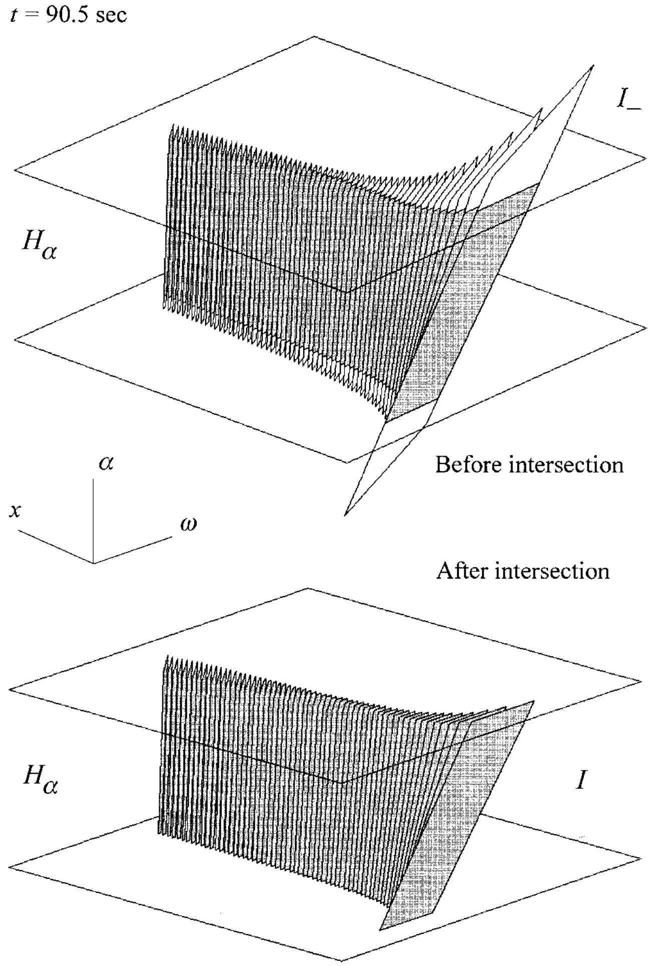

Fig. 4b. Informational sets in the final stage of homing; case of game disturbances.

The effectiveness of the control method U_s is estimated via statistical simulation. It is carried out as follows. Take some subsets B_0, D_0 of the initial uncertainty sets B, D and fix the method of construction of the informational error. Further, we simulate the SV-to-DSO approach a representative number of times. In each run, we perform the dissipation of the initial coordinates and velocities in the sets B_0, D_0 by means of the random number generator with the uniform distribution. In each run, the values of the true miss distance and impulse control expenditure are obtained. Using the set of simulation runs, we obtain the integral probability distribution law of the miss distance. The expenditure of the impulse control is described by its integral probability distribution law. These distribution laws characterize the effectiveness of the U_s control method for initial states and velocities from the sets B_0 and D_0. Note that the sets B_0 and D_0 are not known in the SV onboard control system and cannot be used there. The control system is aware of only the sets B and D.

Figure 5 represents the results of the statistical simulation for the case when

$$B_0 = [-1.0 \text{ km}, 1.0 \text{ km}] \times [5701.3 \text{ km}, 5901.3 \text{ km}],$$

$$D_0 = [-0.1 \text{ km sec}^{-1}, 0.1 \text{ km sec}^{-1}] \times [-58.1 \text{ km sec}^{-1}, -57.9 \text{ km sec}^{-1}].$$

Thus, the true dissipation is implemented around the initial state ($z = 0$ km, $x = 5801.3$ km) and the initial velocity ($V_z = 0 \text{ km sec}^{-1}$, $V_x = -58.0 \text{ km sec}^{-1}$). The integral probability laws were constructed using 50 simulation runs of the SV-to-DSO homing process.

The number 1 [3] marks the curves which correspond to the simulation without distance measuring and under the random [game] error in the angle of vision measurements. The number 2 [4] marks the results obtained under the random [game] errors in the relative distance and angle of vision measurements. It is seen that, for the given value of the error in the relative distance measurements, the operation of the relative distance meter does not give essential improvement of the results.

7. Conclusions

In this paper, we considered the linear problem of pursuit in the plane. The problem originates from model formulations and investigations of certain problems of control and homing processes in outer space under incomplete information conditions. In the case under consideration, a pursuer has limited information about an evader. Data available to the pursuer are based on a sequence of measurements with errors. The latter obey certain geometric constraints.

The new approach introduced in the paper is that an informational set is used as the generalized characteristics, which substitutes the exact phase state unknown to the pursuer. Within the simplifications arising from the peculiarities of the problem discussed, the informational set represents the totality of all phase point-states that are consistent with the history of the observation-control process. The feedback control method that constructs a control as a function of the current informational set has been elaborated.

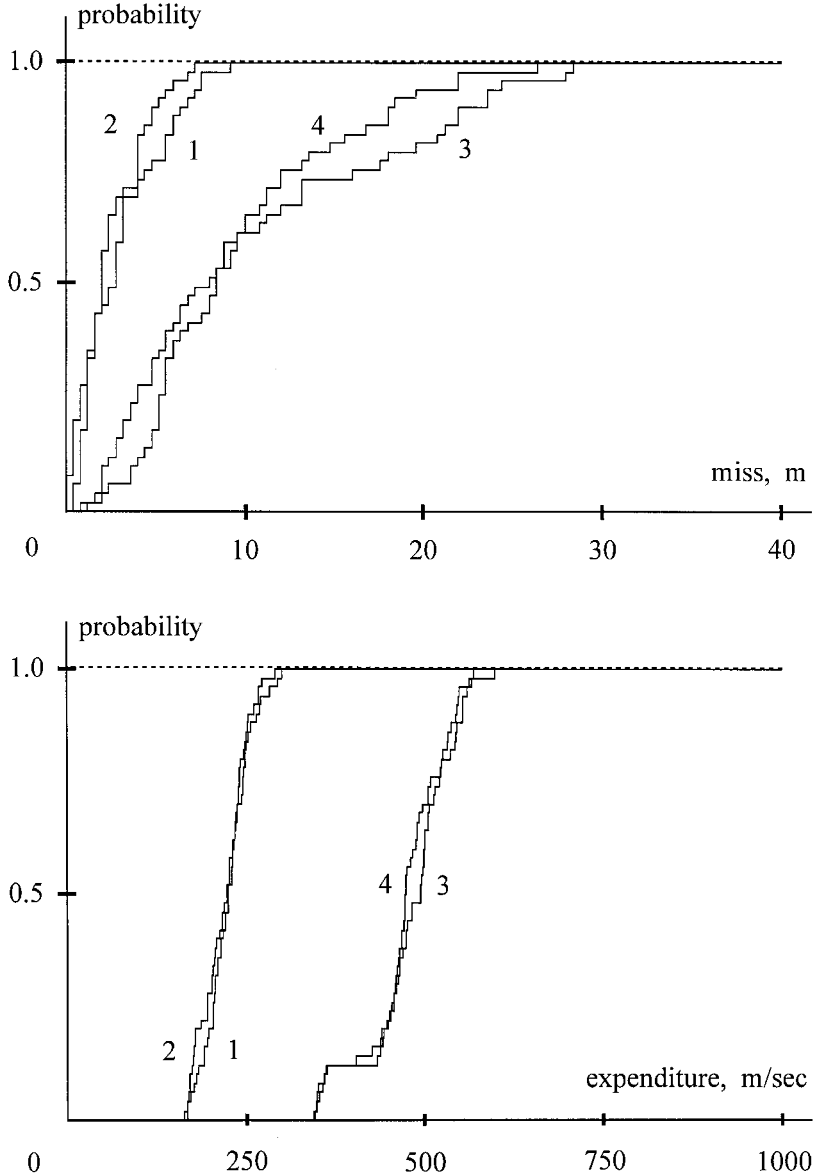

Fig. 5. Statistical simulation results; distribution laws of the miss distance and control expenditure.

To show its advantages, we apply an elaborated method of control to the problem of the SV homing toward an uncontrolled DSO. After the transition to relative coordinates, the nonlinear terms in the dynamic equations become small and are replaced by an uncertain vector parameter bounded in modulus. This uncertain parameter is interpreted as the fictitious evader control, and it is taken into account when building the informational set. The suggested method of the pursuer control is tested in the original nonlinear problem by applying both random disturbances and game informational disturbances.

References

1. KRASOVSKII, N. N., and SUBBOTIN, A. I., *Positional Differential Games*, Nauka, Moscow, Russia, 1974 (in Russian).
2. KURZHANSKII, A. B., *Control and Observation under Conditions of Uncertainty*, Nauka, Moscow, Russia, 1977 (in Russian).
3. CHERNOUSKO, F. L., and MELIKYAN, A. A., *Game Problems of Control and Search*, Nauka, Moscow, Russia, 1978 (in Russian).
4. KRASOVSKII, N. N., and OSIPOV, JU. S., *On the Theory of Differential Games with Incomplete Information*, Soviet Mathematics Doklady, Vol. 15, No. 2, pp. 587–591, 1974.
5. SUBBOTINA, N. N., and SUBBOTIN, A. I., *A Game Problem of Control in the Case of Incomplete Information*, Engineering Cybernetics, Vol. 15, No. 5, pp. 1–10, 1978.
6. MELIKYAN, A. A., and CHERNOUSKO, F. L., *Certain Minimax Control Problems with Incomplete Information*, Journal of Applied Mathematics and Mechanics, Vol. 35, No. 6, pp. 952–961, 1971.
7. PATSKO, V. S., *A Model Example of a Pursuit Game Problem with Incomplete Information, Part 1 and Part 2*, Differentsial'nye Uravneniya, Vol. 7, No. 3, pp. 424–435, 1971 and Vol. 8, No. 8, pp. 1423–1434, 1972 (in Russian).
8. MERZ, A. W., *Stochastic Guidance Laws in Satellite Pursuit–Evasion*, Computers and Mathematics with Applications, Vol. 13, Nos. 1–3, pp. 151–156, 1987.
9. MERZ, A. W., *Noisy Satellite Pursuit–Evasion Guidance*, Journal of Guidance, Control, and Dynamics, Vol. 12, No. 6, pp. 901–905, 1989.
10. KUMKOV, S. I., and PATSKO, V. S., *Impulse Corrections in a Pursuit Problem with Incomplete Information*, International Journal of Computer and Systems Sciences, Vol. 33, No. 5, pp. 99–109, 1995.
11. KUMKOV, S. I., and PATSKO, V. S., *Optimal Strategies in a Pursuit Problem with Incomplete Information*, Journal of Applied Mathematics and Mechanics, Vol. 59, No. 1, pp. 75–85, 1995.
12. KUMKOV, S. I., and PATSKO, V. S., *Control in a Problem on the Basis of Construction of the Sets of Admissible Phase States*, International Journal of Computer and Systems Sciences, Vol. 35, No. 1, pp. 60–66, 1996.

13. KUMKOV, S. I., and PATSKO, V. S., *Model Problem of Impulse Control with Incomplete Information*, Transactions of the Institute of Mathematics and Mechanics, Ural Branch, Russian Academy of Sciences, Ekaterinburg, Russia, Vol. 1, pp. 106–121, 1992 (in Russian).
14. KUMKOV, S. I., and PATSKO, V. S., *Control of Informational Sets in a Pursuit Problem*, Annals of the International Society of Dynamic Games, New Trends in Dynamic Games and Applications, Edited by J. Olsder, Birkhäuser, Boston, Massachusetts, Vol. 3, pp. 191–206, 1995.
15. GEHRELS, T., Editor, *Hazards due to Comets and Asteroids*, Arizona University Press, Tucson, Arizona, 1994.
16. TELLER, E., *Comments on Possible Collision of Asteroids and Comets with Earth*, Proceedings of the International Conference on Problems of Earth Protection against the Impact with Near-Earth Objects (SPE-94), Snezhinsk, Russia, Part 2, pp. 34–37, 1994.
17. MASEVICH, A. G., Editor, *Collisions in the Surrounding Space (Space Debris)*, Kosmosinform, Moscow, Russia, 1995 (in Russian).
18. KUMKOV, S. I., and PATSKO, V. S., *Optimal Strategies in a Pursuit Game with Incomplete Information*, Transactions of the Institute of Mathematics and Mechanics, Ural Branch, Russian Academy of Sciences, Ekaterinburg, Russia, Vol. 3, pp. 104–131, 1995 (in Russian).
19. EHRICKE, K. A., *Space Flight 1: Environment and Celestial Mechanics*, D. Van Nostrand Company, Princeton, New Jersey, 1960.