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Aircraft Landing Control under Wind Disturbances

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Abstract—Results from differential game theory are applied to construct an adaptive control in linear systems with an unknown level of dynamic disturbances. The efficiency of the method is exemplified by a problem of aircraft landing under wind disturbance.
Keywords: adaptive control, differential games, aircraft landing control, wind disturbance, numerical methods.

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INTRODUCTION

Aircraft landing and take-off under wind disturbance are natural examples [1–6] of using modern methods of mathematical control theory and of the theory of differential games in applied problems. Nevertheless, one encounters the following difficulty when formulating problems.

An aircraft has four controls: the thrust force, elevator, rudder, and ailerons. The limits of the possible deviations of the controls from the nominal values are clearly specified. Therefore, when formulating the mathematical problem of aircraft control, we can validly specify a constraint P on the vector control action. The constraint on the wind disturbance is not so easy to specify. Even if we stipulate a disturbance level that is not very high but expect the worst disturbance, then the predicted result of the control will be inadequate. In the case when a weak wind disturbance is realized, the result will be acceptable, but the control actions will switch from one limit position to the other. At the same time, it is clear that a low-level disturbance can be dealt with by using small deviations of the control actions from the nominal values.

Thus, we deal with a control process on a finite time interval, and the level of the dynamic disturbance is *bounded but not known a priori*. Such problems are close to problems on the suppression by a control system of an external bounded disturbance [7–10]. The difference is that, in suppression problems, as a rule, an infinite time interval is considered and there are no constraints on the instantaneous values of the useful control.

Based on the theory of differential games, we proceed as follows. Let us establish some correspondence between the virtual disturbance level and the level of the control that responds to it. Let the constraint for the disturbance be characterized by a set Q_k, where $k \geq 0$ is a numerical parameter. Assume that the set Q_k increases monotonically as k grows. To every value of k, we also assign a set P_k, which is a constraint on the useful control. This set increases with

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the growth of $k \in [0,1]$ and, for $k = 1$, coincides with the constraint P, which cannot be violated. Assume that $P_k = P$ for $k \geq 1$.

To every pair Q_k, P_k, we assign in the space of t, x (time, phase vector) a *stable* [11, 12] tube W_k such that a control chosen from the set P_k can keep the motion within this tube for any disturbance with values in the set Q_k.

Assume that the sets W_k increase monotonically as k grows. Consider the following geometric interpretation: the system of stable tubes expands with the growth of k, each tube corresponding to the constraint P_k on the useful control and to the constraint Q_k on the disturbance. At the instant t, we measure the current phase state $x(t)$, identify the index $\overline{k}$ of the tube $W_{\overline{k}}$ on whose boundary the system is positioned at that instant, and apply on a small time interval the appropriate control taking a value in set $P_{\overline{k}}$. If the motion leaves the tube $W_{\overline{k}}$, this means that the level of disturbance on the time interval under consideration was greater than $Q_{\overline{k}}$. In this case, on the following time interval, we will choose a control from the set $P_{\widetilde{k}}$ related to the tube $W_{\widetilde{k}}$, $\widetilde{k} > \overline{k}$, on whose boundary the system is positioned at the initial moment of the time interval. In the case when the motion goes inside the tube $W_{\overline{k}}$, we apply on the following time interval a control with the level corresponding to the tube $W_{\widetilde{k}}$, where $\widetilde{k} < \overline{k}$, and so on.

It can be said that the level of the control used locally *adjusts* (*adapts*) to the level and "quality" of the acting disturbance. The result obtained in the end of the control process depends on the structure of the system $\{W_k\}$ of stable tubes, the maximal level of disturbance, and the sophistication of its action.

The concept of stable tubes (stable bridges) [11–13] is central to the theory of differential games developed in N.N. Krasovskii's school. In particular, this idea underlies the feedback control method called the *extremal aiming method*, which guarantees the retention of the motion in a stable tube. Thus, when creating a control method that would work in the case when the level of a dynamic disturbance is not known a priori, we can use a well-developed theoretical base and corresponding numerical methods, in particular, methods of constructing *maximal* stable bridges.

Let us specify the principal aspects of the practical implementation of the adaptive control method under consideration: one must have a numerical algorithm for constructing stable tubes, the stable tubes must be nested into each other, and the required tube must be constructed simultaneously with the process based on a small number of special stable tubes stored in the memory.

The simplest algorithms of the numerical construction of maximal stable bridges appear in the case of game problems with linear dynamics and fixed terminal time [14–20]. Tubes are constructed by means of a backward time procedure of passing from one t-section to another starting from the terminal time. In this case, if the terminal set, from which we move back, is taken to be convex, then all the t-sections will also be convex.

The method of constructing a family of nested stable tubes W_k, $k \geq 0$, for problems with linear dynamics was proposed in [21–23]. It is briefly presented in Section 1. The whole system $\{W_k\}$ is generated by two tubes; one of them is included in the system and corresponds to $k = 1$, and the other is auxiliary and is not included in the system. An arbitrary tube W_k is defined by means of linear operations of addition and multiplication by a scalar coefficient of the specified two stable tubes; only these two tubes must be stored in the memory. The application of the extremal aiming method for constructing adaptive control in the case of convex sections $W_k(t)$ is rather simple.

The main part of the paper (Sections 2–4) is devoted to applying the method of adaptive control to the problem of aircraft landing under wind disturbance. The landing process is considered only up to the moment of passing the runway threshold. The landing problem with boundary conditions at the moment of passing the runway threshold was formulated by Kein [1, 24–26]. Boundary conditions are given in the form of two convex sets. One of them bounds the vertical (longitudinal) channel in the following coordinates: the *vertical deviation* from the nominal value, the *velocity of the vertical deviation*. The other set bounds the lateral channel in the following coordinates: the *lateral deviation*, the *velocity of the lateral deviation*. Each of the sets specifies the tolerance. If the tolerance is satisfied, then the final stages of the landing can be carried out successfully (the descent before touching the runway, the run along the runway on the main wheels, and the run on all the wheels).

The nominal motion until the moment of passing the runway threshold follows the descending rectilinear glide path, and the altitude at the passage moment is 15 m. It is reasonable to linearize the nonlinear dynamics along the nominal motion. The adaptive control is formed based on *auxiliary problems with linear dynamics* for the vertical and lateral channels. The predicted moment of passing the runway threshold is corrected during the process. The generated control is sent to the nonlinear system of the aircraft dynamics, which models the motion of the aircraft.

When testing the developed method of control, we use the model of a wind microburst from [27].

This paper makes substantial use of papers [6, 26, 28, 29], in which the study of the landing problem involved a prespecified constraint on the instantaneous values of the wind disturbance. In those papers, sharp switches of the controls from one extreme position to the other were avoided by means of purely engineering techniques not encompassed by any universal mathematical idea. This drawback is overcome in the present paper.

1. ADAPTIVE CONTROL

Let us describe the construction of an adaptive control for linear systems and then apply this construction to the landing problem.

Consider the following system with linear dynamics:

$$\begin{gathered}\dot z = A(t)z + B(t)u + C(t)v,\\ z \in \mathbb{R}^m, \quad t \in T, \quad u \in P \subset \mathbb{R}^p, \quad v \in \mathbb{R}^q.\end{gathered} \tag{1.1}$$

Here, u and v are vector control actions of the first and second players, P is a convex compact constraint on the control of the first player, and $T = [t_0, t_f]$ is the time interval of the control process. Assume that the set P contains the origin of the space $\mathbb{R}^p$. The matrix-valued functions A and C are continuous in t. The matrix-valued function B satisfies the Lipschitz condition on the interval T. There is no specific constraint on the control v.

The first player aims to bring n selected components of the phase vector of system (1.1) to a terminal set M at the instant t_f. We assume that M is a convex compact set in the space of the specified n components of the phase vector z. We also assume that M contains some neighborhood of the origin of this space. Let the origin be the center of the set M. The goal of the first player is to bring the n specified components of the vector z as close to the center of M as possible.

Let us pass to a system that does not contain the phase vector on its right-hand side:

$$\begin{gathered}\dot x = D(t)u + E(t)v,\\ x \in \mathbb{R}^n, \quad t \in T, \quad u \in P \subset \mathbb{R}^p, \quad v \in \mathbb{R}^q.\end{gathered} \tag{1.2}$$

The passage is carried out ([11, p. 160], [12, pp. 89–91]) by means of the relations

$$x(t) = Z_{n,m}(t_f, t)z(t), \quad D(t) = Z_{n,m}(t_f, t)B(t), \quad E(t) = Z_{n,m}(t_f, t)C(t),$$

where $Z_{n,m}(t_f, t)$ is a matrix composed of the n lines of the fundamental Cauchy matrix for the system $\dot{z} = A(t)z$ that correspond to the components of the vector z in the space of which the set M is given. The vector $x(t)$ is the prediction for the instant t_f of the selected components of z along the motion of system (1.1) on the interval $[t, t_f]$ under the controls $u = 0$, $v = 0$.

The first player tries to bring the phase vector of system (1.2) to the set M at the terminal time t_f.

The calculations presented below are performed for system (1.2). The constructed adaptive control $U(t, x)$ as applied to system (1.1) is written in the form $U\big(t, Z_{n,m}(t_f, t)z\big)$.

Denote by $S(t) = \big\{x \in \mathbb{R}^m : (t, x) \in S\big\}$ the section of the set $S \subset T \times \mathbb{R}^n$ at the instant $t \in T$. Denote by $O(\varepsilon) = \{x \in \mathbb{R}^n : |x| \leqslant \varepsilon\}$ the ball of radius ε in the space $\mathbb{R}^n$ centered at the origin.

Stable bridges. On the interval $[t_0, t_f]$, we consider the zero-sum differential game with the terminal set $\mathcal{M}$ and geometric constraints $\mathcal{P}$, $\mathcal{Q}$ on the players' controls

$$\begin{aligned} &\dot{x} = D(t)u + E(t)v, \\ &x \in \mathbb{R}^n, \quad t \in T, \quad \mathcal{M} \subset \mathbb{R}^n, \quad u \in \mathcal{P} \subset \mathbb{R}^p, \quad v \in \mathcal{Q} \subset \mathbb{R}^q. \end{aligned} \tag{1.3}$$

Here, the matrices $D(t)$ and $E(t)$ are the same as in system (1.2). The sets $\mathcal{M}$, $\mathcal{P}$, and $\mathcal{Q}$ are assumed to be convex and compact. They are regarded as parameters of the game.

Let $u(\cdot)$ and $v(\cdot)$ be measurable functions of time with values in the sets $\mathcal{P}$ and $\mathcal{Q}$, respectively. We denote the motion of system (1.3) (and, consequently, of system (1.2)) starting from a point x_* at an instant t_* subject to controls $u(\cdot)$ and $v(\cdot)$ by $x\big(\cdot; t_*, x_*, u(\cdot), v(\cdot)\big)$.

Following [11, 12], we define the notions of a stable and maximal stable bridges.

A set $W \subset T \times \mathbb{R}^n$ will be called a *stable bridge* for system (1.3) and some fixed sets $\mathcal{P}$, $\mathcal{Q}$, and $\mathcal{M}$ if $W(t_f) = \mathcal{M}$ and the following stability property is satisfied: for any position $(t_*, x_*) \in W$ and any control $v(\cdot)$ of the second player, the first player can choose his control $u(\cdot)$ so that the position $\big(t, x(t)\big) = \big(t, x(t; t_*, x_*, u(\cdot), v(\cdot))\big)$ stays in the set W at any instant $t \in (t_*, t_f]$. A set $W \subset T \times \mathbb{R}^n$, $W(t_f) = \mathcal{M}$, that is maximal with respect to inclusion and satisfies the stability property is called a *maximal stable bridge*.

A maximal stable bridge is [11, 12] a closed set. Its t-sections are convex [12, p. 87] since system (1.3) is linear and the set $\mathcal{M}$ is convex.

Construction of a system of stable bridges. 1°. Let us choose a set $Q_{\max} \subset \mathbb{R}^q$, which is interpreted as the maximal constraint on the control of the second player that the first player agrees to regard as "reasonable" when taking system (1.2) to the set M. We assume that the set $Q_{\max}$ contains the origin of its space. Denote by W_{main} the maximal stable bridge for system (1.3) corresponding to the parameters $\mathcal{P} = P$, $\mathcal{Q} = Q_{\max}$, and $\mathcal{M} = M$. We will call it the *main* bridge for brevity.

In addition, we assume that the set $Q_{\max}$ is chosen so that the inclusion

$$O(\varepsilon) \subset W_{\text{main}}(t) \tag{1.4}$$

holds for some $\varepsilon > 0$ and any $t \in T$. The number ε is assumed to be fixed.

Thus, W_{main} is a closed tube in the space $T \times \mathbb{R}^n$, which terminates at the instant t_f on the set M. Each of its t-sections $W_{\text{main}}(t)$ is convex and contains the origin of the space $\mathbb{R}^n$ together with a certain neighborhood.

2°. Let us introduce an *additional* closed tube $W_{\text{add}} \subset T \times \mathbb{R}^n$, each section $W_{\text{add}}(t)$ of which is the reachable set of system (1.3) at the instant t with the initial set $O(\varepsilon)$ taken at the instant t_0. In constructing the tube W_{add}, we assume that the first player is absent ($u \equiv 0$), and the control of the second player is subject to the constraint $Q_{\max}$. It is easy to see that W_{add} is the maximal stable bridge for system (1.3) for

$$\mathcal{P} = \{0\}, \quad \mathcal{Q} = Q_{\max}, \quad \mathcal{M} = W_{\text{add}}(t_f).$$

For any $t \in T$, the section $W_{\text{add}}(t)$ is convex and the following inclusion holds:

$$O(\varepsilon) \subset W_{\text{add}}(t). \tag{1.5}$$

3°. Consider the family of tubes $W_k \subset T \times \mathbb{R}^n$, $k \geqslant 0$, with the sections $W_k(t)$ defined as follows:

$$W_k(t) = \begin{cases} kW_{\text{main}}(t), & 0 \leqslant k \leqslant 1, \\ W_{\text{main}}(t) + (k-1)W_{\text{add}}(t), & k > 1. \end{cases}$$

The sets $W_k(t)$ are compact and convex. For any numbers $0 \leqslant k_1 < k_2 \leqslant 1 < k_3 < k_4$, the strict inclusions

$$W_{k_1}(t) \subset W_{k_2}(t) \subset W_{k_3}(t) \subset W_{k_4}(t)$$

hold by virtue of relations (1.4) and (1.5).

In papers [30,31], the following important properties are established. The tube W_k for $0 \leqslant k \leqslant 1$ is the maximal stable bridge for system (1.3) corresponding to the constraint kP on the control of the first player, constraint $kQ_{\max}$ on the control of the second player, and terminal set kM. For $k > 1$, the set W_k is a stable bridge (which is, generally speaking, not maximal) for the parameters

$$\mathcal{P} = P, \quad \mathcal{Q} = kQ_{\max}, \quad \mathcal{M} = M + (k-1)W_{\text{add}}(t_f).$$

Thus, we have an expanding system of stable bridges, in which each larger bridge corresponds to a larger constraint on the control of the second player. This system of bridges is generated by the two bridges W_{main} and W_{add} by means of the algebraic operations of addition and multiplication by a nonnegative numerical parameter.

Feedback control. The adaptive control $(t, x) \mapsto U(t, x)$ is constructed as follows.

Fix a number $\xi > 0$.

Consider an arbitrary position (t, x). If $|x| \leq \xi$, we set $U(t, x) = 0$. In the case $|x| > \xi$, we find a positive number k^* defining the bridge W_{k^*} whose section $W_{k^*}(t)$ is at a distance of ξ from the point x. On the boundary of the set $W_{k^*}(t)$, we calculate the point x^* nearest to x. We have $|x^* - x| = \xi$. We specify a vector $u^* \in P_{k^*}$ from the extremum condition

$$(x^* - x)' D(t) u^* = \max\big\{(x^* - x)' D(t) u : u \in P_{k^*}\big\}. \tag{1.6}$$

Set $U(t, x) = u^*$.

Thus, the control U is formed by means of the extremal aiming rule, which is widely known in the theory of differential games [11–13].

We apply the control U in a *discrete scheme* [11–13] with a time step Δ. The control action is chosen at the initial point of each interval of length Δ and is held constant to the end of the interval.

In [23], a theorem on the guarantee provided by the control U was formulated and proved.

2. MATHEMATICAL MODEL OF AN AIRCRAFT'S DYNAMICS

2.1. The main part of the dynamics. The motion of an aircraft is described by the following system of differential equations of the 12th order [6, 26, 32, 33]:

$$
\begin{aligned}
\dot{x}_g &= V_{xg},\\
\dot{V}_{xg} &= \big[(p\cos\sigma - qsc_x)\cos\psi\cos\vartheta + (p\sin\sigma + qsc_y)(\sin\psi\sin\gamma - \cos\gamma\cos\psi\sin\vartheta)\\
&\quad + qsc_z(\sin\psi\cos\gamma + \cos\psi\sin\vartheta\sin\gamma)\big]/m,\\
\dot{y}_g &= V_{yg},\\
\dot{V}_{yg} &= \big[(p\cos\sigma - qsc_x)\sin\vartheta + (p\sin\sigma + qsc_y)\cos\vartheta\cos\gamma - qsc_z\cos\vartheta\sin\gamma\big]/m - g,\\
\dot{z}_g &= V_{zg},\\
\dot{V}_{zg} &= \big[(p\cos\sigma - qsc_x)(-\sin\psi\cos\vartheta) + (p\sin\sigma + qsc_y)(\cos\psi\sin\gamma + \sin\psi\sin\vartheta\cos\gamma)\\
&\quad + qsc_z(\cos\psi\cos\gamma - \sin\psi\sin\vartheta\sin\gamma)\big]/m, \qquad (2.1)\\
\dot{\vartheta} &= \omega_z\cos\gamma + \omega_y\sin\gamma,\\
\dot{\omega}_z &= \big[I_{xy}(\omega_x^2 - \omega_y^2) - (I_y - I_x)\omega_x\omega_y + M_z\big]/I_z,\\
\dot{\psi} &= (\omega_y\cos\gamma - \omega_z\sin\gamma)/\cos\vartheta,\\
\dot{\omega}_y &= \big[(I_y - I_z)I_{xy}\omega_y\omega_z + (I_z - I_x)I_x\omega_x\omega_z + I_xM_y + I_{xy}M_x + I_{xy}\omega_z(I_x\omega_y - I_{xy}\omega_x)\big]/J,\\
\dot{\gamma} &= \omega_x - (\omega_y\cos\gamma - \omega_z\sin\gamma)\tan\vartheta,\\
\dot{\omega}_x &= \big[(I_y - I_z)I_y\omega_y\omega_z + (I_z - I_x)I_{xy}\omega_x\omega_z + I_yM_x + I_{xy}M_y + I_{xy}\omega_z(I_{xy}\omega_y - I_y\omega_x\big]/J.
\end{aligned}
$$

The phase variables have the following sense: x_g, y_g, and z_g are the coordinates of the aircraft's center of mass in the *ground* coordinate system (see Fig. 1); V_{xg}, V_{yg}, and V_{zg} are the absolute velocities; ϑ, ψ, and γ are the pitch, yaw, and roll angles; and ω_x, ω_y, and ω_z are the angular velocities in the *bound* coordinate system: the x axis is directed along the aircraft's construction line, the y axis lies in the symmetry plane and is directed upwards, and the z axis completes the right-hand triple.

The dynamic pressure q is calculated by the formula

$$q = \rho\widehat{V}^2/2.$$

The aerodynamic moments are defined by the relations

$$M_x = qslm_x, \quad M_y = qslm_y, \quad M_z = qsbm_z.$$

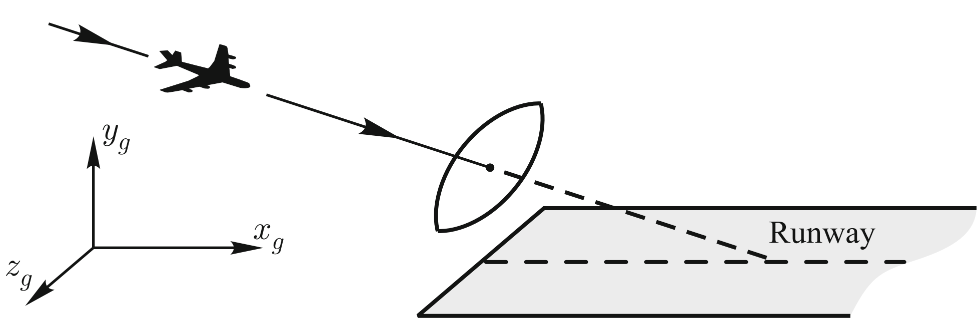

Fig. 1. Aircraft landing.

The value J is determined in terms of the moments of inertia I_x, I_y, and I_{xy}:

$$J = I_x I_y - I_{xy}^2.$$

The remaining variables and constants are explained below.

The aircraft is controlled by means of the thrust force p and the deviations δ_e, δ_r, and δ_a of the elevator, rudder, and ailerons, respectively. Note that the values δ_e, δ_r, and δ_a influence the aerodynamic coefficients of forces c_x, c_y, and c_z and moments m_x, m_y, and m_z. The aerodynamic coefficients depend also on the angle of attack α the gliding angle β, which are computed by the following formulas [32, 33]:

$$\alpha = \arcsin\Big\{\big[-\widehat{V}_{xg}(\sin\psi\sin\gamma - \cos\psi\sin\vartheta\cos\gamma) - \widehat{V}_{yg}\cos\vartheta\cos\gamma$$

$$- \widehat{V}_{zg}(\cos\psi\sin\gamma + \sin\psi\sin\vartheta\cos\gamma)\big]/(\widehat{V}\cos\beta)\Big\},$$

$$\beta = \arcsin\Big\{\big[\widehat{V}_{xg}(\sin\psi\cos\gamma + \cos\psi\sin\vartheta\sin\gamma) - \widehat{V}_{yg}\cos\vartheta\sin\gamma$$

$$+ \widehat{V}_{zg}(\cos\psi\cos\gamma - \sin\psi\sin\vartheta\sin\gamma)\big]/\widehat{V}\Big\}.$$

The wind velocity components w_{xg}, w_{yg}, and w_{zg} influence the components $\widehat{V}_{xg}$, $\widehat{V}_{yg}$, and $\widehat{V}_{zg}$ of the airspeed vector:

$$\widehat{V}_{xg} = V_{xg} - w_{xg}, \quad \widehat{V}_{yg} = V_{yg} - w_{yg}, \quad \widehat{V}_{zg} = V_{zg} - w_{zg}.$$

2.2. Numerical characteristics of the aircraft. We will use the following numerical data, which correspond to the Tupolev Tu-154 aircraft:

$$s = 201\ \mathrm{m}^2, \qquad l = 37.55\ \mathrm{m}, \qquad b = 5.285\ \mathrm{m},$$
$$I_x = 2.5 \times 10^6\ \mathrm{kg\,m}^2, \qquad I_y = 7.5 \times 10^6\ \mathrm{kg\,m}^2, \qquad I_z = 6.5 \times 10^6\ \mathrm{kg\,m}^2,$$
$$I_{xy} = 0.5 \times 10^6\ \mathrm{kg\,m}^2, \qquad m = 75 \times 10^3\ \mathrm{kg}, \qquad \sigma = 1.72^\circ.$$

Here, m is the mass of the aircraft; s is the wing area; l is the wing span; b is the average aerodynamic chord; I_x, I_y, I_z, and I_{xy} are the moments of inertia; and σ is the thrust inclination. The constants g and ρ, which characterize the acceleration of gravity and the air density, are given in the form

$$g = 9.81\ \mathrm{m\,s}^{-2}, \quad \rho = 1.207\ \mathrm{kg\,m}^{-3}.$$

2.3. Aerodynamic coefficients. Using [6, 26, 33], we specify formulas for the coefficients of aerodynamic forces and moments.

The coefficients c_x, c_y, and c_z of aerodynamic forces in system (2.1) should be taken in the bound coordinate system. They are expressed in terms of the coefficients $\widetilde{c}_x$, $\widetilde{c}_y$, and $\widetilde{c}_z$ in the semibound system by means of the relations

$$c_x = \widetilde{c}_x\cos\alpha - \widetilde{c}_y\sin\alpha, \quad c_y = \widetilde{c}_y\cos\alpha + \widetilde{c}_x\sin\alpha, \quad c_z = \widetilde{c}_z.$$

In the *semibound system*, the x axis is directed along the projection of the airspeed onto the aircraft's symmetry plane, the z axis coincides with the same axis of the bound system, and the y axis lies in the symmetry plane and completes the right-hand triple.

The coefficients in the semibound system are as follows:

$$\widetilde{c}_x = 0.21 + 0.004\alpha + 0.47 \times 10^{-3}\alpha^2,$$
$$\widetilde{c}_y = 0.65 + 0.09\alpha + 0.003\delta_e,$$
$$\widetilde{c}_z = -0.0115\beta - (0.0034 - 6 \times 10^{-5}\alpha)\delta_r.$$

Here and below, angular values are measured in degrees.

The coefficients m_x, m_y, and m_z of aerodynamic moments are specified by the following expressions. For the moment about the x axis (the aircraft's construction axis):

$$m_x = m_x^\beta\beta + m_x^r\delta_r + m_x^a\delta_a + (l/(2\widehat{V}))(\pi/180)(m_x^x\omega_x + m_x^y\omega_y),$$
$$m_x^\beta = -0.0035 - 0.0001\alpha, \quad m_x^r = -0.0005 + 0.00003\alpha,$$
$$m_x^a = -0.0004, \quad m_x^x = -0.61 + 0.004\alpha, \quad m_x^y = -0.3 - 0.012\alpha.$$

For the moment about the y axis:

$$m_y = m_y^\beta\beta + m_y^r\delta_r + m_y^a\delta_a + (l/(2\widehat{V}))(\pi/180)(m_y^x\omega_x + m_y^y\omega_y),$$
$$m_y^\beta = -0.004 - 0.00005\alpha, \quad m_y^r = -0.00135 + 0.000015\alpha,$$
$$m_y^a = 0, \quad m_y^x = 0.015\alpha, \quad m_y^y = -0.21 - 0.005\alpha.$$

For the moment about the z axis:

$$m_z = 0.033 - 0.017\alpha - 0.013\delta_e + 0.047\delta_{st} - 1.29\omega_z/\widehat{V}.$$

Here, δ_{st} is the pitch angle of the tailplane.

2.4. Dynamics of the actuators. Let the change of the thrust force be described by the relation

$$\dot{p} = -k_p p + \overline{k}_p(\delta_{ps} + \overline{\delta}_p), \tag{2.2}$$

$$k_p = 1 \text{ s}^{-1}, \quad \overline{k}_p = 3538 \text{ N s}^{-1}\text{deg}^{-1}, \quad \overline{\delta}_p = -41.3^\circ,$$

$$47^\circ \le \delta_{ps} \le 112^\circ. \tag{2.3}$$

Here, δ_{ps} is the position of the engine controller. Substituting the extreme values $\delta_{ps} = 47^\circ$ and $\delta_{ps} = 112^\circ$ into the right-hand side of (2.2), we obtain the stationary values $p \approx 2 \times 10^4$ N and $p \approx 25 \times 10^4$ N for the equation $\dot{p} = 0$. If the initial value of p lies in the range $[2 \times 10^4,\ 25 \times 10^4]$, then the value of p will remain in this range.

We describe the dynamics of the controlling servomechanisms by the following simple equations. For the elevator:

$$\dot{\delta}_e = k_e(\delta_{es} - \delta_e), \quad k_e = 4 \text{ s}^{-1}, \tag{2.4}$$

$$|\delta_{es}| \le 10^\circ; \tag{2.5}$$

for the rudder:

$$\dot{\delta}_r = k_r(\delta_{rs} - \delta_r), \quad k_r = 4 \text{ s}^{-1}, \tag{2.6}$$

$$|\delta_{rs}| \le 10^\circ; \tag{2.7}$$

for the ailerons:

$$\dot{\delta}_a = k_a(\delta_{as} - \delta_a), \quad k_a = 4 \text{ s}^{-1}, \tag{2.8}$$

$$|\delta_{as}| \le 10^\circ. \tag{2.9}$$

The values δ_{es}, δ_{rs}, and δ_{as} are the control positions of the elevator, rudder, and ailerons.

2.5. Complete nonlinear system. Adding relations (2.2), (2.4), (2.6), and (2.8) to main system (2.1), we obtain the differential system in the vector form

$$\dot{\xi} = f(\xi, u, w), \quad \xi \in \mathbb{R}^{16}, \tag{2.10}$$

where ξ is the phase vector and the vectors $u = (\delta_{ps}, \delta_{es}, \delta_{rs}, \delta_{as})'$ and $w = (w_{xg}, w_{yg}, w_{zg})'$ are the control and disturbance. The control variables (*control positions*) δ_{ps}, δ_{es}, δ_{rs}, and δ_{as} are upper and lower bounded by relations (2.3), (2.5), (2.7), and (2.9).

3. LINEARIZED SYSTEMS FOR THE VERTICAL AND LATERAL CHANNELS: FORMING THE ADAPTIVE CONTROL

For the application of the adaptive control method presented in Section 1, one should linearize the aircraft's nonlinear dynamics about the nominal motion. The nominal motion is the motion along the rectilinear descending glide path with a constant velocity and without rotation. To calculate the parameters of the nominal motion, we specify the "average" values of the components w_{xg0}, w_{yg0}, and w_{zg0} of the wind velocity along the axes of the ground coordinate system. On the nominal motion, the gliding angle β_0 is assumed to be zero. Initial data also include the glide slope angle Θ and the nominal value $\widehat{V}_0$ of the airspeed.

Having calculated the parameters of the nominal motion, we linearize nonlinear dynamics (2.10). The linearized system is virtually *decomposed into two subsystems* of the vertical and lateral channels. We neglect the weak mutual influence of the systems. The decomposition of a linearized system into two subsystems is a standard approach in the aviation engineering practice.

We take the initial data for computing the nominal motion and linearized systems in the form

$$\Theta = 2^\circ 40', \quad \widehat{V}_0 = 72.2 \text{ m/s}, \quad w_{xg0} = -5 \text{ m/s}, \quad w_{yg0} = w_{zg0} = 0.$$

The obtained nominal values are as follows:

$$\begin{array}{llll} V_{xg0} = 67.13 \text{ m/s}, & V_{yg0} = -3.13 \text{ m/s}, & \alpha_0 = 5.42^\circ, & \vartheta_0 = 2.94^\circ, \\ p_0 = 124\,500 \text{ N}, & \delta_{st} = -1.26^\circ, & \delta_{ps0} = 76.5^\circ. & \end{array}$$

The values γ_0, ψ_0, ω_{x0}, ω_{y0}, ω_{z0}, δ_{e0}, δ_{r0}, δ_{a0}, δ_{es0}, δ_{rs0}, and δ_{as0} are equal to zero.

3.1. The case of the inertialess wind disturbance. The linear system of the vertical channel is described by the equation

$$\dot{x}^V = A^V x^V + B^V u^V + C^V w^V. \tag{3.1}$$

Here,

$$x^V = (\Delta x_g, \Delta V_{xg}, \Delta y_g, \Delta V_{yg}, \Delta\vartheta, \Delta\omega_z, \Delta\delta_e, \Delta p/m)',$$

$$u^V = (\Delta\delta_{ps}, \Delta\delta_{es})', \qquad w^V = (\Delta w_{xg}, \Delta w_{yg})'.$$

Let us give the numerical values of the matrices A^V, B^V, and C^V:

$$A^V = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.0501 & 0 & -0.0973 & -2.6422 & 0 & 0.0628 & 0.9971 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0.2409 & 0 & -0.6387 & 45.2782 & 0 & 1.4479 & 0.0813 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0.0003 & 0 & 0.0069 & -0.5008 & -0.5263 & -0.3830 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix},$$

$$B^V = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \end{bmatrix}',$$

$$C^V = \begin{bmatrix} 0 & 0.0501 & 0 & -0.2409 & 0 & -0.0003 & 0 & 0 \\ 0 & 0.0973 & 0 & 0.6387 & 0 & -0.0069 & 0 & 0 \end{bmatrix}'.$$

For the lateral channel:

$$\dot{x}^L = A^L x^L + B^L u^L + C^L w^L, \tag{3.2}$$

where

$$x^L = (\Delta z_g, \Delta V_{zg}, \Delta\psi, \Delta\omega_y, \Delta\gamma, \Delta\omega_x, \Delta\delta_a, \Delta\delta_r)',$$

$$u^L = (\Delta\delta_{rs}, \Delta\delta_{as})', \qquad w^L = \Delta w_{zg}.$$

The numerical values of the matrices A^L, B^L, and C^L:

$$A^L = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.0769 & -5.5553 & 0 & 9.2719 & 0 & -1.4853 & 0 \\ 0 & 0 & 0 & 1.0013 & 0 & 0 & 0 & 0 \\ 0 & -0.0129 & -0.9339 & -0.2588 & -0.0883 & -0.0303 & -0.2456 & -0.0460 \\ 0 & 0 & 0 & -0.0514 & 0 & 1 & 0 & 0 \\ 0 & -0.0331 & -2.3865 & -0.9534 & -0.2256 & -1.4592 & -0.2327 & -0.6894 \\ 0 & 0 & 0 & 0 & 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4 \end{bmatrix},$$

$$B^L = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix}',$$

$$C^L = \begin{bmatrix} 0 & 0.0769 & 0 & 0.0129 & 0 & 0.0331 & 0 & 0 \end{bmatrix}'.$$

Consider the following constraints on the control actions:

$$|\Delta\delta_{ps}| \le 27\frac{\pi}{180} = 27^\circ, \quad |\Delta\delta_{es}| \le 10\frac{\pi}{180} = 10^\circ \tag{3.3}$$

in the vertical channel and

$$|\Delta\delta_{rs}| \le 10\frac{\pi}{180} = 10^\circ, \quad |\Delta\delta_{as}| \le 10\frac{\pi}{180} = 10^\circ \tag{3.4}$$

in the lateral channel.

Thus, the linear dynamics is written for each channel (in terms of deviations from the nominal motion), and constraints on the useful control are specified. For the construction of the adaptive control, it is also required to specify for each channel the terminal set M and the set $Q_{\max}$, which is a "reasonable" constraint for the wind disturbance.

In the vertical channel, we introduce the set M^V on the plane Δy_g, ΔV_{yg}, where Δy_g is the vertical deviation (m) and ΔV_{yg} is the velocity of the vertical deviation (m/s), as a convex hexagon with the vertices

$$(-3,0),\ (-3,1),\ (0,1),\ (3,0),\ (3,-1),\ (0,-1).$$

The orientation of the set M^V can be explained as follows: the vertical deviation at the moment of passing the runway threshold is compensated by a deviation from the nominal value of the vertical velocity with the oppposite sign.

We define the set $Q^V_{\max}$ in the vertical channel by the inequalities

$$|\Delta w_{xg}| \le 6 \text{ m/s}, \quad |\Delta w_{yg}| \le 4 \text{ m/s}. \tag{3.5}$$

In the lateral channel, we take the set M^L in the form of a convex hexagon with the vertices

$$(-6,0),\ (-6,1.5),\ (0,1.5),\ (6,0),\ (6,-1.5),\ (0,-1.5)$$

in the coordinates Δz_g, ΔV_{zg}, where Δz_g is the lateral deviation (m) and ΔV_{zg} is the velocity of the lateral deviation (m/s).

We define the set $Q^L_{\max}$ by the inequality

$$|\Delta w_{zg}| \le 10 \text{ m/s}. \tag{3.6}$$

Since the convex set M^V is given in the space of the two coordinates of linear system (3.1), the phase vector of the system of form (1.2) of the vertical channel has second order in the phase variable. Therefore, the sections $W^V_{\text{main}}(t)$ and $W^V_{\text{add}}(t)$ of the stable tubes W^V_{main} and W^V_{add} are *convex two-dimensional* sets. The same is true for the sections $W^L_{\text{main}}(t)$ and $W^L_{\text{add}}(t)$ of the tubes W^L_{main} and W^L_{add} of the lateral channel. For the numerical construction of stable tubes, we use the algorithm from [34].

The deviations Δw_{xg}, Δw_{yg}, and Δw_{zg} of the components of the wind velocity from the nominal values enter linear systems (3.1) and (3.2) of the vertical and lateral channels as disturbance control actions. To calculate tubes of the main bridges W^V_{main} and W^L_{main}, we specify constraints (3.5) and (3.6) on the disturbance. The results of the numerical construction show that tubes of the main bridges degenerate quickly when they are constructed backwards from the terminal time (the t-sections become empty) even under these not very "wide" constraints on the disturbance. This happens because we admit discontinuous (in time) changes in the components of the wind disturbance.

3.2. Linear systems for the inertial wind disturbance. To take into account the *inertia* of the change in the wind velocity, we add the relations

$$\begin{aligned} \Delta\dot{w}_{xg} &= 0.5(\Delta w_{xg} - v_{xg}), \\ \Delta\dot{w}_{yg} &= 0.5(\Delta w_{yg} - v_{yg}) \end{aligned} \tag{3.7}$$

to vector equation (3.1) of the linear dynamics of the vertical channel and the relation

$$\Delta\dot{w}_{zg} = 0.5(\Delta w_{zg} - v_{zg}) \tag{3.8}$$

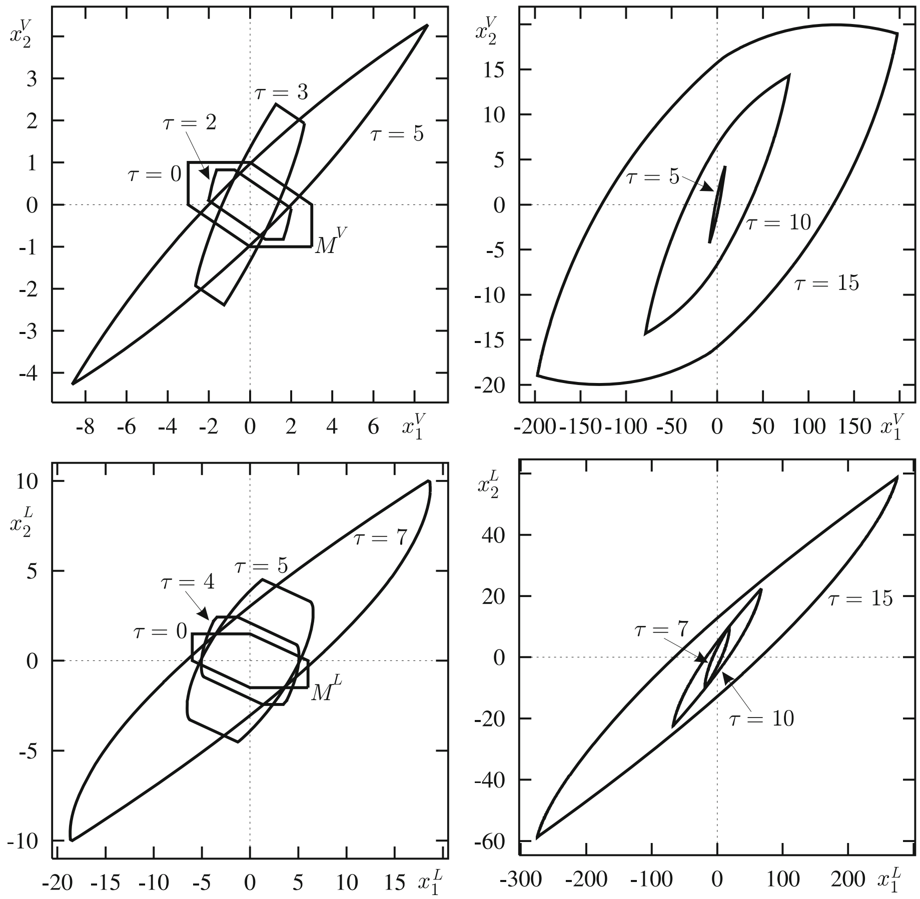

Fig. 2. Several sections of the main bridges W^V_{main} and W^L_{main} of the vertical and lateral channels, respectively.

to equation (3.2) of the lateral channel.

Thus, in the expanded linear system (3.1), (3.7) of the vertical channel, the values Δw_{xg} and Δw_{yg} become phase variables and the values v_{xg} and v_{yg} are disturbance control actions. Let us impose the constraints

$$|v_{xg}| \le 6 \text{ m/s}, \quad |v_{yg}| \le 4 \text{ m/s}, \tag{3.9}$$

which are similar to constraints (3.5). In expanded linear system (3.2), (3.8) of the lateral channel, the value Δw_{zg} is a phase variable and v_{zg} is a disturbance action. We impose the following constraint:

$$|v_{zg}| \le 10 \text{ m/s}. \tag{3.10}$$

Now, the main bridge W^V_{main} for system (3.1), (3.7) of the vertical channel with constraints (3.3) on the useful control and constraints (3.9) on the disturbance action does not degenerate. The same is true for the main bridge W^L_{main} for system (3.2), (3.8) of the lateral channel with constraints (3.4) on the useful control and constraint (3.10) on the disturbance. Figure 2 shows sections of the bridges W^V_{main} and W^L_{main} for several moments of the inverse time $\tau = t_f - t$. For small values of τ, the sections become narrow; then, the direction of elongation changes; and, as τ grows further, the sections grow with a small change in the elongation direction.

3.3. Rough and exact adaptive control. The altitude of 60 m is critical for a descending aircraft ([1, p. 167], [33, p. 116]). Below this altitude, the control of the aircraft must be especially accurate. The time of descending from the altitude of 60 m to the moment of passing the runway threshold is about 15 s. Therefore, we restrict ourselves to computing the main bridges W^V_{main} and W^L_{main} on the inverse time interval of 15 s. This means that, when preparing the main bridges, we set $t_f - t_0 = 15$ s. The additional bridges W^V_{add} and W^L_{add} are also computed on the interval of length 15 s. To this aim, for the vertical (lateral) channel, we use the construction of the tube of the reachable set of system (3.1), (3.7) (system (3.2), (3.8), respectively) under the *zero* useful control and constraint (3.9) (constraint (3.10), respectively) on the disturbance action.

We use the adaptive control in a discrete control scheme with step Δ in the following way. We assume that the t-sections of the main and additional bridges for each channel are calculated in advance on the interval $[t_0, t_f] = [0, 15]$ with the same step Δ. Let $d(t)$ be the distance along the x_g axis to the runway threshold at the current instant $t \geq t_*$, where t_* is the initial instant of the simulation. Then, $a(t) = d(t)/V_{xg0}$ is the predicted time to the passage of the runway threshold. In the case $a(t) > 15$, we construct the adaptive control using the sections of the bridges corresponding to $\tau = 15$ s. If $a(t) \leq 15$, then we use the sections corresponding to the instant $\tau = a(t)$. Thus, our control is formed in a simplified way (with a digression from the exact rule of adaptive control) if $d(t) > 15V_{xg0} \approx 1000$ m. If $d(t) \leq 15V_{xg0}$, we follow the adaptive control scheme described in Section 1.

4. SIMULATION RESULTS

For the purposes of simulation, we consider the disturbance caused by a wind microburst [35] as the wind disturbance. A *wind microburst* is a natural phenomenon appearing when a descending air current strikes the ground and flows off horizontally with whirls being formed. When passing a microburst zone, the aircraft first enters a current of head wind, which changes to a descending wind during a short period of time—dozens of seconds—and, after that, to a tail wind. A head wind increases the airspeed and, hence, the lifting force, while a descending wind or tail wind has the opposite effect. A sharp change in the wind direction from a head wind to a tail wind leads to a sharp fall in the lifting force.

Let us describe the model of a microburst that we use [27]. A torus is given in space, see Fig. 3. Turbulence is formed outside the torus, and the wind velocity decreases proportionally inside the

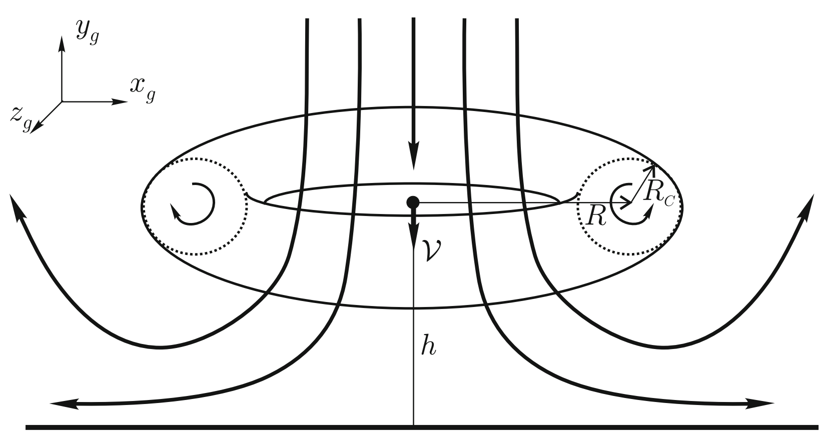

Fig. 3. Wind microburst model.

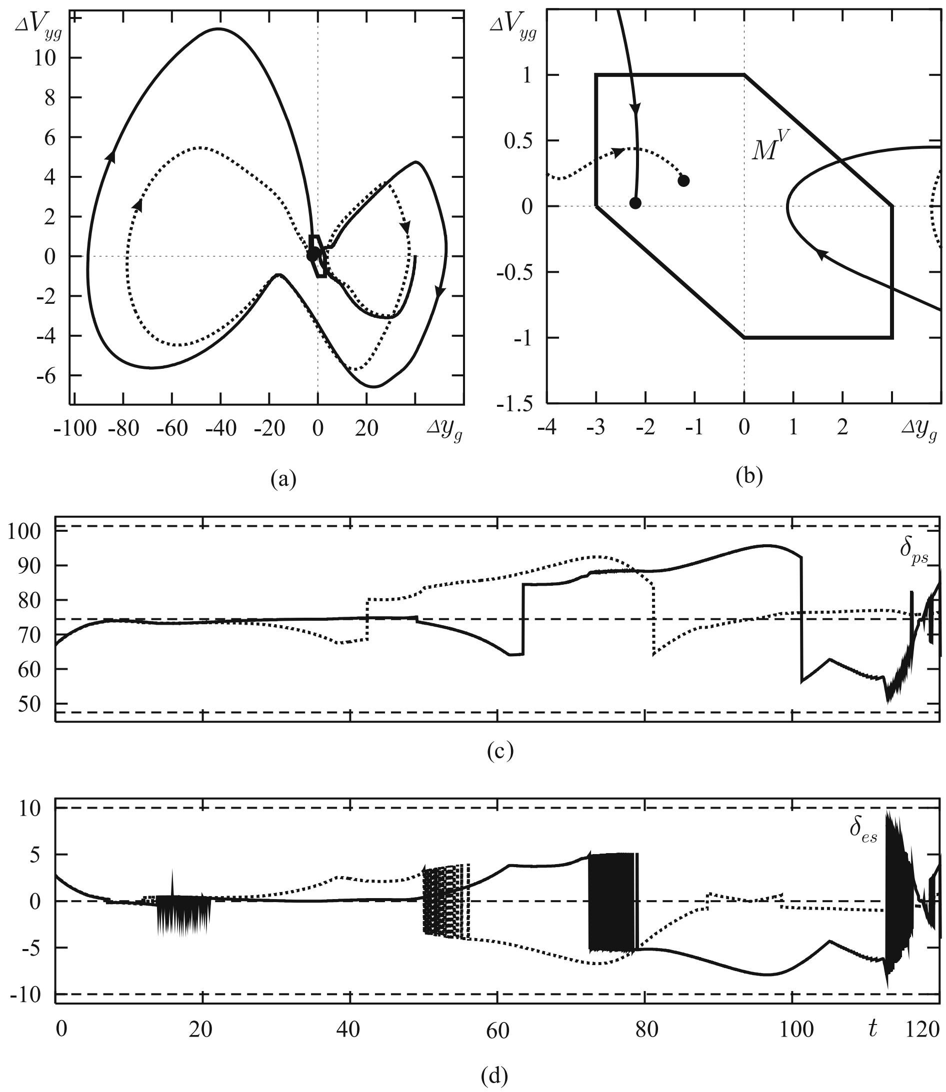

Fig. 4. Simulation with a wind microburst: (a) trajectories in the phase plane $\Delta y_g \times \Delta V_{yg}$ of the vertical channel; (b) an enlarged fragment near the terminal set; (c) graphs of the control positions δ_{ps} (deg) of the engine controller; (d) graphs of the control positions δ_{es} (deg) of the elevator. The dotted line corresponds to microburst 1, and the solid line corresponds to microburst 2.

torus as one moves closer to the center of the tube. The microburst is characterized by the following parameters: $\mathcal{V}$ is the wind speed at the central point (this speed is not maximal; the wind speed near the torus can be greater up to a factor of 2); h is the altitude of the central point; R is the distance from the central point to the center of the tube; $R_C = 0.8h$ is the radius of the tube; and $\widetilde{x}_0$, $\widetilde{z}_0$ is the projection of the central point onto the ground plane.

We assume that the wind velocity vector with components w_{xg}, w_{yg}, w_{zg} at the point of the geometric position of the aircraft is composed of the nominal velocity vector (in our case, $w_{xg0} = -5$ m/s, $w_{yg0} = w_{zg0} = 0$) and an additive, which is caused by the microburst. The values w_{xg}, w_{yg}, w_{zg} are passed to nonlinear system (2.10) of the aircraft motion.

Let us present the results of simulation for two variants of the microburst. Microburst 1 has the following parameters: the wind speed at the central point is 10 m/s; the distance from the

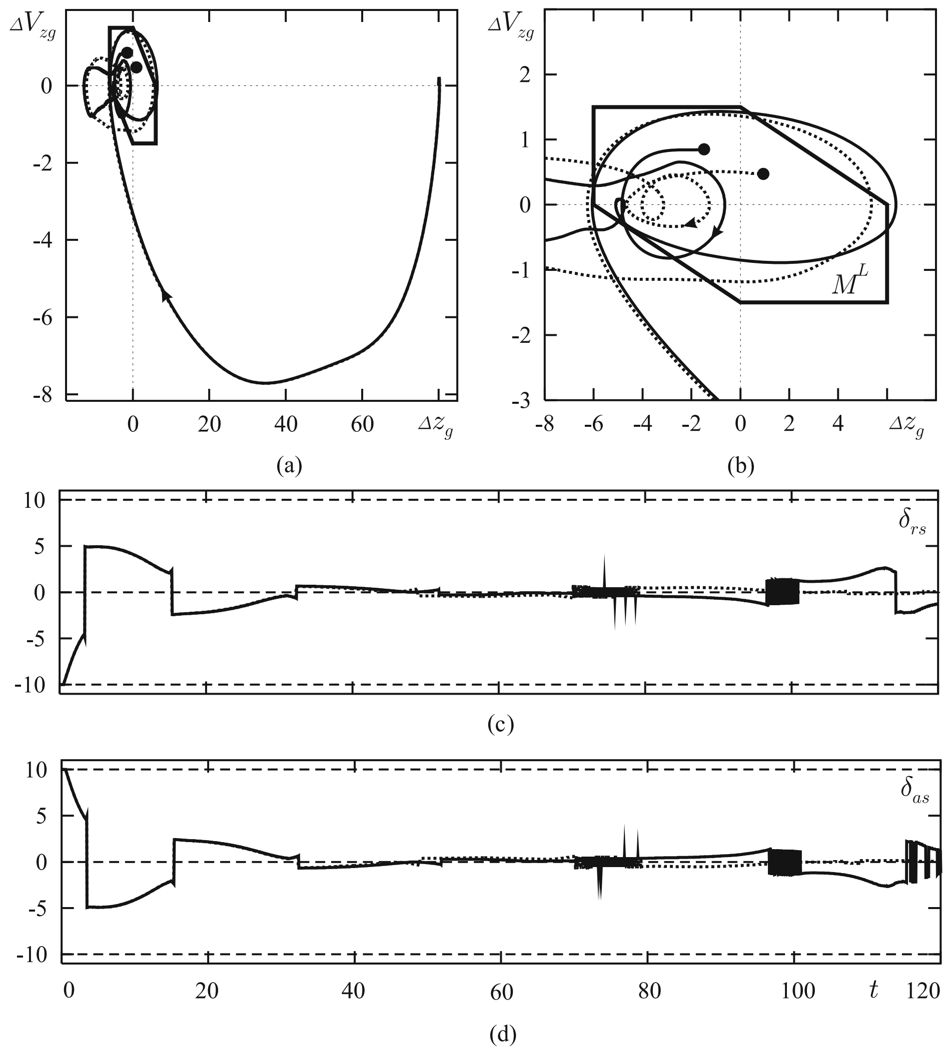

Fig. 5. Simulation with a wind microburst: (a) trajectories in the phase plane $\Delta z_g \times \Delta V_{zg}$ of the lateral channel; (b) an enlarged fragment near the terminal set; (c) graphs of the control positions δ_{rs} (deg) of the rudder; (d) graphs of the control positions δ_{as} (deg) of the ailerons. The dotted line corresponds to microburst 1, and the solid line corresponds to microburst 2.

central point to the center of the tube is 1200 m; the altitude of the central point is 600 m; the distance to the runway in the longitudinal coordinate (along the glide path) is 4000 m; and the lateral deviation from the glide path line is 500 m. Microburst 2 is stronger; it has the greater wind speed 15 m/s and the smaller distance 2500 m to the runway.

The initial position of the aircraft is at 8000 m along the x_g axis from the runway threshold; its deviation from the nominal position is 40 m upwards and 80 m sideway.

In forming the control, we assume that all the phase variables in the description of the nonlinear dynamics of the aircraft are *measured exactly* during the motion process. The step Δ of the discrete scheme of control is 0.05 s.

In Figs. 4–7, the dotted lines denote trajectories and graphs generated by microburst 1 and the solid lines correspond to microburst 2.

First assume that current components of the wind velocity are *measured exactly*. The deviations

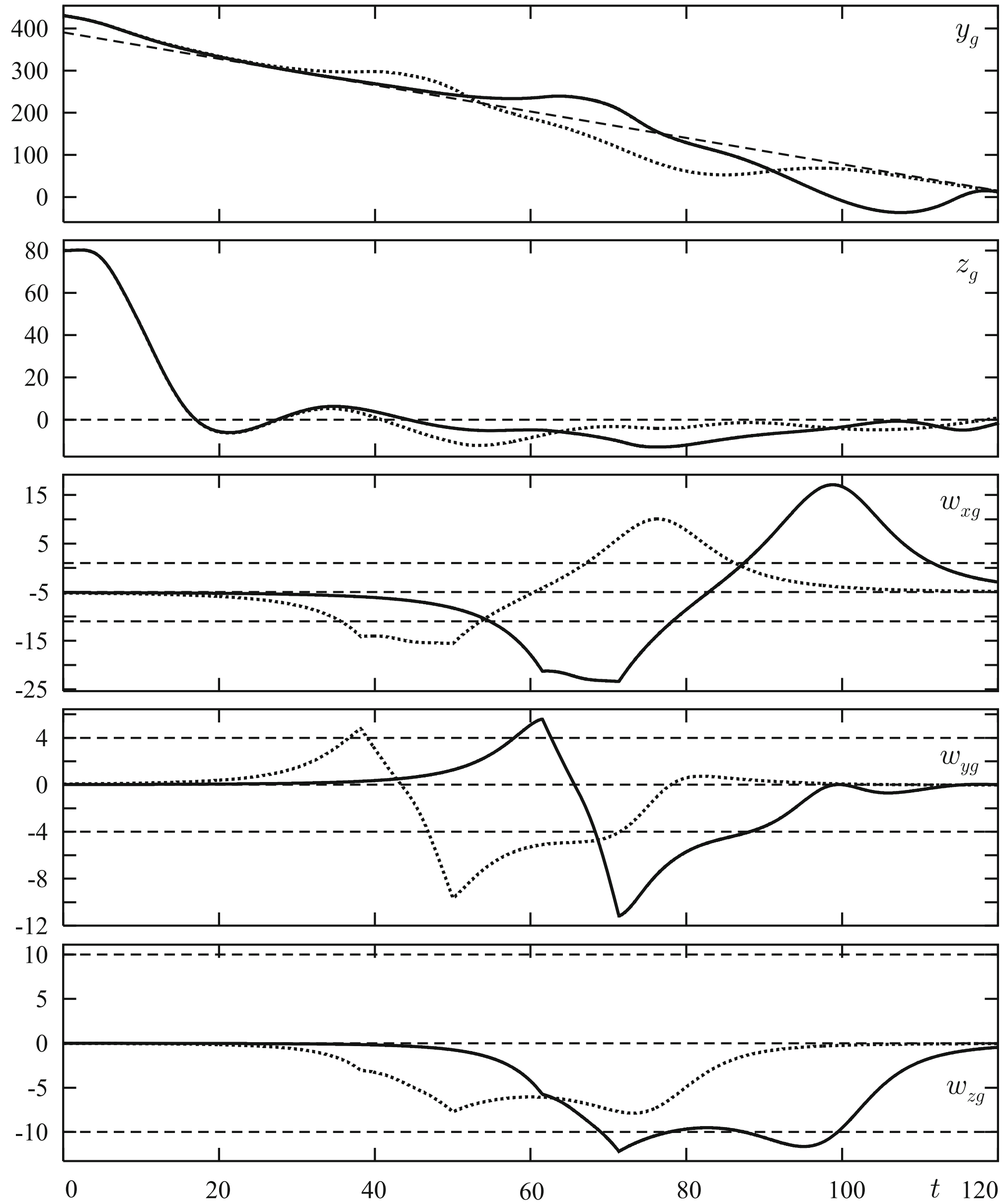

Fig. 6. Graphs of the altitude y_g (m); lateral deviation z_g (m); and longitudinal w_{xg}, vertical w_{yg}, and lateral w_{zg} components (m/s) of the wind velocity. The dotted lines correspond to microburst 1, and the solid lines correspond to microburst 2.

$\Delta w_{xg}, \Delta w_{yg}$ (Δw_{zg}) are used to calculate the phase state of linear system (3.1), (3.7) of the vertical channel (system (3.2), (3.8) for the lateral channel) and, thereby, to calculate the adaptive control in the vertical (lateral) channel.

Figures 4 and 5 show the results for the vertical and lateral channels, respectively. In the graphs of the useful control, the dashed lines denote the nominal and maximal allowable values. It is seen that the realized control does not reach its extreme allowable values. Note that control actions are actually control positions (signals). They are smoothed by the inertia of the actuators.

Graphs of the variation in the altitude y_g and of the lateral deviation z_g, as well as graphs of the components w_{xg}, w_{yg}, and w_{zg} of the wind velocity are presented in Fig. 6. In the graphs of the

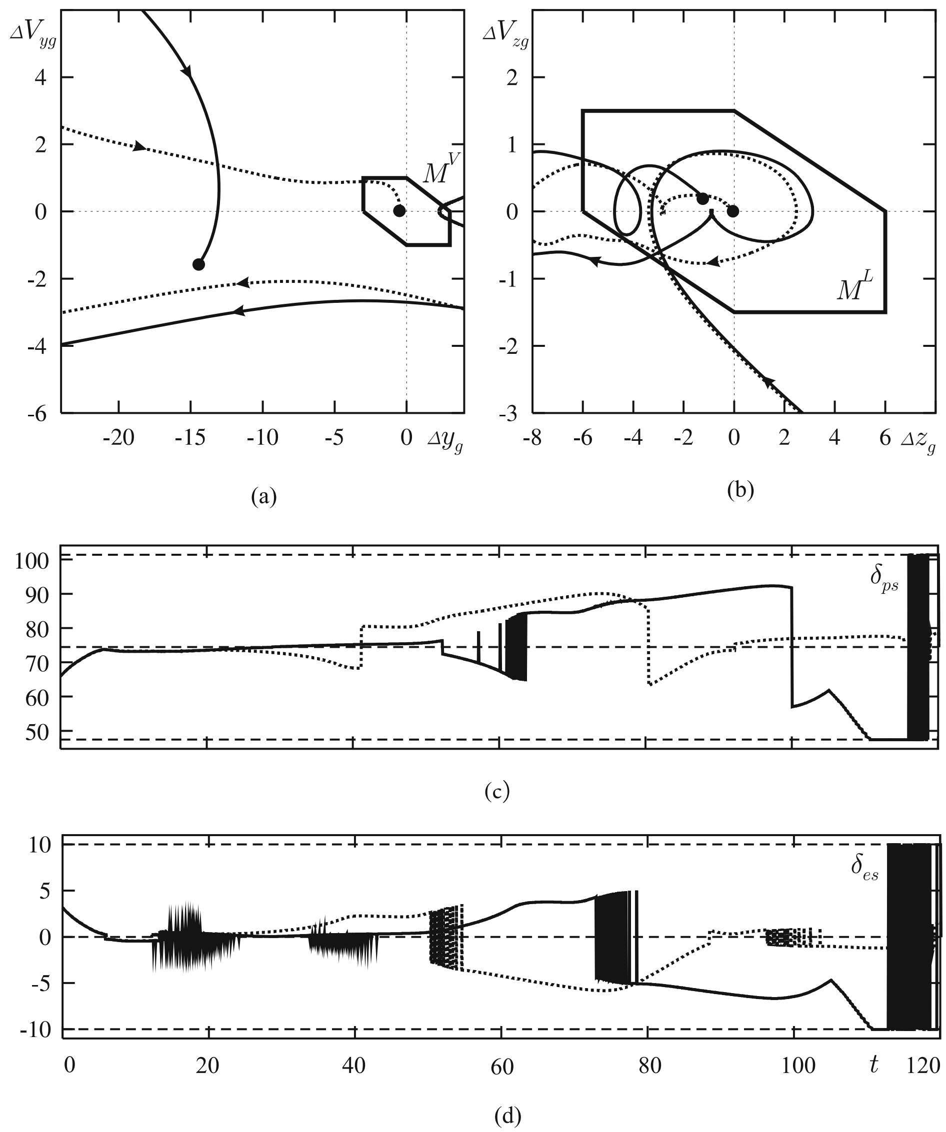

Fig. 7. Simulation results for the case when the components of the wind velocity are constant: (a) fragments of phase trajectories of the vertical channel; (b) fragments of phase trajectories of the lateral channel; (c, d) graphs of control actions of the vertical channel.

wind disturbance, the dashed lines show the expected maximal values and the nominal values; in each of the graphs of the phase coordinates, there is only one additional line, which is the nominal straight line.

Note that wind velocity on some time intervals was substantially greater than the predicted values. Nevertheless, in the case of the weak microburst, the control could successfully handle the disturbance and the motions reached the terminal sets in both channels. In the case of the strong microburst, the terminal conditions are also satisfied formally. However, let us examine the graph of the altitude variation. For the strong microburst, the result is unacceptable: the aircraft hit the ground approximately 20 s before passing the runway threshold. This is explained by the fact that a substantial change in the longitudinal and vertical components of the wind velocity happened at a small altitude. This has also revealed a drawback of the method: we cannot directly take into account the phase constraint concerning the altitude.

The assumption about the measurement of the components of the current wind velocity can hardly be realistic. In this connection, a simulation was carried out, in which *zero values* were fed to the adaptive control scheme instead of the phase variables Δw_{xg}, Δw_{yg}, and Δw_{zg} of linear systems (3.1), (3.7) and (3.2), (3.8). The results are presented in Fig. 7. For microburst 1, they are no worse than in the case of the measurement of the wind velocity. Good results are also obtained in the lateral channel in the case of microburst 2. However, in the vertical channel, the results are poor: there is a large error with respect to the terminal set M^L at the moment of passing the runway threshold and a gliding regime with the extreme control actions of the thrust force and elevator in proximity to the runway threshold.

CONCLUSIONS

The landing stage investigated in this paper (until the moment of passing the runway threshold) is peculiar in that the system linearized about the nominal motion is decomposed into the subsystems of the vertical and lateral channels. Due to the substantial velocity of the longitudinal motion, auxiliary control problems with a fixed terminal time can be used as a base and numerical methods of the theory of differential games developed for such problems can be applied. In this approach, boundary conditions at the terminal time are specified for each channel in the form of convex sets on the plane of only two phase variables that are most important for that channel. Thus, it becomes possible to use the values of these variables predicted for the terminal time, which makes the numerical procedures of control construction very simple. The adaptive control method considered in this paper is adjusted to the current level of wind disturbance, preserving the calculated guarantee under a strong disturbance and smoothly decreasing the level of the control action if the disturbance level recedes.

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