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Vertical Passage of Obstacles by an Aircraft under Wind Disturbance

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Abstract—The adaptive control method is applied to the problem of the vertical passage of obstacles by an aircraft under wind disturbance. Constructions of the theory of differential games are used.
Keywords: adaptive control, differential games, aircraft control, wind disturbance, numerical constructions.

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INTRODUCTION

The paper is devoted to constructing a control in the problem of the vertical passage of obstacles by an aircraft under unknown bounded wind disturbance. The case of a guaranteed detour of two obstacles is considered. The arguments are easily transferred to the case of several obstacles. The suggested method is based on ideas from the theory of differential games [1, 2] and on the method of adaptive control presented in [3, 4].

It is assumed the aircraft moves horizontally in a steady regime before detecting the obstacles. Some distance is fixed at which obstacles lying ahead are detected. After the detection, the aircraft starts the maneuver of a vertical passage of the obstacles. The control that guarantees the passage of the first obstacle is constructed with the help of an auxiliary differential game. When the next obstacle is detected, the second game is considered. The method of choosing a control is suggested that guarantees achieving the goal (i.e., the passage of the two obstacles) in both games.

In each of the games, the control is constructed as follows. A rectilinear reference motion is constructed in the vertical plane, and the nonlinear dynamics is linearized along this motion. An auxiliary linear differential game with fixed terminal time is constructed, in which the terminal time is taken to be the moment of the nominal passage of the obstacle. The vector useful control has two components: one of them governs the deviation of the thrust force from the nominal value, while the other is a fictitious control of the elevator, which has the sense of the target pitch angle. The disturbance is presented by two components (longitudinal and vertical) of the wind velocity. Geometric constraints are imposed on the useful control and disturbance.

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The auxiliary game is reduced to the form in which the phase variable is one-dimensional. It is calculated based on the current state with the use of the Cauchy fundamental matrix of the linearized system. This game characterizes the prediction of the altitudinal deviation from the nominal value at the time of passing the obstacle. The one-dimensionality allows one to easily construct in the game space two curves such that the location of the phase variable between these curves is guaranteed by the useful control regardless of the action of the wind disturbance within the specified limits. If the realized level of the disturbance is lower than the level specified in the description of the auxiliary game, then it is compensated by a useful control that is also small. Auxiliary differential games are used to choose the control, while the motion is modeled within the original nonlinear system.

The problem of the vertical passage of obstacles by an aircraft under unpredictable wind gusts is close to problems of take-off and of abort landing under wind disturbances. The application of modern methods from mathematical control theory and from the theory of differential games to such problems was considered in [5–12].

Section 1 is devoted to the problem statement and to the description of the aircraft's dynamics. In Section 2, we consider the application of the adaptive control in the case of one obstacle. In Section 3, we extend the control method to the case of several obstacles. Simulation results are presented in Section 4.

1. PROBLEM STATEMENT AND DESCRIPTION OF THE NONLINEAR DYNAMICS

We consider the problem of preventing a collision of an aircraft with two surface obstacles (Fig. 1). We consider the altitudinal evasion and, accordingly, the aircraft's motion in the vertical plane. Sudden wind disturbances are allowed.

The positions and heights of the two obstacles are given. We assume that the aircraft moves initially along a horizontal trajectory, which can be assumed to be the initial reference trajectory. We are also given the horizontal distance D at which an obstacle is detected. After detecting the first obstacle, we draw a reference trajectory from the current point to some point above the obstacle. The aim of the control is to guarantee that the aircraft is above this point at the moment of passing the obstacle. When the second obstacle is detected, we draw the second reference trajectory. The aim of the control is to guarantee the detour of both obstacles.

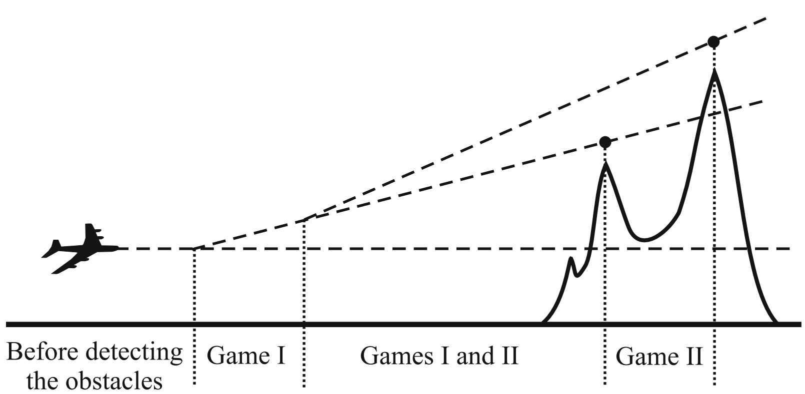

Fig. 1. Problem of the passage of two obstacles.

The motion is modeled by an eight-dimensional nonlinear system [13–16] describing the vertical channel of the aircraft's motion:

$$\begin{aligned}
\dot{x}_g &= V_{xg}, \\
\dot{V}_{xg} &= [(P\cos\sigma - qSc_x)\cos\vartheta(P\sin\sigma + qSc_y)\sin\vartheta)]/m, \\
\dot{y}_g &= V_{yg}, \\
\dot{V}_{yg} &= [(P\cos\sigma - qSc_x)\sin\vartheta\text{-}(P\sin\sigma + qSc_y)\cos\vartheta]/m - g, \\
\dot{\vartheta} &= \omega_z, \\
\dot{\omega}_z &= M_z/I_z, \\
\dot{P} &= k_p(\delta_{ps} - P), \\
\dot{\delta}_e &= k_e(\delta_{es} - \delta_e).
\end{aligned} \tag{1.1}$$

Here, x_g is the longitudinal coordinate of the aircraft's center of mass, m; y_g is the altitude, m; V_{xg}, V_{yg} are the longitudinal and vertical components of the velocity vector, m/s; ϑ is the pitch angle, rad; ω_z is the angular velocity, rad/s; P is the thrust force, N; δ_e is the deviation of the elevator, rad; δ_{ps} is the command position of the thrust force; and δ_{es} is the command position of the elevator.

The system also includes the following values:

$q = \rho\widehat{V}^2/2$ is the dynamic pressure;

$\widehat{V} = (\widehat{V}_{xg}^2 + \widehat{V}_{yg}^2)^{1/2}$ is the magnitude of the airspeed vector, m/s;

$\widehat{V}_{xg} = V_{xg} - W_{xg}$, $\widehat{V}_{yg} = V_{yg} - W_{yg}$ are the components of airspeed vector, m/s;

W_{xg}, W_{yg} are the components of the wind velocity, m/s;

$\rho = 1.207$ is the air density, kg/m^3;

$g = 9.81$ is the acceleration of gravity, m/s^2;

$c_x = \widetilde{c}_x\cos\alpha - \widetilde{c}_y\sin\alpha$, $c_y = \widetilde{c}_y\cos\alpha + \widetilde{c}_x\sin\alpha$ are the aerodynamic coefficients of the resistance force in the bound [15, 17] coordinate system;

$\alpha = \arcsin\{(-\widehat{V}_{xg}\sin\vartheta - \widehat{V}_{yg}\cos\vartheta)/\widehat{V}\}$ is the angle of attack;

$M_z = qSbm_z$ is the moment of force, N m.

We write the remaining values in the numerical form corresponding to the Tupolev Tu-154 aircraft:

$\widetilde{c}_x = 0.21 + 0.004\alpha + 0.47\times10^{-3}\alpha^2$, $\widetilde{c}_y = 0.65 + 0.09\alpha + 0.003\delta_e$ are the coefficients of the resistance force in the semibound [15, 17] coordinate system;

$m_z = 0.033 - 0.017\alpha - 0.013\delta_e + 0.047\delta_{st} - 1.29\omega_z/\widehat{V}$ is the aerodynamic coefficient of the moment in the bound coordinate system;

$S = 201$ is the wing area, m^2;

$b = 5.285$ is the average aerodynamic chord, m;

$m = 75\times10^3$ is the mass of the aircraft, kg;

$\sigma = 1.72$ is the thrust inclination, deg;

$I_z = 6.5\times10^6$ is the moment of inertia, kg m^2;

δ_{st} is the pitch angle of the tailplane, deg;

$k_p = 1$, $k_e = 4$ are the inertia factors of the thrust force and elevator, s^{-1}.

In the formulas for $\widetilde{c}_x$, $\widetilde{c}_y$, and m_z all the angular values are taken in degrees. The stabilizer is a control organ equivalent to the elevator, and its deviation is used to set the nominal value of the elevator to zero. The stabilizer is put to its nominal position according to the chosen reference trajectory.

The control is implemented by means of command positions of the thrust force and of the elevator. The disturbances acting on the system are the longitudinal and vertical components of the wind velocity.

Let the control of the elevator be given by the linear law

$$\delta_{es} = -k_1\big(u_\vartheta - \vartheta - k_2(\widehat{V} - \widehat{V}^0)\big) + k_3\omega_z. \tag{1.2}$$

Here, $k_1 = 1$, $k_2 = 0.0075$, and $k_3 = 0.2$ are coefficients; $\widehat{V}$ and $\widehat{V}^0$ are the current and nominal (on the corresponding reference trajectory) airspeeds; and u_ϑ is a fictitious control, which has the sense of the target pitch angle. The new control is introduced because of the following considerations. The potential of using the elevator is rather large, and it is far more important to keep the pitch angle within the specified limits. However, since the pitch angle is a phase variable in the original system and it is impossible to control it directly, the one-dimensional differential game introduced below involves the target pitch angle (subject to some constraints). In this game, a control law for the elevator is constructed, which brings the real pitch angle close to the target one.

Let the control of the thrust force be subject to the inequalities

$$P^0 \le \delta_{ps} \le P^0 + 1.2m, \tag{1.3}$$

where P^0 is the nominal thrust force corresponding to the chosen reference trajectory. The lower bound for δ_{ps} means that the thrust cannot be dropped below the nominal value during the maneuver of passing an obstacle. The upper bound corresponds to the thrust reserve that can be used in the passage maneuver. Naturally, the upper bound should not exceed some global constraint on the thrust force.

For the control of the target pitch angle, we specify the constraints

$$0^\circ\frac{\pi}{180^\circ} \le u_\vartheta \le 20^\circ\frac{\pi}{180^\circ}. \tag{1.4}$$

We assume that the nominal value of the wind disturbance on the whole interval of motion is zero. Possible deviations of the components of the wind velocity are taken in the form

$$|W_{xg}| \le 10 \text{ m/s}, \quad |W_{yg}| \le 5 \text{ m/s}. \tag{1.5}$$

The assumption about the zero nominal wind is not essential. Constraints (1.5) on the deviations from the nominal value are chosen from some "sensible" considerations and are used to construct an adaptive control in linearized problems and to calculate the guarantee provided by the control.

2. CONTROL IN THE CASE ONE OBSTACLE

Let us consider the case of one obstacle and describe the construction of a control in the auxiliary differential game (from the time of detecting the obstacle to the passage of this obstacle).

Linearization with respect to the reference straight line. At the start of the evasion maneuver, a reference straight line is constructed in the geometric coordinates x, y. This line passes through the point (x_{g0}, y_{g0}) where the aircraft is located at the moment of detecting the obstacle and through the point (x_M, y_M) lying at a given height above the obstacle. The altitude reserve is defined by technical conditions. From the mathematical point of view, the value y_M can be assumed to be the height of the obstacle.

We calculate parameters of the nominal motion along the reference straight line. Let V_{xg}^0 be the nominal velocity along the axis x_g. The vector z of difference coordinates, which is the deviation of the current position from the vector of nominal coordinates on the reference straight line corresponding to a current time, is specified. The third coordinate z_3 of the vector z is the altitudinal deviation from the nominal value.

Nonlinear system (1.1) is linearized along the reference straight line:

$$\dot{z} = \mathbf{A}z + \mathbf{B}u + \mathbf{C}v, \quad u \in \Omega_u, \quad v \in \Omega_v. \tag{2.6}$$

In the linear system, the control vector u consists of two components: u_p is the deviation of the command position of the thrust force from the nominal value ($u_p = \delta_{ps} - P^0$), and u^* is the deviation of the target pitch angle from the nominal pitch angle ($u^* = u_\vartheta - \vartheta^0$). The restricting set, which follows from (1.3) and (1.4), is denoted by Ω_u. Relations (1.5) define the set Ω_v, which limits the disturbance. We have $0 \in \Omega_u$, $0 \in \Omega_v$.

Consider an auxiliary game with fixed terminal time $T = (x_M - x_{g0})/V_{xg}^0$. The first player chooses the control u, aiming to provide the inequality $z_3(T) \geq y_M$ at time T. Although no well-defined upper bound is specified for the value $z_3(T)$, the difference $z_3(T) - y_M$ should not be too large. The second player, who is responsible for the action v, applies some a priori unknown control with values in the set Ω_v.

One-dimensional differential game. Using the standard technique of passing to the dynamics without the phase variable on the right-hand side [1, 2], we obtain

$$\begin{gathered} \dot{\xi} = B(\tau)u + C(\tau)v, \quad u \in \Omega_u, \quad v \in \Omega_v, \\ \xi = Z_3(\tau)z, \quad B(\tau) = Z_3(\tau)\mathbf{B}, \quad C(\tau) = Z_3(\tau)\mathbf{C}. \end{gathered} \tag{2.7}$$

Here, $Z_3(\tau)$ is the third row of the Cauchy fundamental matrix of the system $\dot{z} = \mathbf{A}z$, and $\tau = T - t$ is the backward time. The phase variable ξ has the sense of the altitudinal prediction of the position of system (2.6) at time t for time T for $u = v \equiv 0$. The first player is interested in providing the inequality $\xi(0) \geq 0$.

Construction of the control. In the space $\xi \times \tau$ of differential game (2.7), we construct two families of semipermeable curves (Fig. 2). A semipermeable curve [18] is defined as a curve for each point of which there exists an action of the first player such that the motion of the system does not intersect this curve from + to − whatever action the second player chooses. For the second player, a semipermeable curve is defined similarly but the intersection is not allowed from − to +. Curves of the same family differ from each other by a vertical translation only. For curves from family I (II), the sign + is above (below).

Any two nonintersecting semipermeable curves (one from family I and the other from family II) generate a maximal stable bridge [1,2]: the curve from family I is its lower boundary and the curve from family II is its upper boundary.

Let us define the main stable bridge. We take the semipermeable curve from family I passing through the point $\tau = 0$, $\xi = 0$ as the lower boundary. Denote this curve by $\Gamma(\tau)$. Let us take a horizontal straight line $\widetilde{z}(\tau)$ at some distance ε from the lower boundary (see Fig. 2). The

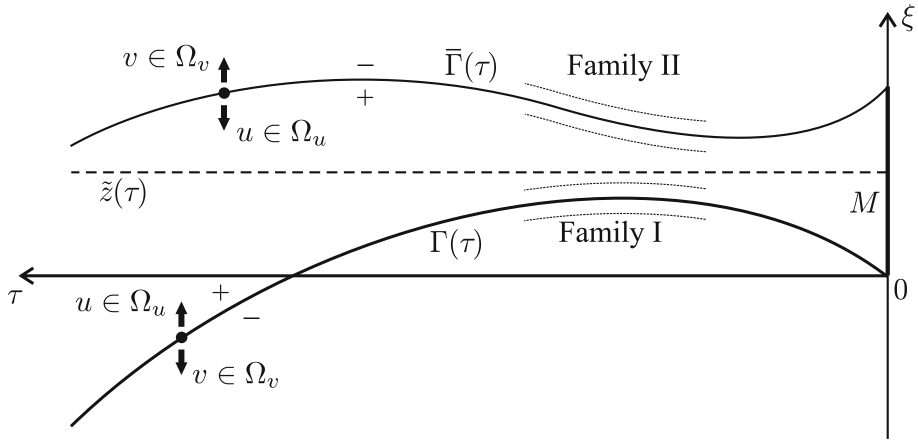

Fig. 2. Two families of semipermeable curves and the main stable bridge.

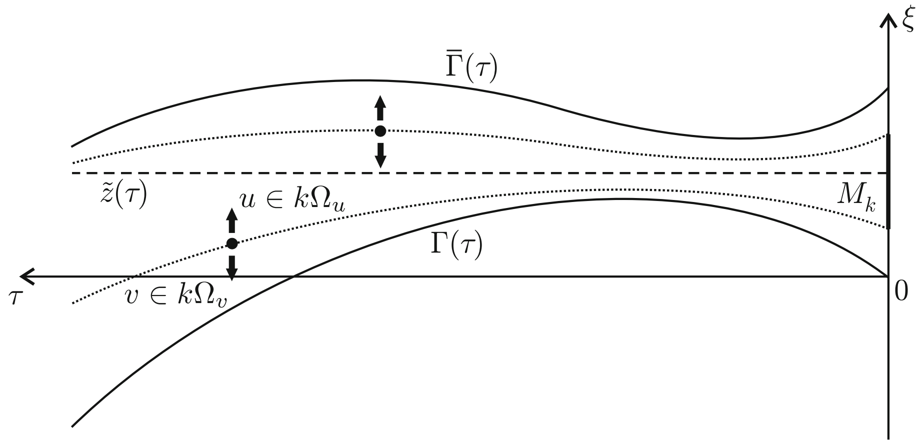

Fig. 3. The adaptive control idea: the proportional change of t-sections of the bridges.

semipermeable curve from family II lying at the distance of ε above the straight line $\widetilde{z}(\tau)$ is taken as the upper boundary of the main bridge and is denoted by $\overline{\Gamma}(\tau)$.

The section of the main bridge at time $\tau = 0$ will be called the terminal segment M. The segment M is defined by the dynamics of game (2.7) and by the chosen value of the parameter ε. The length of the segment characterizes the upward deviation from the point $\xi = 0$ that the first player allows at time $\tau = 0$.

It follows from the theory of differential games as applied to problems with one-dimensional phase variable that, for a current position on the lower boundary of the bridge, one should choose a control in system (2.7) that yields a velocity vector that is maximally directed upward. Accordingly, for a current position on the upper boundary, one should choose a control that directs the velocity vector maximally downward. This strategy guarantees that, if the initial position is within the main bridge and the wind disturbance does not exceed the specified constraints, then the motion stays inside and comes to the segment M. Hence, at terminal time $\tau = 0$, the motion is no lower than the point $\xi = 0$ but, at the same time, not very high.

On the whole, the construction of the control is based on the ideology of adaptive control [3,4]. If the main bridge formed by two semipermeable curves is decreased proportionally by multiplying by a coefficient $0 \leq k < 1$ with respect to $\widetilde{z}(\tau)$ in every time section, then the obtained set is also a maximal stable bridge. The new bridge (Fig. 3) corresponds to the constraints $k\Omega_u$, $k\Omega_v$ on the control and disturbance and to the terminal set M_k obtained by compressing the set M with the coefficient k with respect to the point $\widetilde{z}(0)$. Moreover, the straight line $\widetilde{z}(\tau)$ is a degenerate bridge for the zero constraints. We choose an extreme control on the boundary curves of the main bridge and outside the bridge and the zero control on the line $\widetilde{z}(\tau)$. This is why the line $\widetilde{z}(\tau)$ will be called the switching line. Within the main bridge below and above the switching line, we use the proportionally changed control.

In our computations, we take the predicted backward time $\tau = \big(x_M - x_g(t)\big)/V_{xg}(t)$ as the backward time τ.

3. CONTROL IN THE PRESENCE OF TWO OBSTACLES

In the case when several obstacles are detected sequentially, it is required to construct a control guaranteeing the detour of each of them. Let us consider how this is done in the case of two obstacles with overlapping effective zones.

We assume that the reference straight lines are drawn in the process of motion and linearized systems (2.6) and one-dimensional differential games (2.7) are obtained. Each individual game yields its control vector. It is required to choose from the two controls a control that would guarantee the achievement of the goal in both games. This can be done rather easily, taking into account the one-dimensionality of the differential games and the fact that the main goal in the game is to avoid the intersection of the lower boundary of the main bridge from + to −, while the intersection of the upper boundary is, generally speaking, admissible according to the sense of the problem.

Let us introduce the operation of comparing the bridges obtained in the two games by multiplying the main bridge by a scalar coefficient k. The bridge that is obtained by multiplying by a smaller coefficient will be called the smaller of the two bridges, while the other bridge will be called the larger bridge.

Consider the variants of the location of current positions in the two games (Fig. 4). Let us emphasize that we speak about two positions corresponding to the same state of nonlinear system (1.1) at the moment under consideration.

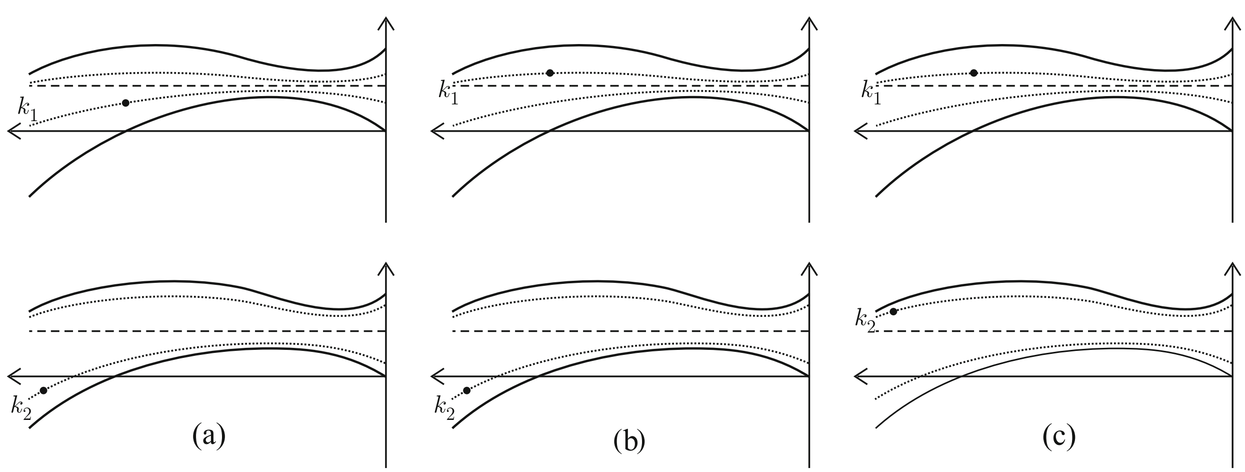

Fig. 4. Different variants of the location of current positions in the two games.

(a) Current positions in both games are below the switching line. Consequently, the vector $B(\tau)u$ on the right-hand side of system (2.7) is directed upward in each game but the modules are different: for the first game, the control is taken from the boundary of the set $k_1\Omega_u$ and, for the second game, the control is taken from the boundary of $k_2\Omega_u$. If we choose the maximum of these two controls, i.e., the control corresponding to the larger bridge, then we preserve the guaranteed result in both games, because we only increase the module of the acting vector in one of the games.

(b) One position is above the switching line and the other position is below the switching line. In this case, we choose the control from the game in which the position is below the switching line. Thus, we preserve the result in this game. However, the possibility appears of intersecting the upper boundary of the bridge in the second game.

(c) Both positions are above the switching line. We will take the control corresponding to the smaller bridge due to the following considerations. Variant (c) can arise from variant (b) when the lower point crosses the switching line. If we take the control corresponding to the larger bridge, this will lead to a sharp change of the control signal. In our case, the sign of the control is changed but its level remains small.

Let us summarize the three considered cases. We always choose the control that directs the motion of the system upward as much as possible. This control corresponds to the larger bridge if the current points are below the switching line and to the smaller bridge if they are above the switching line.

4. SIMULATION RESULTS

In simulating the motion of nonlinear system (1.1), we use the wind disturbance caused by a wind microburst [19]. Let us describe the model of a microburst that we use [20, 21]. A torus is given in the three-dimensional space x_g, y_g, z_g (Fig. 5). Turbulence is formed outside the torus, and the wind velocity decreases proportionally inside the torus as one moves closer to the center of the torus tube. The microburst is characterized by the following parameters: $\mathcal{V}$ is the wind velocity at the central point (this velocity is not maximal; the wind velocity near the torus can be greater up to a factor of 2); h is the altitude of the central point; R is the distance from the central point to the center of the tube; $R_C = 0.8h$ is the radius of the tube; and $\widetilde{x}_0$, $\widetilde{z}_0$ is the projection of the central point onto the ground plane.

When passing a microburst zone, the aircraft first enters a current of head wind, which changes to a descending wind during a short period of time and, after that, to a tail wind. A head wind

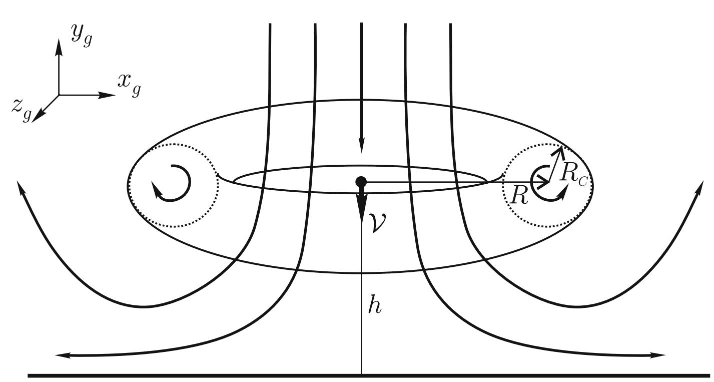

Fig. 5. Wind microburst model.

increases the airspeed and, hence, the lifting force, while a descending wind or tail wind has the opposite effect.

Simulation was carried out for two variants of the microburst. Microburst 1 has the following parameters: $\mathcal{V} = 5\,\mathrm{m/s}$, $h = 600\,\mathrm{m}$, $R = 1200\,\mathrm{m}$, and $R_C = 480\,\mathrm{m}$. The center of the microburst (in projection to the ground plane) is located at the distance of 500 m from the point where the motion starts: $\widetilde{x}_0 = 500\,\mathrm{m}$, $\widetilde{z}_0 = 0$. Microburst 2 is stronger; the wind velocity at the central point is $\mathcal{V} = 8\,\mathrm{m/s}$.

We assume that the obstacle is detected at the distance $D = 1400$ m.

The parameter ε in the games with dynamics (2.7) is chosen so that the length of the terminal segment M is approximately 30 m.

Let us first present simulation results for the first variant of the microburst.

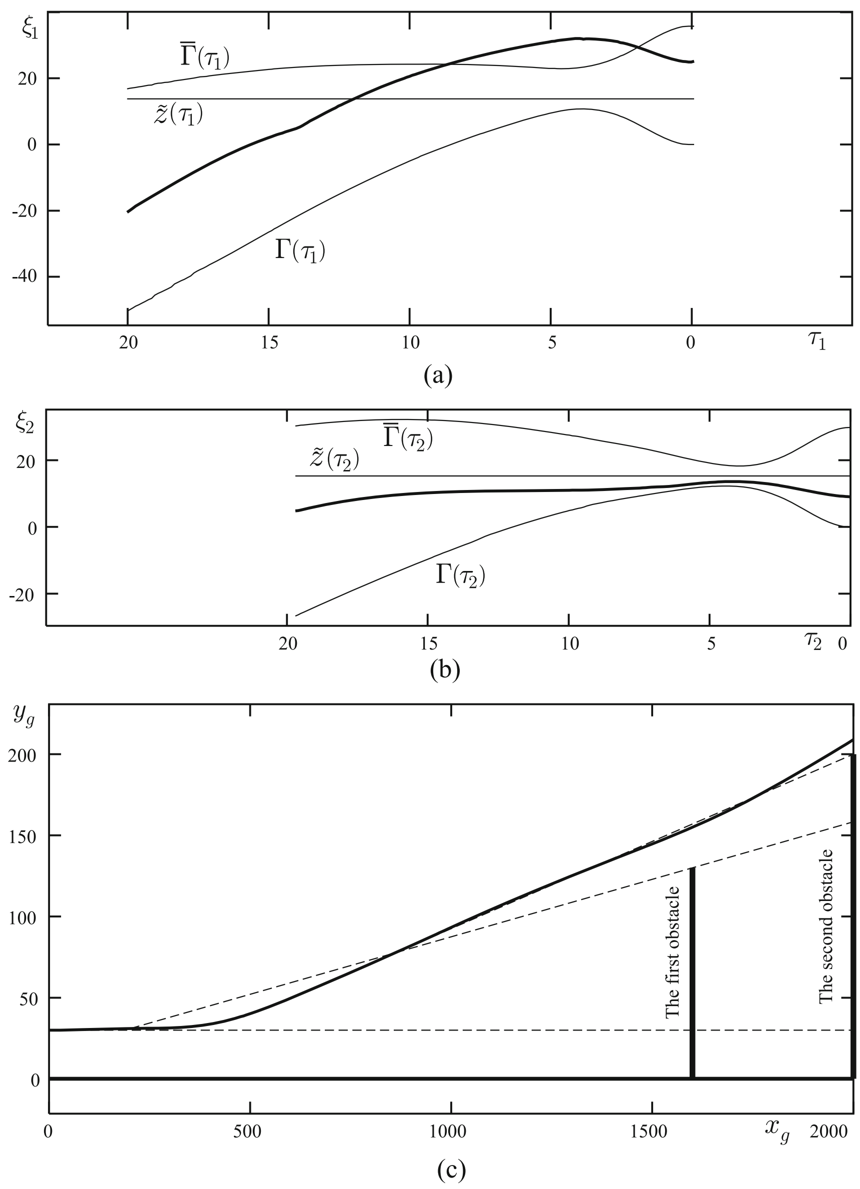

Fig. 6. Microburst 1: (a, b) the graphs of motions in the two differential games; (c) graph of the altitude. Time and distances are measured in seconds and meters, respectively.

Figure 6 shows the graphs of motions in the coordinates of two differential games (2.7) and the graph of altitude change. In the graphs of motions, the horizontal axis is the backward time τ in the corresponding game and the vertical axis is the coordinate ξ (we use the notation τ_1, ξ_1 for the first game and τ_2, ξ_2 for the second game). The graphs are shifted in time so that the starting and terminal moments of the games can be seen. The fine lines show the boundaries of the main bridge and the switching line. The altitude graph is given in the coordinates x_g, y_g. The ground line, obstacles (in the form of vertical segments), and three reference straight lines are shown.

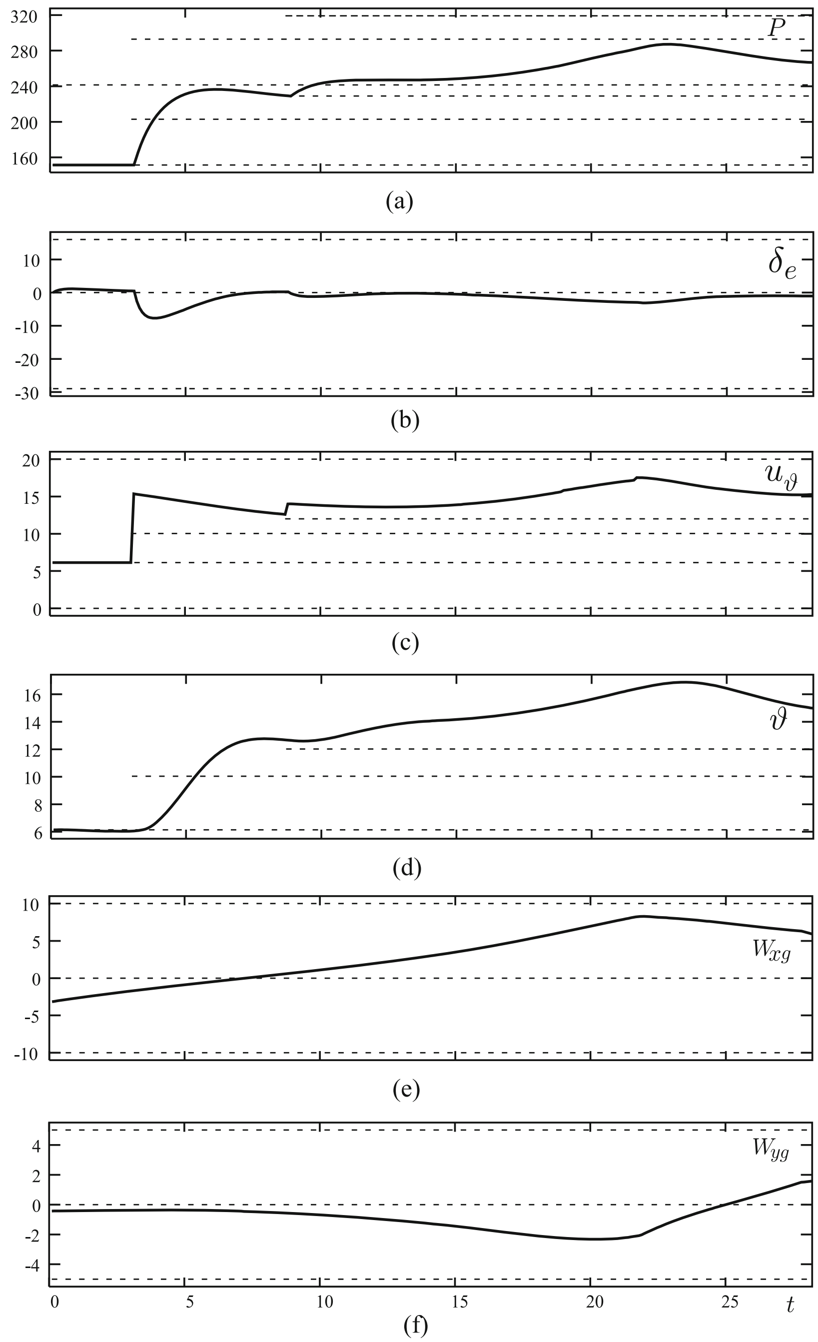

Fig. 7. Microburst 1: the graphs of (a) the thrust force P, $\times 1000$ N; (b) the change of the elevator δ_e, deg; (c) the target pitch angle u_ϑ, deg; (d) the pitch angle ϑ, deg; (e, f) the longitudinal W_{xg} and vertical W_{yg} components of the wind velocity, m/s.

It is seen that the motion in the first game intersects the upper boundary of the main bridge and goes above it on some interval. This implies that the control is taken from the second game and, consequently, the second obstacle plays a key role here.

Figure 7 shows the graphs of the thrust force, elevator, target pitch angle, pitch angle, and realization of the wind disturbance. Time in seconds is laid off along the horizontal axis. In the graph of the thrust force, the fine dashed lines show three nominal values of the thrust force corresponding to the three reference straight lines and three maximal values the thrust force that are at 1.2 m from the nominal values. In the graph of the deviation of the elevator, the dashed lines show the zero value and the maximal admissible deviations of the elevator (they are wide enough and we do not use them in the algorithm of control construction). In the graph of the target pitch angle, the maximal admissible values and three nominal values are shown. In the graph of the pitch angle, three nominal values are shown.

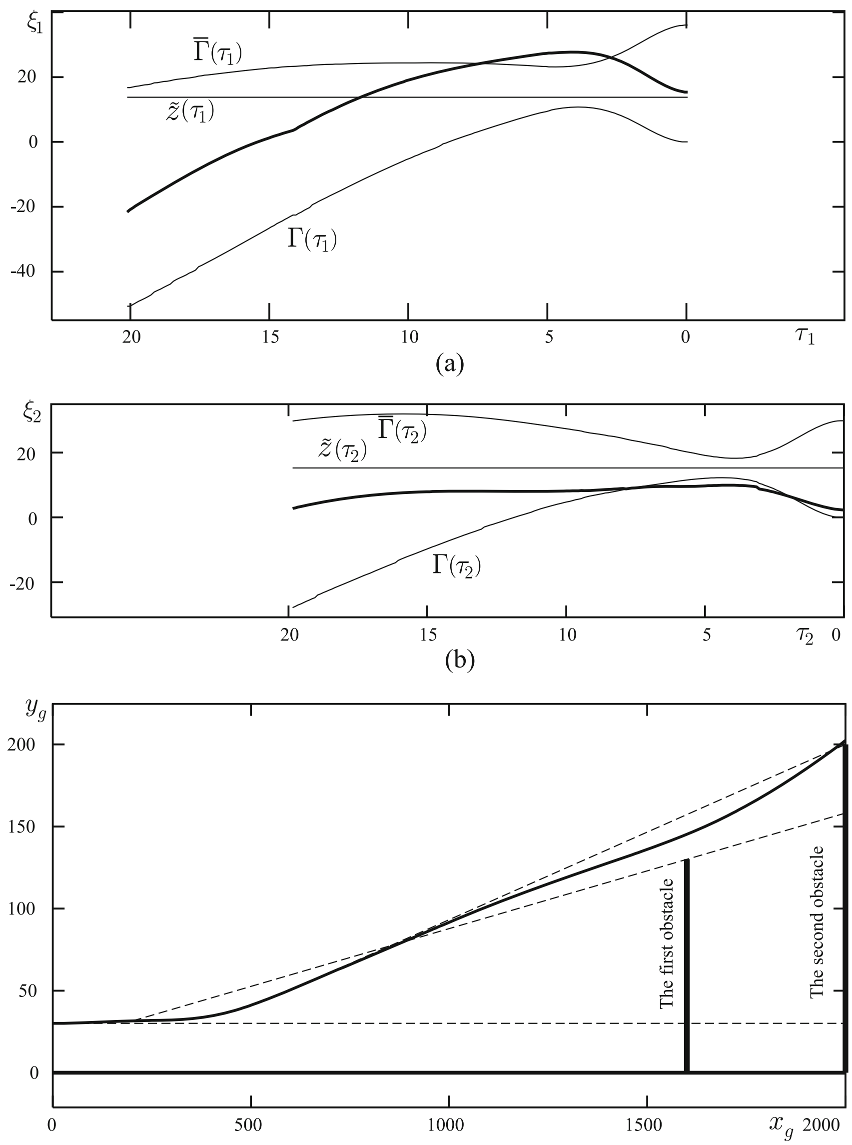

Fig. 8. Microburst 2: (a, b) the graphs of motion in the two differential games, (c) the altitude graph. Time and distances are measured in seconds and meters, respectively.

On the initial segment before the detection of the first obstacle, the value of the thrust force P was constant and equal to the calculated nominal value. The target pitch angle u_ϑ was also constant. Its value coincided with the nominal value of the pitch angle. The influence of the wind disturbance was compensated by the control of the elevator according to formula (1.2).

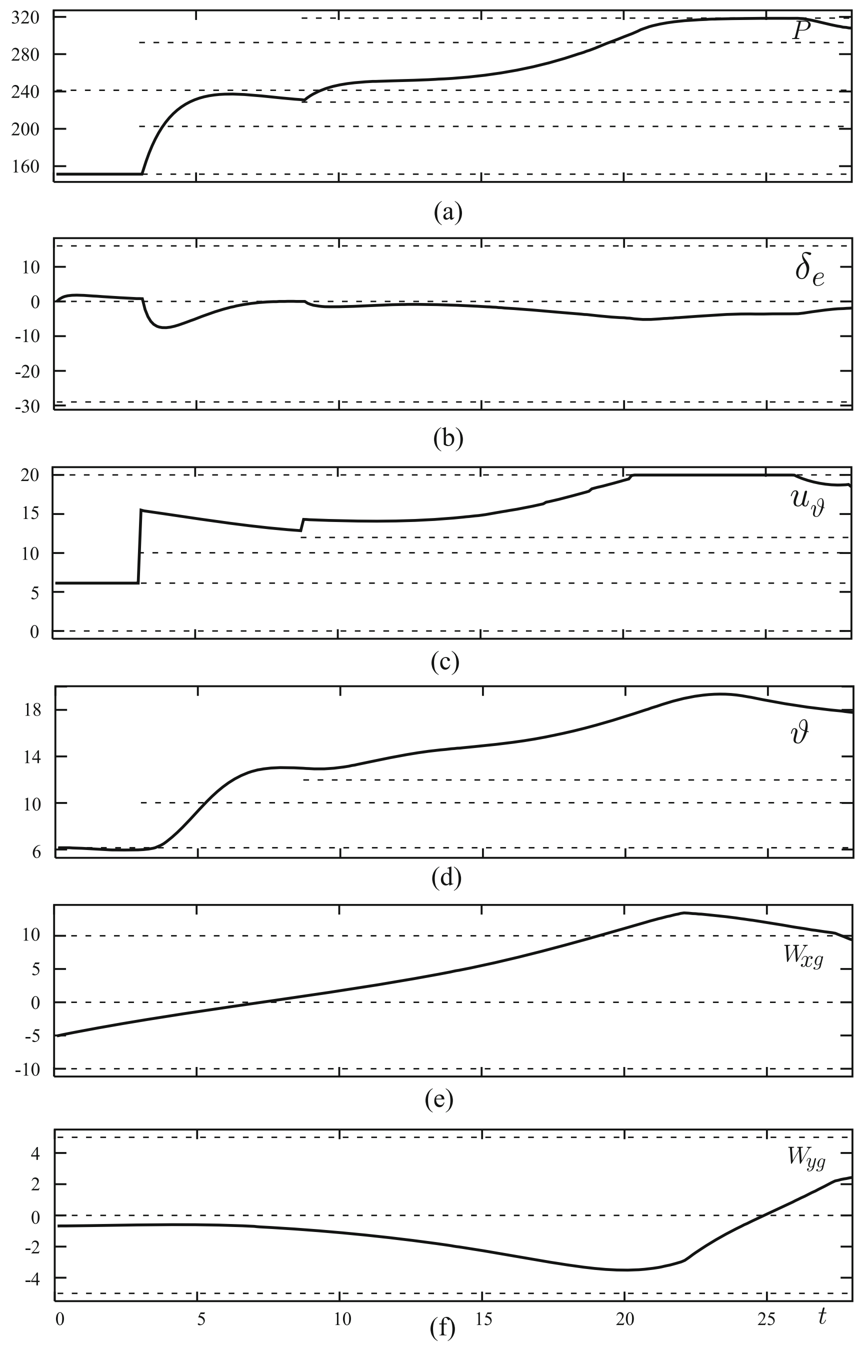

Fig. 9. Microburst 2: the graphs of: (a) the thrust force P, $\times 1000$ N; (b) the change of the elevator δ_e, deg; (c) the target pitch angle u_ϑ, deg; (d) the pitch angle ϑ, deg; (e, f) the longitudinal W_{xg} and vertical W_{yg} components of the wind velocity, m/s.

Since the realizations of the wind velocity lie within the constraints of 10 m/s for W_{xg} and 5 m/s for W_{zg}, the one-dimensional phase variable goes above the lower curve $\Gamma(\tau)$ of the main bridge in each of the auxiliary differential games. The value P of the thrust force and the value u_ϑ of the target pitch angle do not reach their maximal admissible values.

Simulation results for the stronger microburst are shown in Figs. 8 and 9. Here, the maximal wind velocity in the x coordinate gets out of the specified constraints on some segment. On this segment, the current position in the second game transgresses the lower boundary of the main bridge, and the controls of the thrust force and of the target pitch angle become maximal. Later, when the wind velocity goes back within the specified limits, the motion returns inside the bridge and the goal of the game is reached.

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